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A VARIATIONAL LOCALIZED-SOURCE EFFECTIVE DISTURBANCE FIELD MODEL FOR SINGLE-PARTICLE DOUBLE-SLIT INTERFERENCE

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ABSTRACT

The double-slit experiment remains a central demonstration of quantum interference, including in experiments in which particles are detected individually. Standard quantum mechanics describes the observed distribution through a quantum amplitude and the Born probability rule. The present work examines a more restricted phenomenological question: whether qualitative single-particle interference can be represented by a localized particle associated with an extended disturbance field.

A Variational Localized-Source Effective Disturbance Field (EDF) is introduced as a mathematical construct distinct from the quantum-mechanical wavefunction, the electromagnetic field, and any material medium. The field dynamics are obtained from a local quadratic Lagrangian by Hamilton's variational principle. The resulting equation is an inhomogeneous wave equation containing a localized source. In the source-free region it reduces to a linear homogeneous wave equation, so superposition follows directly. A retarded Green-function representation is given for the source-driven field. For a two-slit geometry, equal-amplitude monochromatic solutions give an interference modulation proportional to $\cos^2(\Delta\phi/2)$. The relation between this disturbance intensity and particle detection probability is explicitly identified as a phenomenological postulate rather than a consequence of the variational equation.

The model is intentionally modest in scope. It does not replace quantum mechanics, derive the Born rule, or specify the microscopic origin of the disturbance field. Its purpose is to formulate and analyse a localized-source variational framework that preserves particle localization while reproducing the qualitative spatial structure of single-particle double-slit interference.

I. INTRODUCTION

The double-slit experiment is one of the clearest demonstrations of interference in physics. Young's early optical experiments established the interference of light [1], while later experiments demonstrated interference with individual electrons and with increasingly complex microscopic objects [7, 8]. These experiments make a useful distinction between a localized detection event and the spatial distribution accumulated after many events.

In standard quantum mechanics, interference is described by a complex probability amplitude whose modulus squared gives the corresponding probability density. The development of wave mechanics by Schrödinger [2] and the statistical interpretation associated with Born [3] provide the standard mathematical framework. Theoretical and computational modelling provides an important framework for analysing complex physical phenomena, where analytical formulations can provide physical insight while numerical approaches enable the investigation of systems with more complicated field configurations and interactions [4]. Its experimental success is not in question here. The issue considered in this paper is more limited: whether the same qualitative spatial interference structure can be represented by an alternative phenomenological field construction while retaining a localized particle.

The distinction is relevant to several historical and modern approaches. Bohmian mechanics retains particle trajectories while using a guiding wavefunction and a quantum potential [6]. De Broglie's early wave-mechanical programme associated localized corpuscular behaviour with an extended wave description [5]. Hydrodynamic pilot-wave experiments later showed that a localized droplet can interact with a self-generated surface wave and display diffraction- and interference-like statistics [9, 10]. These systems are physically different from quantum

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particles, but they demonstrate that a localized object and an extended field can coexist in a mathematically and experimentally meaningful model.

The present paper does not attempt to select between interpretations of quantum mechanics. Instead, it investigates a narrower mathematical question: can an extended disturbance field, generated by a localized source and governed by a variational wave equation, reproduce the qualitative interference structure of a two-slit experiment? Recent computational approaches increasingly employ structured data-processing and modelling frameworks to extract meaningful patterns from complex datasets and support quantitative analysis [11]. In this context, computational modelling provides a framework for quantitatively examining the spatial structure generated by the proposed disturbance field and comparing its resulting interference pattern with the qualitative features of the double-slit configuration.

A. Research Gap

The literature contains several descriptions in which a localized object is associated with an extended wave or guiding field. However, the particular construction studied here is deliberately restricted to a local quadratic action containing an explicit source term for a localized particle. The field equation is derived from Hamilton's principle rather than postulated directly, and the source-free limit is used to separate field generation from subsequent propagation.

The aim is therefore not to claim that particle-associated waves or particle-field descriptions are unprecedented. Rather, the contribution is to formulate and analyse this specific localized-source variational construction and to determine what follows from it without importing the quantum wavefunction into the derivation. The resulting model is assessed only at the level of qualitative interference and is not presented as a complete alternative to quantum theory.

II. VARIATIONAL LOCALIZED-SOURCE EFFECTIVE DISTURBANCE FIELD

The Effective Disturbance Field is introduced as a phenomenological scalar field

$$\Phi_D(r,t), \tag{1}$$

associated with a localized moving particle. It is not identified with the quantum-mechanical wavefunction, the electromagnetic field, a gravitational field, or a material medium. It is an effective mathematical degree of freedom used to examine the propagation of a particle-associated disturbance.

