ISSN: 2752-3829 Vol. 3 No.2, (December, 2023)

Stochastic Modelling and Computational Sciences

GRAPH COLORING AND LABELING IN BHARATANATYAM PARAVAL ADAVU

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ABSTRACT
Graph theory has widespread applications across various fields such as science, medicine, and technology. This paper explores its innovative application within the art form of Bharatanatyam, one of the most prominent classical dances of India. Specifically, the study develops a control diagram for the Paraval Adavu in Bharatanatyam, which is color-coded and labeled for clarity. The analysis reveals that the chromatic number of the control diagram for each Paraval Adavu is consistently two. Additionally, it is observed that for all vertices in the control diagram, both the indegree and outdegree are equal to one. This research highlights the intersection of mathematics and dance, offering a novel perspective on the structural elements of Bharatanatyam through the lens of graph theory.

1. INTRODUCTION
Graph theory is a branch of mathematics that deals with networks of points connected by edges. It originated in 1735 with the famous Königsberg Bridge problem, which led to the foundation of the field. The problem, solved by Swiss mathematician Leonhard Euler, involved determining whether it was possible to traverse all seven bridges spanning a river and its island without crossing any bridge more than once. Additionally, graph coloring is a specific form of graph labeling that assigns colors to various components of a graph. The first significant result in graph coloring focused on planar graphs through map coloring. Francis Guthrie formulated the four-color conjecture while attempting to color a map of England's counties, observing that four colors were enough to ensure no two adjacent counties shared the same color. In 1879, Alfred Kempe published a paper claiming to prove the conjecture. Building on Kempe's work, Percy Heawood demonstrated in 1890 that any planar map could be colored using no more than five colors. The four-color problem was ultimately proven in 1976 by Kenneth Appel and Wolfgang Haken. Graph labeling is a technique of assigning integers to vertices, edges, or both, based on specific conditions. The concept of graph labeling emerged in the mid-1960s. Over the past 50 years, nearly 200 different graph labeling methods have been explored in more than 2,000 research papers.[1, 3, 4, 6-9]

Bharatanatyam is one of the oldest and most renowned dance forms in India, originating from the southern state of Tamil Nadu.[5] The name "Bharatanatyam" can be broken down as follows: *Bha* for *Bhavam* (expression), *Ra* for *Ragam* (melody), *Ta* for *Talam* (rhythm), and *Natyam* meaning dance. This classical dance form embodies the essence of the four Vedas and integrates words, gestures, music, and emotions. Bharatanatyam is a composite art form that serves as a confluence of various Indian artistic disciplines, including music, sculpture, painting, literature, and architecture, which seamlessly blend to create a mesmerizing and dynamic performance. Traditionally, it was also regarded as a sacred form of worship in temples. Dance has the profound ability to elevate consciousness beyond the dualities of life, leading to a state of spiritual bliss. This art form consists of two fundamental aspects: form and sentiment (or emotion). The dancer crafts a visual representation through abstract dance movements, transforming the performance into a display of visually expressive shapes and dimensions. Bharatanatyam is distinguished by movements that are conceptualized in space, primarily following straight lines and triangular formations. The dancer intricately weaves a series of triangles along with various geometric patterns, highlighting the spatial aspect of the dance. This structured movement reflects the physicality of Bharatanatyam. In Bharatanatyam, *Angika Abhinaya* pertains to the physical expression of the art form. The dance's physicality is closely linked to mathematical concepts such as geometry, arithmetic, and proportion.[2] The repeated use of triangular formations and other geometric patterns further emphasizes the spatial dimension of Bharatanatyam, creating a harmonious blend of movement and structure.

