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STUDY OF FUZZY METRIC SPACES AND FIXED POINT THEOREMS

Anita Singh Banafar

Department of Applied Mathematics Jabalpur Engineering College, Jabalpur
Email: anita.banafar1@gmail.com

Abstract: In this paper the notion of product of probabilistic metric spaces is extended to some family of fuzzy metric spaces. This paper also study the characteristics of product of fuzzy metric spaces in topological expect and a fixed point theorem.

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1. Introduction

The theory of fuzzy sets was introduced by Zadeh in 1965 [18]. Then Deng Zi-Ke and M. A. Erceg have introduced separately the concepts of fuzzy metric in different ways [3, 5]. In particular, Kramosil and Michalek [8] generalized the concept of probabilistic metric space given by Menger [14] to the fuzzy framework. In [7, 8] George and Veeramani modified the concept of fuzzy metric space introduced by Kramosil and Michalek and obtained a Hausdoff and first countable topology on this modified fuzzy metric space. On the other hand, the study on product spaces in the probabilistic framework was initiated by Istratescu and Vaduva [11], and subsequently by Egbert [4], Alsina [1] and Alsina and Schweizer [2]. Recently, Lafuerza –Guillen[13] has studied finite products of probabilistic spaces and proved some interesting results. fixed point the orems given by Gregori and Sapena [10] for complete fuzzy metric space in the sense of George and Veeramani [7]. Grabiec [9] proved a fuzzy Banach contraction theorem whenever fuzzy metric space was considered in the sense of Kramosil and Michalek and was complete in Grabiec's sense. Meanwhile, Gregori and Sapena [10] gave fixed point theorems for complete fuzzy metric space in the sense of George and Veeramani and also for Kramosil and Michalek's fuzzy metric space which are complete in Grabiec's sense. Recently, Zikic [19] proved that the fixed point theorem holds under general conditions: theory of countable extension of a t −norm.

The main purpose of this paper is to introduce the product spaces in the fuzzy framework. Section 3, generalize the concepts of product of probabilistic metric spaces studied by Egbert [4]. Section 4, contains the study on the fixed point theorem on this newly developed fuzzy product spaces by generalizing the fixed point theorem .

2. Preliminaries

Definition 2.1: A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:
(TN-1) $*$ is associative and commutative
(TN-2) $*$is continuous
(TN-3) $p * 1 = p$ for all $p \in [0,1]$
(TN-4) $p * q \leq r * s$ whenever $p \leq r$ and q$\leq s$, for each $p, q, r, s \in [0,1]$.

Two typical examples of continuous t-norm are
$p * q = pq$ and $p * q = \mathrm{Min}(p, q)$.

Definition2.2: [7] A fuzzy Metric Space is an ordered triplex $(X, \mathcal{M}, *)$ such that X is a nonempty set, $*$ is a continuous t-norm and $\mathcal{M}: X \times X \times [0, \infty) \to [0,1]$ is a mapping (called fuzzy metric) which satisfies the following conditions: for every $x, y, z \in X$ and $t, s > 0$

(FM1) $\mathcal{M}(x,y,t) > 0$
(FM2) $\mathcal{M}(x,y,t) = 1 \; if \; and \; only \; if \; x = y$
(FM3) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$
(FM4) $\mathcal{M}(x,z,t+s) \geq \mathcal{M}(x,y,t) * \mathcal{M}(y,z,s)$
(FM5) $\mathcal{M}(x,y,\cdot)$: $[0,\infty) \to [0,1]$ is continuous.

Definition 2.3:[15] Let Δ and $*$ are continuous t-norms. We say that Δ is stronger than $*$, if for each $x_1, x_2, y_1, y_2 \in [0,1]$, $(x_1,y_1)\Delta(x_2,y_2) \geq (x_1 \Delta\, x_2) * (y_1 \Delta y_2)$.

Lemma 2.4: [9] $\mathcal{M}(x,y,\cdot)$ is non decreasing for all $x, y \in X$.

