Stochastic Modelling and Computational Sciences

AN INNOVATIVE OPTIMIZATION TO SOLVE SEQUENCING PROBLEM UNDER FUZZY APPROACH

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ABSTRACT

In this paper, we introduce a fuzzy sequencing approach applicable to modern engineering and industrial manufacturing systems. These systems frequently encounter various challenges related to job processing times, machine setup durations, availability of raw materials, manpower limitations, energy consumption, and customer-driven constraints. In real production scheduling environments, numerous internal and external uncertainties arise. Owing to incomplete information and fluctuating production conditions, it becomes challenging to represent all scheduling parameters using precise, crisp values. When a new job enters the system, the inherent vagueness and uncertainties can be effectively addressed through the application of fuzzy logic theory.

We propose a simple and effective approach for solving fuzzy job-sequencing problems under a fuzzy environment, where processing times are represented using pentagonal fuzzy numbers (PFNs). The optimal job sequence and the corresponding idle times for each machine are obtained by converting the fuzzy model into an equivalent crisp sequencing problem. In this procedure, the fuzzy sequencing data are defuzzified using Pascal's triangular graded mean–based pentagonal fuzzy ranking index method. The fuzzy problem is thereby transformed into a crisp one by employing linguistic variables, after which Johnson's Algorithm is applied to determine the optimal sequence. The use of linguistic variables enables conversion of qualitative information into quantitative form, making the method suitable for handling sequencing problems characterized by vagueness and imprecision. Finally, the practicality and effectiveness of the proposed approach are demonstrated through a numerical example.

Keywords: *Fuzzy sequencing problem, Pentagonal fuzzy number, Pascal's triangular graded mean technique and linguistic variable.*

1. INTRODUCTION

Sequencing and scheduling is a form of decision-making that plays a crucial role in manufacturing and service industries. In the current competitive environment effective sequencing and scheduling has become a necessity for survival in the market place. Companies have to meet shipping dates; they have to schedule activities in such a way as to use the resources available in an efficient manner. Otherwise, it will impinge upon reputation of a business. As a result, good scheduling algorithms are needed.

A sequencing problem is to decide the order in which required tasks are to be done so as to minimize the total elapsed time taken for all the tasks. In Operations Research (O.R.), Johnson's rule [4], [2] has given a method of scheduling jobs in two work centers. Its primary objective is to find an optimal sequence of jobs to reduce makespan (the total amount of time it takes to complete all jobs). It also reduces the amount of idle time between the two work centres. Furthermore, the method finds the shortest makespan in the case of three work centres if additional constraints are met. But in reality, it is observed that the processing times during performance of the job are imprecise.

Yager's ranking method [5, 6] is one of the robust ranking techniques which are used to solve fuzzy sequencing problems involving fuzzy numbers. Generally, in job sequencing problems, the processing times are precise valued. But in reality, it is observed that the processing times during performance of the job are imprecise. To handle impreciseness fuzzy set theory plays an important role as fuzzy set is a best mathematical way for representing impreciseness (or) vagueness.

The optimal solution and idle time for each machine is obtained by solving corresponding sequencing problems using the modified method. Finally, to illustrate the modified method, a numerical example is solved and results are presented.

This paper is schedule as follows: In Section *2:* Basic definitions and preliminaries are given. In Section 3 and 4: Pentagonal fuzzy numbers and fuzzy linguistic variables is discussed, In Section 5: the Mathematical formulation for solving fuzzy sequencing problem is explained, In Section 6 and 7: Pascal's triangular graded mean for pentagonal fuzzy number is given and working rule for solving fuzzy sequencing problem is explained. In Section: 8: A numerical example is solved, and in section 9: Conclusion is given.

2. SOME DEFINITIONS (PRELIMINARIES)

The concept of fuzzy sets was proposed by Zadeh. The impreciseness occurring in scheduling problems is categorized as fuzzy-scheduling problems. [14, 10] has described a fuzzy production system for multi-objective scheduling. A Heuristic algorithm is proposed to find the optimal sequences of different single objectives as the production rules, and the fuzzy min-operator with non-linear membership function as the test criterion. We review the fundamental of fuzzy set theory, initiated by Bellman and Zadeh, [14].