Adavu represents the foundation of Bharatanatyam, signifying the authenticity and structure of the dance. It is also associated with measurement, as the Tamil word *Adavu* refers to the act of moving from one place to another. In Bharatanatyam, the dancer transitions through different spatial dimensions, creating geometrical formations. *Adavu* serves as the fundamental lesson in Bharatanatyam, forming the basis for the dancer's technique. The primary *Adavus* include *Tattadavu, Nattadavu, Paraval Adavu, Kuduthamettadavu, Korvai Adavu, Kuth Adavu, Sarukkal Adavu, Thatimettu Adavu, Paichal Adavu, Uthpalavana Adavu, Mandi Adavu, Sarika Adavu, Karthari Adavu,* and *Theermana Adavu* (also known as *Makuta Adavu*). The core posture of Bharatanatyam is *Araimandi*, which maintains the dancer's balance and grace. Each *Adavu* has unique variations of *Sollukattu* (rhythmic syllables), further enhancing the intricacy of the dance. *Paraval Adavu* is the third fundamental lesson in Bharatanatyam.[5] The term *Paraval* in Tamil means "to spread," reflecting the dancer's movement across space. This Adavu is also known as *Pakka Adavu*, *Meetu Adavu*, and *Marditha Adavu*. In Bharatanatyam, the dancer performs in harmony with a structured rhythmic pattern called *Solkattu*, which consists of instrumental beats and vocalized syllables to create a rhythmic musical framework. The *Sollukattu* used for Paraval Adavu is *Tha Thei Thei Tha Thith Thei Thei Tha*. This Adavu has four variations, and its movements are characterized by circular formations, adding to the spatial dynamics of the dance.

2. BASIC AND PRELIMINARY CONCEPTS

Definition 2.1

A graph G is defined as a pair (V, E) where V and E are finite sets. The elements of V are referred to as vertices, points, or nodes, while the elements of E represent edges, lines, or arcs that connect pairs of vertices.

Definition 2.2

A graph $G = (V, E)$ is called a directed graph if each edge in G is assigned a specific direction, meaning the edges are represented as ordered pairs of distinct vertices.

Definition 2.3

The $indegree$ of a vertex v in a graph refers to the number of edges that terminate at and is represented as $indeg\,(v)$ Similarly, the $outdegree$ of a vertex v is the number of edges that originate from v and is denoted as $outdeg\,(v)$.

Definition 2.4

In a graph, edges that connect the same pair of vertices are known as *parallel edges*.

Definition 2.5

An edge that connects a vertex to itself is called a *self-loop*.

Definition 2.6

A graph that contains no parallel edges or *self-loops* is known as a simple graph.

Definition 2.7

In a graph, the degree of a vertex v refers to the number of edges connected to it. It is represented as $d(v)$ or $deg\,(v)$.

Definition 2.8

A vertex is called an even vertex if its degree is an even number; otherwise, it is referred to as an odd vertex.

Definition 2.9

Let $G = (V,E)$ be a graph. A walk in G is a finite, loop-free, alternating sequence of vertices and edges:v_0, e_1, v_1, e_2, v_2, e_3, …, v_{n-1}, e_n, v_n, starting and ending with a vertex. Each edge e_i connects the consecutive vertices v_{i-1} and v_i, where $1 \le i \le n$, and all edges in the sequence are distinct. The vertices v_0 and v_n are known as the terminal vertices of the walk. In a walk, vertices may appear multiple times.

Definition 2.10
An open walk is called a trail if it does not repeat any edges, though vertices may appear more than once.

Definition 2.11
An open walk in which each vertex appears only once is called a path.

Definition 2.12
A simple path from vertex u to vertex v is a path in which no edge is repeated.

Definition 2.13
The length of a path is defined as the number of edges it contains.

Definition 2.14
A circuit is a closed trail or walk in which no edge is repeated.

Definition 2.15
A cycle is a circuit where no vertex is repeated, except for the starting vertex, which is also the ending vertex of the sequence.

Definition 2.16
A graph G is considered connected if there is at least one path between every pair of vertices; otherwise, it is classified as disconnected.

Definition 2.17
A disconnected graph G consists of two or more connected subgraphs, which are referred to as the components of G.

Definition 2.18
A coloring of a graph G is the process of assigning colors to its vertices so that no two adjacent vertices share the same color.

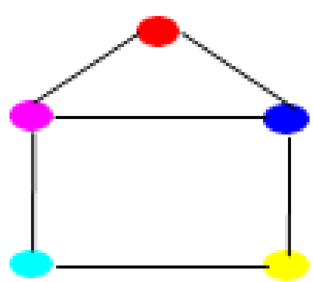

Figure: coloring graph

Definition 2.19
The chromatic number of a graph is the minimum number of colors needed for a proper coloring, ensuring that no two adjacent vertices share the same color.