Remark 2.5: [7] In a fuzzy metric space $(X,\mathcal{M},*)$, for any $r \in (0,1)$ there exists an $s \in (0,1)$ such that $s * s \geq r$.

Remark 2.6: [7] Every fuzzy metric $\mathcal{M}$ on X generates a topology $\tau_{\mathcal{M}}$ on X which has a base the family of sets of the form $\{B_x(r,t): x \in X, r \in (0,1), t > 0\}$ where $B_x(r,t) = \{y \in X: \mathcal{M}(x,y,t) > 1-r\}$ is a neighborhood of $x \in X$ for all $r \in (0,1)$ and $t > 0$.

Theorem 2.7: [7] Let $(X,\mathcal{M},*)$ be a fuzzy metric space and $\tau_{\mathcal{M}}$ be the topology induced by the fuzzy metric $\mathcal{M}$. Then for a sequence (x_n)in X, $x_n \to x$ if and only if $\mathcal{M}(x_n,x,t) \to 1$ as $n \to \infty$.

Theorem 2.8: [9] Let $(X,\mathcal{M},*)$ be a complete fuzzy metric space such that $lim_{x\to\infty}\mathcal{M}(x,y,t) = 1$ for all $x, y \in X$. Let $f: X \to X$ be a continuous mapping then f is a unique fixed point.

Definition 2.9: [7] A sequence (x_n) in a fuzzy metric space $(X,\mathcal{M},*)$ is a Cauchy sequence iff for each $\epsilon > 0$ and $t > 0$ there exists $\eta_0 \in \mathbb{N}$ such that $\mathcal{M}(x_n,x_m,t) > 1-\epsilon$ for all $n,m \geq \eta_0$, ie $lim_{n,m\to\infty}\mathcal{M}(x_n,x_m,t) = 1$ for every $t > 0$.

A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space.

3. Product of Fuzzy Metric Space

Definition 3.1: Let $(X,\mathcal{M}_x,*)$ and $(Y,\mathcal{M}_y,*)$ are two fuzzy metric spaces defined with same continuous $t-$norms $*$. Let Δ be a continuous $t-$norms . The $\Delta-$product of $(X,\mathcal{M}_x,*)$ and $(Y,\mathcal{M}_y,*)$ is the product space $(X \times Y, \mathcal{M}_\Delta, *)$ where $X \times Y$ is the cartesian product of the sets X and $\mathcal{M}_\Delta$ is the mapping from $(X \times Y \times (0,1)) \times (X \times Y \times (0,1))$ into $[0,1]$ defined by

$$\mathcal{M}_\Delta(a,b,t+s) = \mathcal{M}_1(x_1,x_2,t)\, \Delta \mathcal{M}_2(y_1,y_2,s) \ldots\ldots\ldots (1)$$

for every $a = (x_1,y_1)$ and $b = (x_2,y_2)$ in $X \times Y$ and $t + s \in (0,1)$.

Theorem 3.2. If $(X,\mathcal{M}_x,*)$ and $(Y,\mathcal{M}_y,*)$ are fuzzy metric spaces under the same continuous t-norm $*$, then their $*$-product $(X \times Y, \mathcal{M}*,*)$ is a fuzzy metric space under $*$.

noted that for a metric space (X,d_x) if $a * b = ab$ (or $a * b = Min(a,b)$)

For all $a,b \in [0,1]$ and $\mathcal{M}_{dx}(x_1,x_2,t) = \frac{kt^n}{kt^n + m dx(x_1,x_2)}$ for each $x_1, x_2 \in X$ and $k,m,n > 0$, then $(X,\mathcal{M}_{dx},*)$ is a fuzzy metric space induced by the metric dx.

Example 3.3: Let (X,d_x) and (Y,d_y) are metric spaces and $(X \times Y, d)$ be their product with $d(p,q) = \mathcal{M}ax\{d_x(x_1,x_2), d_y(y_1,y_2)\}$ for each $p = (x_1,y_1)$ and $q = (x_2,y_2)$ in $X \times Y$.Where $a\Delta b \epsilon [0,1]$ and let

$\mathcal{M}_d(p,q,t) = \frac{t}{t+d(p,q)}$

Then $(x \times Y, \mathcal{M}_d, *)$ is $a\Delta$ −product of (X, d_x) ,(Y, d_y).