Three assumptions define the model.

First, the particle is treated as localized throughout its propagation. Second, the particle acts as a localized source of the disturbance field. Third, away from the source the disturbance obeys a linear field equation. The last assumption is important because it permits superposition when the disturbances associated with different paths overlap. Thus, in a source-free region,

$$\Phi_D(\mathbf{r}, t) = \sum_i \Phi_i(\mathbf{r}, t), \tag{2}$$

provided each Φ_i satisfies the same homogeneous linear equation.

These assumptions are phenomenological. In particular, the model does not derive the existence of the disturbance field from a more fundamental theory.

III. VARIATIONAL FORMULATION

A. Action and Lagrangian density

The field dynamics are obtained from Hamilton's principle,

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$$\delta S = 0, \quad (3)$$

$$\text{With} \quad S = \int L d^3r \, dt. \quad (4)$$

We choose the local quadratic Lagrangian density

$$L = \frac{\alpha}{2v_D^2} \left(\frac{\partial \Phi_D}{\partial t} \right)^2 - \frac{\alpha}{2} (\nabla \Phi_D)^2 + \alpha J(\mathbf{r}, t) \Phi_D, \quad (5)$$

where v_D is the characteristic propagation speed of the disturbance, $\alpha > 0$ is a phenomenological constant, and $J(\mathbf{r}, t)$ is the source function.

The first two terms are the standard quadratic kinetic and gradient terms of a scalar wave field. The last term couples the field linearly to the prescribed source. No microscopic interpretation of α or J is assumed in this work.

B.Euler–Lagrange equation

For a scalar field, the Euler–Lagrange equation is

$$\frac{\partial L}{\partial \Phi_D} - \frac{\partial}{\partial t} \left[\frac{\partial L}{\partial (\partial_t \Phi_D)} \right] - \nabla \cdot \left[\frac{\partial L}{\partial (\nabla \Phi_D)} \right] = 0. \quad (6)$$

The required derivatives are

$$\frac{\partial L}{\partial \Phi_D} = \alpha J, \quad (7)$$

$$\frac{\partial L}{\partial (\partial_t \Phi_D)} = \frac{\alpha}{v_D^2} \frac{\partial \Phi_D}{\partial t}, \quad (8)$$

$$\frac{\partial L}{\partial (\nabla \Phi_D)} = -\alpha \nabla \Phi_D \quad (9)$$

Substitution into Eq. (6) gives

$$\boxed{\nabla^2 \Phi_D - \frac{1}{v_D^2} \frac{\partial^2 \Phi_D}{\partial t^2} = -J(\mathbf{r}, t)} \quad (10)$$

as the governing inhomogeneous disturbance equation.

The important point is that Eq. (10) is obtained from the assumed local action. The field equation itself is therefore not introduced as an independent postulate once the Lagrangian has been chosen.

C.Source-free propagation

In any region in which the localized source is absent,

$$J(\mathbf{r}, t) = 0, \quad (11)$$

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and Eq. (10) becomes

$$\nabla^2 \Phi_D - \frac{1}{v_D^2} \frac{\partial^2 \Phi_D}{\partial t^2} = 0 \quad (12)$$

Because Eq. (12) is linear, the sum of any two solutions is also a solution. This is the mathematical origin of the superposition used later in the double-slit analysis. It is important, however, that linear superposition of the field does not by itself establish a quantum interpretation; it is simply a property of the chosen field equation.

IV. LOCALIZED SOURCE AND RETARDED PROPAGATION

To represent a localized particle following a trajectory $r_p(t)$, the source may be idealized as

$$J(r, t) = J_0 \delta^{(3)}[r - r_p(t)], \quad (13)$$

where J_0 is a phenomenological source strength. Equation (13) is a mathematical idealization of a localized source; the present work does not specify a microscopic theory that determines

J_0 .

For the wave operator in Eq. (10), the retarded Green function satisfies

$$\left(\nabla^2 - \frac{1}{v_D^2} \frac{\partial^2}{\partial t^2} \right) G(\mathbf{r}, t; \mathbf{r}', t') = -\delta^{(3)}(\mathbf{r} - \mathbf{r}') \delta(t - t') \quad (14)$$

The retarded solution can consequently be written formally as

$$\Phi_D(r, t) = \int d^3r' dt' G_{\text{ret}}(r, t; r', t') J(r', t'). \quad (15)$$

For the three-dimensional wave operator,

$$G_{\text{ret}} = \frac{\delta(t - t' - R/v_D)}{4\pi R}, \quad R = |\mathbf{r} - \mathbf{r}'| \quad (16)$$

For the point source in Eq. (13), the field is determined by the source position at retarded time. Writing

$$R_r = |\mathbf{r} - \mathbf{r}_p(t_r)|, \quad t_r = t - \frac{R_r}{v_D},$$

the moving-source solution takes the familiar retarded-source form

$$\Phi_D(r, t) = \frac{J_0}{4\pi R_r |1 - \mathbf{R}_r \cdot \mathbf{v}_p(t_r)/v_D|}, \quad (17)$$

for a source velocity v_p below the disturbance propagation speed. For a stationary or sufficiently slow source, the final factor approaches unity and the familiar $1/R_r$ dependence is recovered.