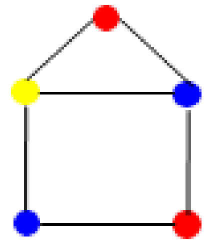

Figure 2: minimal coloring

3. Graph Coloring and Labeling In Bharatanatyam Paraval Adavu

This section begin by conceptualizing dance as a sequence of fundamental movements performed in a continuous flow. To formally represent this concept, we introduce a graph model. Let A denote a set of node labels, where the number of possible values is finite. Based on the hand movements of Bharatanatyam's Paraval Adavu, we construct a graph and apply various concepts, including graph coloring, graph labeling, fuzzy coloring, fuzzy labeling, and automata.

Definition 3.1

A labeled and directed graph is a system $G = (V, E, s_r, t_r, I)$, where:

- V and E are finite sets representing nodes and edges, respectively.
- $s_r, t_r : E \to V$ are functions that assign each edge a source node and a target node, respectively.
- $I : V \to A$ is a function that labels each node with an element from the set A

Definition 3.2

A control diagram is a directed and labeled graph with a set of node labels $A \cup \{I, F\}$ that satisfies the following conditions:

- There is exactly one initial node labeled I and one final node labeled F.
- The initial node I has no incoming edges, and the final node F has no outgoing edges.

In a control diagram, the nodes are labeled with the *Solkattu* of *Paraval Adavu*, while the edges of the directed graph define the sequence in which movements should be performed.

The control diagram is constructed using Paraval Adavus 1, 2, and 4, all of which start on the right side. This diagram includes the full Adavu 2 and Adavu 4, while only the first half of Adavu 1 is incorporated. The Aramandi posture appears as both the first and last node in the diagram, symbolizing that Bharatanatyam begins and ends with this posture, as depicted in Figure 1. Additionally, the vertices are labeled from 1 to 40, and the $indegree$ and $outdegree$ for each vertex are calculated. For the Paraval Adavu, both the $indegree$ and $outdegree$ are equal to one, i.e., $indeg(V_i) = outdeg(V_i) = 1$, where $i = 1, 2, 3, \ldots, 40$.

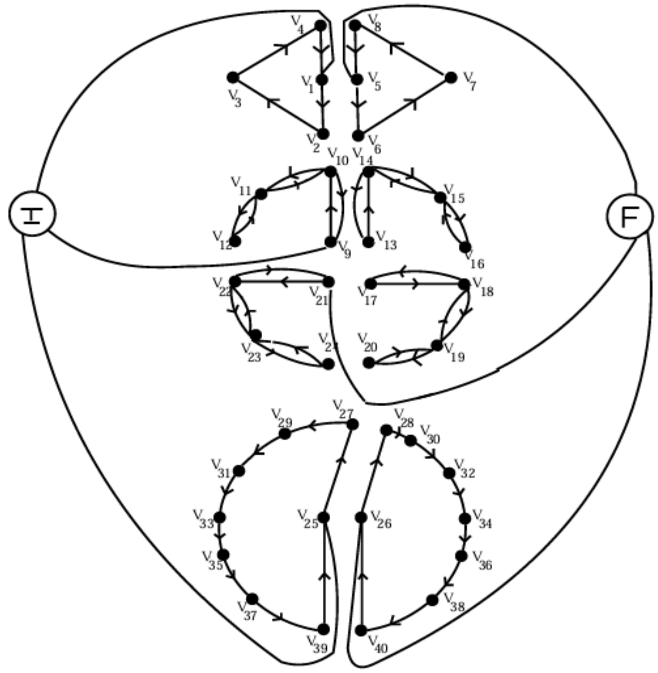

Figure 1: Control diagram and vertices

3.1 Graph Coloring in Paraval Adavu

Consider a graph $G = (V, E, s_r, t_r, I)$ where V represents the set of vertices and E represents the edges of the graph. This graph contains 40 vertices, with both the initial and final nodes included. We apply the coloring principle to the control diagram and aim to determine the chromatic number $\chi(G)$ of the graph shown in Figure 1. Based on the coloring scheme in Figure 2, we observe that only two colors are used. From this coloring pattern, it is evident that all vertices in odd positions are colored the same, and all vertices in even positions are colored the same. This indicates that the minimum number of colors required to color the graph is two, meaning the chromatic number is $\chi(G) = 2$.