Proof: It is sufficient to show that the condition (1) is satisfied

$$\mathcal{M}_d(p,q,t) = \frac{t}{t + d(p,q)}$$

$$= \frac{t}{t + Max\{d_x(x_1,x_2), d_y\ (y_1,y_2)\}}$$

$$= \frac{t}{Max\{t + d_x(x_1,x_2), d_y\ (y_1,y_2)\}}$$

$= Min\left(\frac{t}{t+d_x(x_1,x_2)} + \frac{t}{t+d_y\ (y_1,y_2)}\right) = \left(\frac{t}{t+d_x(x_1,x_2)}\right) \Delta \left(\frac{t}{t+d_y\ (y_1,y_2)}\right)$ where

$$\mathcal{M}_d(p,q,t) = \mathcal{M}_{dx} \Delta \mathcal{M}_{dy}$$

Lemma 3.4: If Δ is stronger than $*$ then $\Delta \geq *$.

Proof: From Definition 2.3, if $x_2, = y_1 = 1; (x_1 \Delta y_2) \geq (x_1 \Delta\ y_2)$ implies $\Delta \geq *$.

Theorem 3.5: If $(X, \mathcal{M}_x, *)$ and $(Y, \mathcal{M}_y, *)$ are fuzzy metric spaces under the same continuous t-norm $*$ and $\Delta \geq *$ then the Δ −product $(X \times Y, \mathcal{M}_\Delta, *)$ is a fuzzy metric space under $*$.

Proof: Here it is sufficient to show that properties FM4 and FM5 are satisfied. Let $p = (a_1, b_1)$,q=(a_2, b_2),$r = (a_1, b_2)$, are in $X \times Y$.

Then

$$\mathcal{M}_\Delta(\mathrm{p,r},2\alpha) = \mathcal{M}_x\{(a_1,a_3,\alpha)\Delta\ \mathcal{M}_y\ (b_1,b_3,\alpha)\}$$

$$= \mathcal{M}_x\left\{\left(a_1,a_2,\frac{\alpha}{2}\right) * \mathcal{M}_x\left(a_2,a_3,\frac{\alpha}{2}\right)\right\} \Delta \left\{\mathcal{M}_y\ \left(b_1,b_2,\frac{\alpha}{2}\right) * \mathcal{M}_y\ \left(b_2,b_3,\frac{\alpha}{2}\right)\right\}$$

$$\geq \mathcal{M}_x\left\{\left(a_1,a_2,\frac{\alpha}{2}\right) \Delta \mathcal{M}_x\left(a_2,a_3,\frac{\alpha}{2}\right)\right\} * \left\{\mathcal{M}_y\ \left(b_1,b_2,\frac{\alpha}{2}\right) \Delta \mathcal{M}_y\ \left(b_2,b_3,\frac{\alpha}{2}\right)\right\}$$

$$= \mathcal{M}_\Delta(\mathrm{p,q},\alpha) * \mathcal{M}_\Delta(\mathrm{q,r},\alpha).$$

And the continuity of the t −norm implies the function $\mathcal{M}_\Delta(\mathrm{p,r},.): (0,\infty) \to [0,1]$ is continuous. Hence proof is completed.

Corollary 3.6: If $(X, \mathcal{M}_x, *_1)$ and $(Y, \mathcal{M}_y, *_2)$ are fuzzy metric spaces and if there exists a continuous t −norm Δ stronger than $*_1$and $*_2$ then their Δ − product is a fuzzy metric space under Δ.

Theorem 3.7: If $(X_1, \mathcal{M}_1, *)$ and $(X_2, \mathcal{M}_2, *)$ are fuzzy metric spaces under the same continuous t −norm $*$. Let U and V denote the neighborhood system in $(X_1 \times X_2, \mathcal{M}_*, *)$, consisting of the Cartesian products $B_{x_1}(r,s) \times B_{x_2}(r,s)$ where $x_1 \in X_1$, $x_2 \in X_2$ and $r \in (0,1)$ and $t > 0$.Then U and V induce the same fuzzy topology on $(X_1 \times X_2, \mathcal{M}_*, *)$.