Definition: 2.1 (Fuzzy Set) Let X be a set. A fuzzy set A on X is defined to be a function $A : X \rightarrow [0\,;\,1]$ or $A\mu : X \rightarrow [0\,;\,1]$. Equivalently, a fuzzy set A is defined to be the class of objects having the following representation $A = \{(x,\, A\mu\, x) : x \varepsilon X)\}$ where $A\mu : X \rightarrow [0\,;\,1]$, is a function called the membership function of A.

Definition: 2.2 (Fuzzy Number) The fuzzy number A is a fuzzy set whose membership function $A\mu\,(x)$ satisfies the following characteristics:

a) $A\mu\,(x)$ is piecewise continuous;

b) A fuzzy set A of the universe of discourse X is convex;

c) A fuzzy set of the universe of discourse X is called a normal fuzzy set if $\exists\, x, \in X, A\mu\,(x_i) : 1$.

Definition: 2.3 (Processing order) [16], [3] It is a sequence in which various machines are needed for completion of the job.

Definition: 2.4 (Processing Time). It represents the time by a job on each machine.

Definition: 2.5 (Idle Time). It is the time for which a machine does not have a job to process.

Definition: 2.6 (Total Elapsed Time). It is the time interval between starting of the first job and completion of the last job including ideal time in a particular order by the given set of machines.

3. PENTAGONAL FUZZY NUMBER

A Pentagonal Fuzzy Number (PFN) of a fuzzy set A is defined as $\bar{A}_p = \{a_1\,, a_2\,, a_3\,, a_4\,, a_5\}$, where $a_1\,, a_2\,, a_3\,, a_4\,, a_5$ are real numbers and its membership function is given by [1], [7] [15]:

$$\mu \bar{A}_p(x) = \begin{cases} 0 & , \quad for\ x < a_1 ; \\ \dfrac{(x-a_1)}{(a_2-a_1)} & , \quad for\ a_1 \le x \le a_2 ; \\ \dfrac{(x-a_2)}{(a_3-a_2)} & , \quad for\ a_2 \le x \le a_3 ; \\ 1 & , \quad for\ x = a_3 ; \\ \dfrac{(a_4-x)}{(a_4-a_3)} & , \quad for\ a_3 \le x \le a_4 ; \\ \dfrac{(a_5-x)}{(a_5-a_4)} & , \quad for\ a_4 \le x \le a_5 ; \\ 0 & , \quad for\ x > a_5 . \end{cases} \tag{1}$$

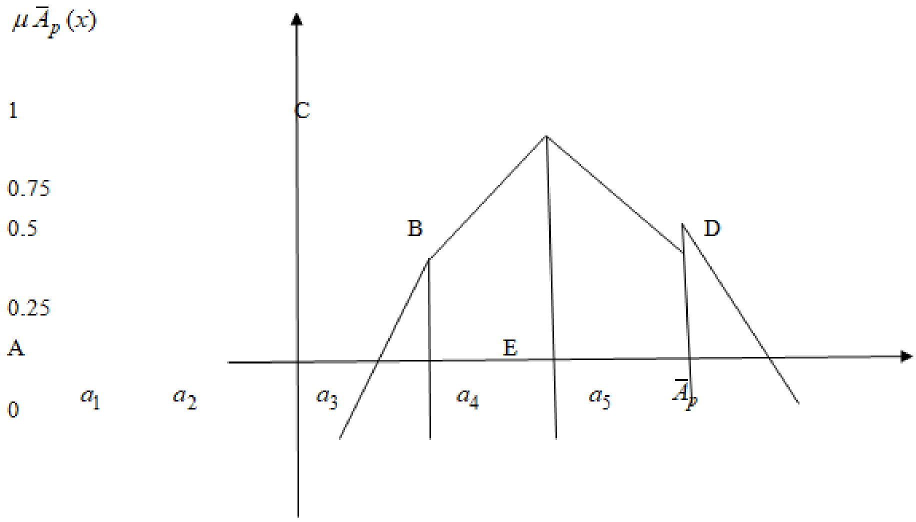

Fig.1. Graph of a Pentagonal Fuzzy Number (Source: JCMS Vol. 10 (6) P. 1255)

3.1. Condition on Pentagonal Fuzzy Number

A Pentagonal Fuzzy Number $\bar{A}_p$ should satisfy the following conditions:

a) $\mu \bar{A}_p(x)$ is a continuous function in the interval [0,1].

b) $\mu \bar{A}_p(x)$ is strictly increasing and continuous function on $[a_1, a_2]$ and $[a_2, a_3]$.

c) $\mu \bar{A}_p(x)$ is strictly decreasing and continuous function on $[a_3, a_4]$ and $[a_4, a_5]$.