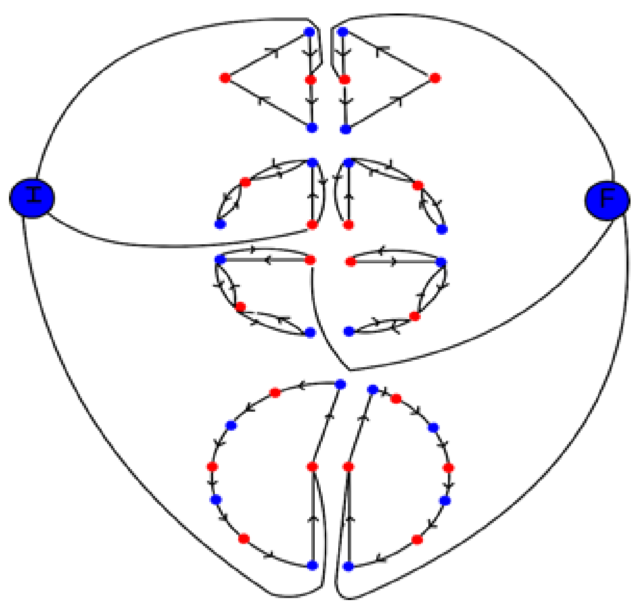

Figure 2: Graph Coloring in Paraval Adavu

The thalam of the *paraval adavu* in Bharatanatyam follows the *aadhi thalam*, which consists of 8 counts (i.e., the adavu has an even count). Each adavu forms a cyclic graph on its own. When we analyze each *paraval adavu* separately within the control diagram, we obtain an even-length cyclic graph. For cyclic graphs with even lengths, the chromatic number is 2. Therefore, for these cyclic graphs, the chromatic number is $\chi(G) = 2$. This chromatic number remains the same whether we consider the control diagram as a whole or examine the individual separated graphs.

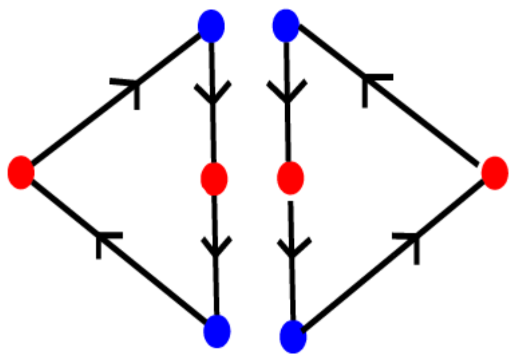

Figure 3.1

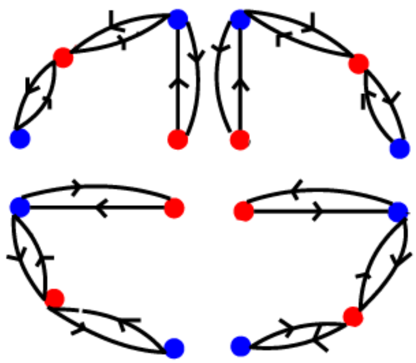

Figure 3.2

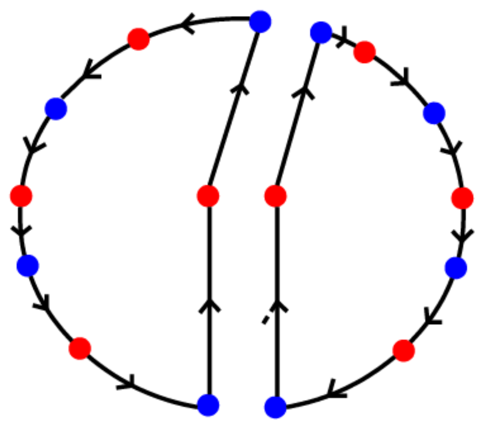

Figure 3.3

3.2 Graph Labeling in *Paraval Adavu*

Let $G = (V, E, s_r, t_r, I)$ be a graph where the vertex set is labeled with the paraval adavu solkattu (i.e., "Tha Thei Thei Tha Thith Thei Thei Thei Tha"). Each paraval adavu consists of eight vertices, so the vertex set is $V\ (G)\ =\ \{Tha, Thei, Thei, Tha, Thith, Thei, Thei, Tha\}$.