Proof: Here U and V are bases for their respective topology, since $*$ is continuous.So, it is suffices to prove that for each $V' \in V$ there exist a $U' \in U$ such that $U' \leq V'$ and conversely.

Let $A_1 \times A_2 \in V$. There exist neighborhoods $B_{x_1}(r,s)$ and $B_{x_2}(r,s)$ contained in A_1 and A_2 respectively. Let $r =$

$Min(r_1, r_2)$ and $s = Min(s_1, s_2)$, and let $y = (y_1, y_2) \in B_x(r, s)$. Then

$$\mathcal{M}_1(x_1, y_1, s_1) = \mathcal{M}_1(x_1, y_1, s_1) * 1 \geq \mathcal{M}_1(x_1, y_1, s_1) * \mathcal{M}_1(x_2, y_2, s_2)$$

$\geq \mathcal{M}_1(x_1, y_1, s) * \mathcal{M}_1(x_2, y_2, s) = \mathcal{M}(x, y, s) > 1 - r \geq 1 - r_1$

And similarly $\mathcal{M}_2(x_2, y_2, s_2) \geq 1 - r_2$

Therefore $y_1 \in B_{x_1}(r_1, s_1)$ and $y_2 \in B_{x_2}(r_2, s_2)$ implies that $B_x(r, s) \in A_1 \times A_2$.

Conversely, let $B_x(r, s) \in U$.And since $*$ is continuous, there exixts $\delta \in (0,1)$ such that $(1 - \delta) * (1 - \delta) > 1 - r$. Let $y \in (y_1, y_2) \in B_{x_1}(\delta, s) \times B_{x_2}(\delta, s)$. Then

$$\mathcal{M}_*(x, y, s) = \mathcal{M}_1(x_1, y_1, s) * \mathcal{M}_2(x_2, y_2, s) \geq (1 - \delta) * (1 - \delta) > 1 - r$$

Therefore $y \in B_x(r, s)$ and $B_{x_1}(r_1, s_1) * B_{x_2}(r_2, s_2) \subseteq B_x(r, s)$.

4. Fixed Point Theorems

Definition 4.1. Let $(X, \mathcal{M}, *)$ be a fuzzy metric space. A mapping $f: X \to X$ is said to be fuzzy contraction if there exists $k \in (0,1)$ such that $\mathcal{M}(f_x, f_y, s) \geq \mathcal{M}(x, y, s/k)$ for all $x, y \in X$.

Theorem 4.2: Let $(X, \mathcal{M}, *)$ be a fuzzy metric space with $a * b = Min(a, b)$.Let $f_i: X \to X$ be a function with atleast one fixed point x_i ; $i = 1,2,3, \ldots$. and $f_0: X \to X$ be a fuzzy contraction mapping with fixed point x_0. If the sequence (f_i) converges uniformly to f_0, then the sequence (x_i) converges to x_0.

Proof: Let $k \in (0,1)$ and choose a positive number $N \in \mathbb{N}$ such that $i \geq N$implies

$$\mathcal{M}(f_i x, f_0 x, (1 - k)t) > 1 - r$$

where $r \in (0,1)$ and $x \in X$. Then, if $i \geq N$, hence

$$\mathcal{M}(x_i, x_0, t) = \mathcal{M}(f_i x_i, f_0 x_0, t)$$
$$\geq \mathcal{M}(f_i x_i, f_0 x_i, (1 - k)t) * \mathcal{M}(f_0 x_i, f_0 x_0, kt)$$
$$> Min(1 - r, \mathcal{M}(x_i, x_0, t)).$$

Hence, $\mathcal{M}(x_i, x_0, t) \to 1$ as $i \to \infty$. This proves that (x_i) converges to x_0.

In what follows $p_1: X \times Y \to X$ and $p_2: X \times Y \to Y$ will denote the first projection mapping and second projection mapping defined by $p_1(x, y) = x$, while $p_2(x, y) = y$ respectively.

