Stochastic Modelling and Computational Sciences

MATHEMATICAL MODELLING AND COMPUTATIONAL ANALYSIS OF BLOOD FLOW THROUGH STENOSIS

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ABSTRACT
In this paper, we investigate the effects of stenosis on blood flow through arteries. The governing fundamental equations are solved numerically to examine the influence of various parameters on the apparent fluidity, volumetric flow rate, and wall shear stress. The obtained results are presented and analyzed using tables and graphical representations, with particular emphasis on the effects of the viscosity coefficient. Furthermore, the results are compared with those obtained from existing mathematical models to assess the validity and applicability of the proposed model.

1. INTRODUCTION
Stenosis refers to the abnormal narrowing of the lumen of an artery, generally resulting from the growth or accumulation of pathological material within the arterial wall. Such narrowing can significantly alter the normal circulation of blood and may lead to serious cardiovascular complications by restricting the supply of blood to tissues and organs. Stenotic lesions can develop at different locations within the arterial and microcirculatory systems under various pathological conditions. The physiological consequences of stenosis can be better understood by investigating the blood-flow behavior and associated hemodynamic characteristics in the vicinity of the stenotic region.

Blood exhibits complex rheological behavior, particularly when it flows through small arteries and capillaries. Consequently, numerous theoretical, experimental, and mathematical investigations have been carried out to understand the flow characteristics of blood through arteries under both normal and pathological conditions. Fahraeus and Lindqvist (1931) investigated the variation of blood viscosity in narrow capillary tubes, providing an important foundation for the study of blood rheology in the microcirculatory system. Womersley (1955) developed a theoretical framework for determining the velocity, flow rate, and viscous drag in arteries when the pressure gradient is prescribed.

Beavers and Joseph (1967) investigated the boundary conditions at a naturally permeable wall, providing a basis for mathematical treatment of flow near permeable boundaries. Benis et al. (1971) studied the relationship between shear stress and shear rate for blood using Couette viscometry. Davis and Ray (1971) investigated the finite-element analysis of blood-flow dynamics. Copley et al. (1975) examined the microscopic behavior associated with the viscoelasticity of human blood under steady and oscillatory flow conditions, while Chien et al. (1975) investigated the viscoelastic properties of human blood and red-cell suspensions. Thurston (1976) studied the viscosity and viscoelasticity of blood flowing through small-diameter tubes.

Gokhale et al. (1978) applied the finite-element method to the solution of the Navier–Stokes equations for two-dimensional steady flow through a section of a canine aorta. Nigma and Jafrin (1980) investigated blood flow through narrow tubes using a two-phase model. Tiefenbruck and Leal (1982) performed a numerical study of the motion of viscoelastic fluids past rigid spheres and spherical bubbles. Ahmed and Giddens (1983) investigated velocity measurements in steady flow through axisymmetric stenoses at moderate Reynolds numbers. Bitoun and Bellet (1986) studied blood flow through a stenosis in the microcirculation, while Schneditz et al. (1987) investigated the viscoelastic properties of whole blood and the influence of rapidly sedimenting red-cell aggregates.

Nakayama and Sawada (1988) conducted a numerical investigation of the flow of a non-Newtonian fluid through an axisymmetric stenosis. Chakravarty and Dutta (1989) investigated the effect of stenosis on arterial rheology using a mathematical model. Fukushima et al. (1989) studied the visualization and finite-element analysis of pulsatile flow in models of abdominal aortic aneurysms. Perktold et al. (1990) performed a three-dimensional numerical analysis of pulsatile flow and wall shear stress in the carotid artery bifurcation. Perktold et al. (1991) further investigated the characteristics of pulsatile non-Newtonian flow in a three-dimensional model of the human carotid bifurcation.

Chaturani and Palanisamy (1991) investigated the pulsatile flow of blood under periodic body acceleration. Stergiopulos et al. (1992) developed computer simulations of arterial flow with applications to arterial and aortic stenoses. Mishra et al. (1993) proposed a non-Newtonian fluid model for blood flow through arteries under stenotic conditions. Pedersen et al. (1993) investigated two-dimensional velocity measurements in a pulsatile-flow model of the normal abdominal aorta under different hemodynamic conditions. Thurston (1996) further examined the viscoelastic properties of blood and blood analogues. Srivastava (1996) developed a two-phase model of blood flow through a stenosed tube in the presence of a peripheral layer.