The initial and final nodes are also labeled, with $I = F = \{Thalangu\ thakadhiku\ thaka\ thadhinginathom\}$. The first speed of the adavu is labeled in the control diagram. Similarly, we can construct labeled graphs for the second and third speeds of the adavu. For the second and third speeds, the vertex sets are as follows:

For the second speed: $V\ (G)\ = \{Tha\ Thei, Thei\ Tha, Dhit\ Thei, Thei\ Tha\}$

For the third speed: $V\ (G)\ =\ \{Tha\ Thei\ Thei\ Tha, Dhit\ Thei\ Thei$

$Tha, Tha\ Thei\ Thei\ Tha, Dhit\ Thei\ Thei\ Tha, Tha\ Thei\ Thei\ Tha, Dhit\ Thei\ Thei\ Tha\}$

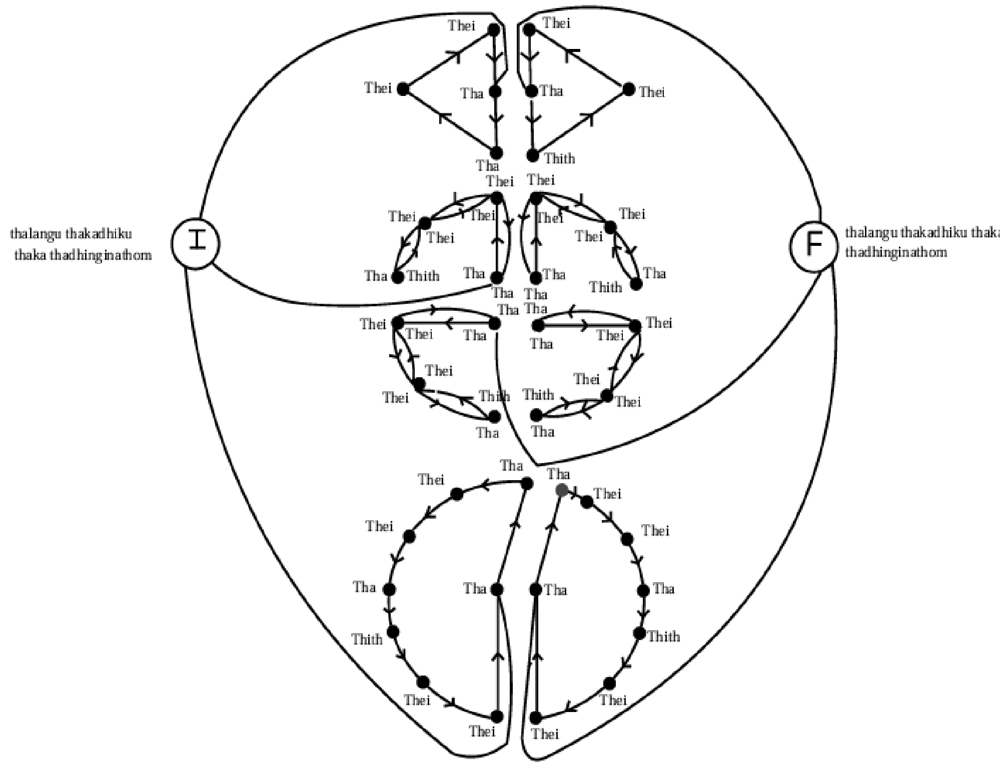

Figure 4: Graph Labeling in Paraval Adavu

Bharatanatyam, a classical Indian dance form, is intricately connected to the principles of mathematical graph theory, creating exciting opportunities for research. Additionally, integrating fundamental mathematical concepts into the teaching of Bharatanatyam can enhance student engagement, sparking a deeper interest in both the dance form and mathematics.

4. CONCLUSION

This study focuses on developing the control diagram for the Paraval Adavu in Bharatanatyam. The control diagram, as outlined in this paper, has been color-coded and properly labeled. Our analysis showed that the chromatic number of the control diagram for each Paraval Adavu is consistently two. Furthermore, we observed that for all vertices in the control diagram, both the indegree and outdegree are equal to one.

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