Definition 4.3. Let $(X, \mathcal{M}, *)$ be a fuzzy metric space and Y be any space. A mapping $f: X \times Y \to X \times Y$ is said to be locally fuzzy contraction in the first variable if and only if for each $y \in Y$ there exists an open ball $B_y(\epsilon), \epsilon \in (0,1)$ containing y and a real number $k(y) \in (0,1)$ such that

$\mathcal{M}(p_1 \circ f(x_1, y), p_1 \circ f(x_2, y), t) \geq \mathcal{M}(x_1, x_2, t/k(y))$ for all $x_1, x_2 \in X$.It is obvious that every fuzzy contraction mapping is locally fuzzy contraction in the first variable.

Remark 4.4: The fuzzy metric space $(X, \mathcal{M}, *)$ has fixed point property (f.p.p) if every continuous mapping $f: X \to X$ has fixed point.

Theorem4.5: Let $(X, \mathcal{M}_x, *)$ be a complete fuzzymetric space and $(Y, \mathcal{M}_y, *)$ be fuzzy metric space with the f.p.p. and let $f: X \times Y \to X \times Y$ be uniformly continuous and a fuzzy contractionmapping in the first variable. Then f has a fixed point.

Proof: For $\in Y$, let $f_y : X \to X$ be defined by $f_y(x) = p_1 \circ f\ (x, y)$ for all $x \in X$. Since, for every $\in Y$, f_y is a fuzzy contraction mapping ,therefore f_y has a unique fixed point (Theorem 2.8). Let $G : Y \to X$ be given by $G(y) = f_y(G(y))$ is the unique fixed point of f_y. Now, let $y_0 \in Y$ and let (y_n) be a sequence of points of Y which converges to y_0. Since f is uniformly continuous, the and let (y_n) be a sequence of points of Y which converges to y_0. Since f is uniformly continuous, the sequence (f_{yn}) converges uniformly to f_{y0} . Hence, by Theorem 2.8, the sequence $(G(y_n))$ converges to $G(y_0)$. This shows that the function G is continuous on Y.

Now, let $g : Y \to Y$ be a continuous function defined via $g(y) = p_2 \circ f\ (G(y), y)$ for each $\in Y$. Since, $(Y, \mathcal{M}_y, *)$ has f.p.p., there is a point z $\in Y$ such that $g(z) = z$, *i.e.*, $z = g(z) = p_2 \circ f\ (G(z), z)$. It follows that $(G(z), z)$ is a fixed point of f.

Theorem 4.6: Let $(X, \mathcal{M}, *)$ be a fuzzy metric space with $a * a \geq a$ for every $a \in [0,1]$ and Y be a fuzzy topological space with f.p.p. Let $\boldsymbol{f} : X \times Y \to X \times Y$ be locally fuzzy contraction in the first variable. Let $x_0 \in X$ and $\in Y$. Define the sequence $(s_n(y))$ in X as follows:

$s_0(y) = x_0$ and $s_n = s_n(y) = p_1 \circ f(s_{n-1(y)}, y)$

Then,

(i) $(s_n$ (y)) is a Cauchy sequence in X.

(ii) If $s_n \to s_y$ then $p_1 \circ f(s_y, y) = s_y$

Proof: (i) Since f is a locally fuzzy contraction mapping in the first variable, there exists a real number $k \in (0,1)$ such that

$$\mathcal{M}(s_n, s_{n+1}, t) = \mathcal{M}(p_1 \circ f(s_{n-1}, y), p_1 \circ f(s_n, y), t)$$

$\geq \mathcal{M}(s_{n1}, s_n, t/k)$; $n \geq 1$

by simple induction $\mathcal{M}(s_n, s_{n+1}, t) \geq \mathcal{M}(s_0, s_1, t/k^n)$

for all n and $t > 0$. We note that, for every positive integer m, n with $m > n$ and $k \in (0,1)$, hence

$$(1-k)(1+k+k^2+k^3+\cdots+k^{m-n-1}) = 1-k^{m-n} < 1$$

$$\Longrightarrow t > (1-k)(1+k+k^2+k^3+\cdots+k^{m-n-1})t$$

Since $\mathcal{M}$ is non decreasing, hence

$$\mathcal{M}(s_n, s_m, t) \geq \mathcal{M}(s_n, s_m, (1-k)(1+k+k^2+k^3+\cdots+k^{m-n-1})t).$$