Equation (17) is not required for the double-slit calculation below; its purpose is to show that the localized-source equation admits a causal retarded solution. The detailed dynamics of the source itself remain outside the scope of the present phenomenological model.

V. APPLICATION TO THE DOUBLE-SLIT CONFIGURATION

Consider two narrow slits, S_1 and S_2 , followed by a detection region. The particle remains localized according to the model, while the associated disturbance field can propagate through the available paths.

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In the source-free propagation region after the slits, Eq. (12) applies. Let the contributions associated with the two paths at an observation point be Φ_1 and Φ_2 . Linearity gives

$$\Phi_D(r,t) = \Phi_1(r,t) + \Phi_2(r,t). \quad (18)$$

For a monochromatic idealization with equal amplitudes,

$$\Phi_1 = A \cos(kr_1 - \omega t), \quad (19)$$

$$\Phi_2 = A \cos(kr_2 - \omega t), \quad (20)$$

Where

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f. \quad (21)$$

Using

$$\cos x + \cos y = 2 \cos \left(\frac{x-y}{2} \right) \cos \left(\frac{x+y}{2} \right),$$

we obtain

$$\Phi_D = 2A \cos \left(\frac{\Delta\phi}{2} \right) \cos \bar{\phi}, \quad (22)$$

where

$$\Delta\phi = k(r_2 - r_1), \quad (23)$$

$$\bar{\phi} = \frac{k(r_1 + r_2)}{2} - \omega t. \quad (24)$$

The time-averaged quadratic disturbance measure is defined phenomenologically as

$$I_D = \langle \Phi_D^2 \rangle_t. \quad (25)$$

For harmonic oscillations, and therefore

$$\cos^2 \bar{\phi} = \frac{1}{2}, \quad (26)$$

$$I_D = 2 A^2 \cos^2 \frac{\Delta\phi}{2}$$

Constructive interference occurs when

$$\Delta\phi = 2m\pi, \quad m \in \mathbb{Z}, \quad (27)$$

whereas destructive interference occurs when

$$\Delta\phi = (2m + 1)\pi. \quad (28)$$

For a far-field screen at distance L from the slits, with slit separation d and transverse

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coordinate y , the path difference is approximately

$$r_2 - r_1 \simeq d \sin \theta \simeq \frac{dy}{L}, \quad (29)$$

so that

$$I_D(y) = 2A^2 \cos^2 \left(\frac{\pi dy}{\lambda L} \right). \quad (30)$$

Equation (30) gives the familiar qualitative fringe modulation. The derivation itself is mathematically the standard two-component interference calculation; the EDF-specific element is the interpretation of the superposed quantities as disturbances associated with a localized particle.

A. Phenomenological detection rule

To connect the field calculation with a distribution of localized detection events, the present model assumes

$$P(r) = C I_D(r), \quad (31)$$

where C is chosen so that

$$\int_A P(r) dA = 1 \quad (32)$$

over the detector area A .

This step is deliberately identified as a phenomenological postulate. It is not derived from the action in Eq. (4), and therefore the present paper does not claim to derive the Born rule. With Eq. (31), the predicted spatial distribution has the same qualitative phase modulation as Eq. (26).

VI. DISCUSSION

A. Source generation and propagation

The mathematical structure of the model separates two regimes. In the neighbourhood of the localized particle, the field obeys the inhomogeneous equation $\mathcal{L}\Phi_D = -J$,

$$\mathcal{L} = \nabla^2 - \frac{1}{v_D^2} \partial_t^2. \quad (33)$$

Away from the source, the same operator gives

$$\mathcal{L}\Phi_D = 0 \quad (34)$$

Thus the model distinguishes source generation from subsequent source-free propagation. Interference is a property of the latter region, where linear superposition is available. This source–propagation distinction is a mathematical consequence of the inhomogeneous field equation rather than an additional physical law.