3.2. Arithmetic Operations on Pentagonal Fuzzy Number

If $\bar{A}_p = \{a_1, a_2, a_3, a_4, a_5\}$ and $\bar{B}_p = \{b_1, b_2, b_3, b_4, b_5\}$ then

Addition : $\bar{A}_p + \bar{B}_p = \{a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5\}$

Subtraction : $\bar{A}_p - \bar{B}_p = \{a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5\}$

Multiplication : $\bar{A}_p \times \bar{B}_p = \{a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5\}$

4. FUZZY LINGUISTIC VARIABLE

Fuzzy linguistic variables [5], [6] are commonly depicted as being spread over the 'Universe of Discourse' by means of overlapping triangular trapezoidal or any other suitable geometric representation. In this paper the r priority is based on the fuzzy linguistic values from the term set {very high, high, medium, low, very low } are employed to explicate the fuzzy linguistic variable 'physical impairment', linguistic values from the term set {very high, high, medium, low, very low} are used for modeling uncertainty due to vagueness. Fuzzy membership functions used to define how well a value fits into a fuzzy set.

5. MATHEMATICAL FORMULATION OF N JOBS THROUGH 2 MACHINES

Let there are n jobs say $A_1, A_2, \ldots, A_n$ be processed through 2 machines say M_1, M_2 in the order M_1 M_2. Let t_{ij} be the fuzzy processing time taken by i^{th} job to be completed by j^{th} Machine. The well-known Johnson method can be extended to this problem, and then we find optimal sequence, total elapsed time and idle time on machines. Job machine fuzzy time for n jobs and 2 machines are given [15] below. Here fuzzy times are considered as pentagonal fuzzy number.

Jobs	Machine M_1	Machine M_2
A_1	$t_{11} = (a_{11}, b_{11}, c_{11}, d_{11}, e_{11}, f_{11})$	$t_{12} = (a_{12}, b_{12}, c_{12}, d_{12}, e_{12}, f_{12})$
A_2	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_{21}, f_{21})$	$t_{22} = (a_{22}, b_{22}, c_{22}, d_{22}, e_{22}, f_{22})$
A_3	$t_{31} = (a_{31}, b_{31}, c_{31}, d_{31}, e_{31}, f_{31})$	$t_{32} = (a_{32}, b_{32}, c_{32}, d_{32}, e_{32}, f_{32})$
A_4	$t_{41} = (a_{41}, b_{41}, c_{41}, d_{41}, e_{41}, f_{41})$	$t_{42} = (a_{42}, b_{42}, c_{42}, d_{42}, e_{42}, f_{42})$
A_5	$t_{51} = (a_{51}, b_{51}, c_{51}, d_{51}, e_{51}, f_{51})$	$t_{52} = (a_{52}, b_{52}, c_{52}, d_{52}, e_{52}, f_{52})$

6. PASCAL'S TRIANGULAR GRADED MEAN FOR PENTAGONAL FUZZY NUMBER

Let $\bar{A}_p = \{a_1, a_2, a_3, a_4, a_5\}$ be a pentagonal fuzzy number, [1], [7] we can take the coefficient of fuzzy numbers from Pascal's triangles and apply the simple probability approach we get the following formula

$$P(A) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4 + a_5}{16} \qquad (2)$$

The coefficient of a_1, a_2, a_3, a_4, a_5 are 1, 4, 6, 4, 1. This procedure is simply taken from the Pascal's triangles. These are useful to taken coefficient of fuzzy variables are Pascal's triangular numbers and we just add and divided by the total of pascal numbers.

7. WORKING RULE FOR SOLVING FUZZY SEQUENCING PROBLEMS

Step 1: Using Pascal's triangular graded mean approach, the fuzzy sequencing problem canbe converted into crisp sequencing problem.