Thus, FM4, we have $m > n$,

$$\mathcal{M}(s_n, s_m, (1-k)(1+k+k^2+k^3+\cdots+k^{m-n-1})t)$$

$$\geq \mathcal{M}(s_n, s_{n+1}, (1-k)t) * \ldots * \mathcal{M}(s_{m-1}, s_m, (1-k)^{m-n-1}t)$$
$$\geq \mathcal{M}(s_0, s_1, (1-k)t/k^n) * \ldots * \mathcal{M}(s_0, s_1, (1-k)^{m-n-1}t/k^{m-1})$$
$$= \mathcal{M}(s_0, s_1, (1-k)t/k^n) * \ldots * \mathcal{M}(s_0, s_1, (1-k)t/k^n)$$

Since $a * a \geq a$ which implies

$$\mathcal{M}(s_n, s_m, t) \geq \mathcal{M}(s_0, s_1, (1-k)t/k^n)$$

Let $n \to \infty$ and $m > n$, hence we get

$$\lim_{n,m\to\infty} \mathcal{M}(s_n, s_m, t) = \lim_{n\to\infty} \mathcal{M}(s_0, s_1, (1-k)t/k^n) = 1$$

Which implies $(s_n(y))$ is a Cauchy sequence in X.

(ii) Let $u = p_1 \circ f(s_n, y)$. By contradiction, suppose that $u \neq s_n$. Then $\mathcal{M}(u, s_n, t) = \epsilon < 1$ for every $t > 0$. Since $f: X \times Y \to X \times Y$ is continuous, there exists an open set $U \times V$ in $X \times Y$ and a real number $\lambda \in (0, \epsilon)$ such that $(s_n, y) \in U \times V$, $U \subseteq B_{s_n}(\lambda, t)$ and $f(U \times V) \subseteq B_u(\lambda, t) \times Y$. Since $s_n \to s_y$, there is a positive number $N \in \mathbb{N}$ such that $s_n \in U$ for all $n \geq N$. But $p_1 \circ f(s_n, y) = s_n \in U$. Therefore $f(s_n, y) \notin B_u(\lambda, t) \times Y$ which contradicts the fact that $f(U \times V) \subseteq B_u(\lambda, t) \times Y$. Therefore our assumption is incorrect.

Theorem 4.7: Let $(X, \mathcal{M}, *)$ be a fuzzy metric space with $a * a \geq a$ for every $a \in [0, 1]$ and Y be a fuzzy topological space with f.p.p. Let $f: X \times Y \to X \times Y$ be locally fuzzy contraction in the first variable, then f is a fixed point.

Proof: Let $x_0 \in X$ and $\in Y$. Define the sequence $(s_n(y))$ as follows:
$s_0(y) = x_0$ and $s_n(y) = p_1 \circ f(s_{n-1}(y), y)$

By Theorem 4.6(i), the sequence $(s_n(y))$ is a Cauchy sequence in X. Since $(X, \mathcal{M}, *)$ is complete, there exists a point $s_y \in X$ such that

$\lim_{n \to \infty} s_y(y) = s_y$.

Now consider a continuous mapping $g: Y \to Y$ defined by

$g(y) = p_2 \circ f(s_y, y)$.

Since Y has the f.p.p., there exists a point $y_0 \in Y$ such that $(y_0) = y_0$. By Theorem 4.6(ii), we have $p_1 \circ f(s_{y_0}, y_0) = s_{y_0}$. But $y_0 = g(y_0) = p_2 \circ f(s_{y_0}, y_0)$. Hence, $f(s_{y_0}, y_0) = (s_{y_0}, y_0)$ which completes the proof.

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