B. Relation to Established Descriptions

The EDF should not be regarded as a replacement for quantum mechanics. Standard quantum mechanics uses a quantum amplitude and the Born rule, while the present construction uses a real phenomenological scalar field and postulates a proportionality between its quadratic measure and detection probability.

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Table I. Qualitative comparison of the mathematical structures discussed in this paper. The entries describe the present formulation and are not intended as a ranking of physical interpretations.

Feature	Standard QM	Bohmian mechanics	de Broglie programme	Prsesnt EDF
Localised particle	Interpretation dependent	Yes	Yes	Yes
Wavefunction fundamental	Yes	Yes	Not necessarily	No
Explicit local source term	No	No	Not explicit here	Yes
Variational field equation	Standard action formulation exist	Not the defining feature	Not explicit here	Yes
Interference mechanism	Quantum amplitude	Guiding wave	Wave structure	Disturbance superposition
Detection rule	Born rule	Born rule	Not derived here	Phenomenological

Bohmian mechanics also retains localized particle trajectories, but the guidance of the particle is formulated through the quantum wavefunction and associated dynamical structure [6]. The EDF formulation instead introduces a source-driven classical-type scalar field equation. No quantum potential is introduced here.

The relation to de Broglie's ideas is historical and conceptual rather than an assertion of equivalence. De Broglie's work associated particle and wave descriptions, while the present paper specifies a local action and source term for an effective disturbance field. The present construction therefore should be viewed as a restricted phenomenological formulation rather than a completion of the double-solution programme [5].

Hydrodynamic pilot-wave systems provide another useful comparison. In the walkingdroplet experiments, a localized droplet interacts with a surface wave that it helps generate, and diffraction- and interference-like statistics can result [9, 10]. The physical mechanism in those experiments is a vibrating fluid, whereas the EDF contains no material medium. The comparison is therefore structural rather than dynamical.

C. What is and is not established by the calculation

The derivation establishes that the chosen quadratic action produces the inhomogeneous wave equation and that its source-free limit permits linear superposition. The two-slit calculation then shows that equal-amplitude monochromatic solutions produce the usual $\cos^2(\Delta\phi/2)$ modulation.

This result should be interpreted carefully. The same trigonometric interference expression is obtained in standard wave theories. Therefore, reproducing the fringe modulation alone does not establish that the EDF is a new physical theory, nor does it distinguish the EDF experimentally from quantum mechanics.

The specific physical origin of the source strength J_0 also remains unspecified. The model assumes a localized source because that is the construction being investigated; it does not yet provide a microscopic particle-field coupling from which J_0 can be calculated.

D. Limitations

Several Limitations are Explicit.

First, the microscopic origin of the EDF is unknown. Second, the relation between disturbance intensity and detection probability is an additional phenomenological assumption. Third, the present analysis is restricted to linear propagation and a stationary two-slit geometry. The paper does not address entanglement, spin, relativistic quantum mechanics, quantum field theory, or many-particle dynamics.

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These limitations are not hidden assumptions to be resolved by interpreting the present result more strongly than the mathematics permits. They define the present scope of the model.

E. Possible future tests

A useful future development would be to construct a microscopic source theory that determines $J(r,t)$ from measurable particle properties rather than leaving its normalization phenomenological. A second direction would be to search for a consequence of the source dynamics that differs from standard quantum predictions. Such a result would be necessary before the EDF could be regarded as more than an alternative phenomenological representation.

VII. CONCLUSIONS

A Variational Localized-Source Effective Disturbance Field model has been formulated for qualitative single-particle double-slit interference. The disturbance field is introduced as a phenomenological scalar field associated with a localized particle. Starting from a quadratic local Lagrangian, Hamilton's principle yields an inhomogeneous wave equation with an explicit localized source.

The source-free limit is a homogeneous linear wave equation, which provides the mathematical basis for superposition. A retarded Green-function representation demonstrates how the localized source can generate a propagating disturbance. In the two-slit geometry, equal-amplitude monochromatic disturbances lead to

$$I_D = 2A^2 \cos^2 \left(\frac{\Delta\phi}{2} \right), \quad (35)$$

and a phenomenological detection rule gives the corresponding qualitative fringe distribution.

The result is intentionally modest. The calculation reproduces a familiar interference modulation and does not by itself distinguish the EDF from standard wave-based quantum descriptions. In particular, the microscopic origin of the disturbance field and the source term remain open questions, and the Born probability rule is assumed rather than derived.

The principal result of the paper is therefore the formulation and analysis of a localized source variational framework, together with a clear separation between source generation, source-free propagation, and phenomenological detection. A physically motivated source mechanism and an experimentally distinguishable prediction are required for any stronger physical claim.

VIII. REFERENCES

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