Step 2: The Optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

Step 3: After finding the optimal sequence. Determine the total elapsed fuzzy time and also the fuzzy idle time on machines.

8. NUMERICAL EXAMPLE

In this section, a numerical example has been considered to illustrate the proposed solution procedure. Consider the fuzzy sequencing problem. Here the processing time of five jobs is given whose elements are fuzzy quantifiers which characterize the linguistic variables that are replaced by pentagonal fuzzy numbers. The problem is then solved by processing n jobs through two machines. The said fuzzy sequencing problem is given below:

Table 1. Quantitative Data

Jobs	**Machine** M_1	**Machine** M_2
J_1	Low	Medium
J_2	Medium	Very Low
J_3	Very Low	Good
J_4	Good	Very Good
J_5	Very Good	Low

The linguistic variables showing the qualitative data is converted into quantitative data using the full – table. As the processing time varies between 0 to 40 the minimum possible values is taken as 0 and the maximum possible value is taken as 40.

Table 2. Problem Table (Linguistic terms and corresponding fuzzy numbers)

VL	(0,1,2,3,4)
L	(6,8,10,12,14)
M	(15,16,17,18,19)
G	(22,24,26,28,30)
VG	(31,33,35,37,39)

Table 3. Quantitative Data

Jobs	M_1	M_2
J_1	(6,8,10,12,14)	(15,16,17,18,19)
J_2	(15,16,17,18,19)	(0,1,2,3,4)
J_3	(0,1,2,3,4)	(22,24,26,28,30)
J_4	(22,24,26,28,30)	(31,33,35,37,39)
J_5	(31,33,35,37,39)	(6,8,10,12,14)

As per steps for solving fuzzy sequence problems,

Step 1: Apply Pascal's triangular graded mean for pentagonal fuzzy number, the fuzzy times can be converted in to crisp times

t_{11} = (6; 8; 10; 12; 14) = 10; t_{12} = (15; 16; 17; 18; 19) = 17

t_{21} = (15; 16; 17; 18; 19) = 17; t_{22} = (0; 1; 2; 3; 4) = 2

t_{31} = (0; 1; 2; 3; 4) = 2; t_{32} = (22; 24; 26; 28; 30) = 26

t_{41} = (22; 24; 26; 28; 30) = 26; t_{42} = (31; 33; 35; 37; 39) = 35

t_{51} = (31; 33; 35; 37; 39) = 35; t_{52} = (6; 8; 10; 12; 14) = 10

Jobs	M_1	M_2
J_1	10	17
J_2	17	2
J_3	2	26
J_4	26	35
J_5	35	10

Step 2: Total Elapsed time and Idle time on machine M_1 and M_2

Jobs	M_1		M_2	
	Time in	Time out	Time in	Time out
$\mathbf{J_3}$	0	2	2	28
$\mathbf{J_1}$	2	12	28	45
$\mathbf{J_4}$	12	38	45	80
$\mathbf{J_5}$	38	73	80	90
$\mathbf{J_2}$	73	**90**	90	**92**

$\therefore$ Optimal Sequence: $J_{3,}\, J_{1,}\, J_{4,}\, J_{5,}\, J_2$

From step 2 it is seen that total expected elapsed time is 92 hours, idle times for machine M_1: 2 hours and machine M_2: 2 hours respectively. This is the optimal solution as compare to a heptagonal fuzzy number in solving fuzzy sequencing problem.

9. CONCLUSION

In this study, we address the job sequencing problem under fuzzy processing conditions by representing processing times with pentagonal fuzzy numbers. A structured solution methodology is proposed to derive the optimal sequence efficiently. The findings demonstrate that the total elapsed time achieved using pentagonal fuzzy numbers is considerably lower than that obtained with heptagonal fuzzy numbers, highlighting the superiority of the proposed representation.

The method has broad practical relevance in various real-world scheduling domains, including classroom and course timetabling, assignment of patients to hospital beds, railway and airline scheduling, and complex manufacturing and engineering systems. Moreover, the present work offers significant scope for future extensions, particularly in integrating data mining and knowledge discovery techniques for enhanced sequencing and scheduling applications.

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