Subsequent studies have considered more sophisticated descriptions of arterial flow and blood rheology. Pozrikidis (1997) investigated shear flow over a protuberance on a plane wall. Yu et al. (1999) numerically investigated steady and pulsatile flow characteristics in axisymmetric abdominal aortic aneurysm models, together with experimental evaluation. Chakravarty, Mandal, and Mandal (2000) developed a mathematical model for pulsatile blood flow in a distensible aortic bifurcation subject to body acceleration. Haik, Pai, and Chen (2001) investigated the apparent viscosity of human blood in a high static magnetic field. El-Khatib and Damiano (2003) considered linear and nonlinear analyses of pulsatile blood flow in a cylindrical tube, while Urquiza et al. (2004) investigated multidimensional modelling of blood flow in the carotid artery.

Kumar et al. (2005) investigated the performance modelling and analysis of blood flow in elastic arteries. Yin and Zhu (2006) studied the oscillatory flow of a viscoelastic fluid in a pipe using the fractional Maxwell model. Kumar and Varshney (2007) investigated techniques for the performance modelling and analysis of blood flow in elastic arteries and their applications. Srivastava and Srivastava (2009) investigated particulate-suspension blood flow through a narrow catheterized artery.

The above studies demonstrate that mathematical modelling provides an effective framework for investigating the influence of arterial geometry, rheological properties, stenosis severity, pulsatility, and other physical parameters on blood-flow characteristics. However, the prediction of hemodynamic quantities such as apparent viscosity, volumetric flow rate, and wall shear stress remains important for understanding the physiological implications of arterial stenosis.

In the present study, the effect of stenosis on blood flow is investigated using a mathematical model. The governing equations are solved numerically, and the influence of the relevant physical parameters on the apparent fluidity, volumetric flow rate, and wall shear stress is analyzed. The numerical results are presented through tables and graphical representations. Particular attention is given to the role of the viscosity coefficient, and the results are compared with those obtained from existing models in order to assess the consistency and applicability of the present formulation.

2. FORMULATION OF THE PROBLEM:

We consider a steady flow of incompressible non-Newtonian Bingham fluid in a stenosed tube to circular cross section. The geometry is most conveniently represented by the cylindrical coordinate system (r, z) where z-axis coinciding with central line of vessel. The Young,s model given as;

$$\frac{R}{R_0} = \begin{cases} 1; & otherwise \\ 1 - \frac{\sigma}{2R_0}[1+\cos\frac{2\pi}{L_O}(z-d-\frac{L_0}{2})], & d \le z \le L_0 + d \end{cases}$$

(1)

Where R is radious of obstructed tube, R_0 the radious of unobstructed tube,

σ the maximum height of stenotic growth, d the location of the stenoses,

L_0 the lenth of stenosis and z is the distance along central axis.

We have the following assumptions in the problem as:

(a) The motion is axially symmetric and in the z-direction.

(b) No body force act on the fluid.

(c) The motion is slow so that inertia term can be neglected.

(d) $R_0 \frac{\sigma}{R_O} << 1$ and $\mathrm{Re}\frac{\sigma}{z_O} << 1$,

Where R_e is the Reynolds number of the flow.

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) - \frac{\partial p}{\partial z} = 0 \quad , \quad \ldots \quad (2)$$

$$\frac{\partial p}{\partial r} = 0 \quad , \quad \ldots \quad (3)$$

$$\frac{\partial v}{\partial z} = 0 \quad (4)$$

Where v is the axial velocity, p the pressure and is τ_{rz} the shear stress normal to r in z- direction.

The equation for the shear stress τ and strain rate $(\dot{\gamma} = \frac{dy}{dr})$ is given as:

$$\tau = \tau_0 + \eta\dot{\gamma}, \tau \ge \tau_0$$

$$\dot{\gamma} = 0, \tau \ge \tau_0 \quad \ldots (.5)$$

Where η is the coefficient of viscosity and τ_0 is the yield stress.

The boundary conditions are given as:

V=0 at r=R (6)

$$\tau_{rz} = \tau_w$$

at r=R (7)

τ is finite at r=0. (8)

Where τ_w is the shear stress at the stenotic wall.

3 MATHEMATICAL ANALYSIS:

Now solving the equation (7.2) using boundary condition (7.8), we have

$$\tau_{rz} = \frac{\Delta pr}{2L} \tag{9}$$

Where Δp is the pressure drop over a length L of the tube. Using equations (9) in (5),

We have:

$$\frac{dv}{dr} = -\frac{1}{\eta}\left(\frac{\Delta pr}{2L} - \tau_0\right) \tag{10}$$

Now solving the equation (10) with boundary conditions (6) and (7). We have:

$$v = \frac{1}{2}\frac{\tau_w R}{\eta}\left[(1-\beta)^2 - \left(\frac{r}{R} - \beta\right)^2\right] \tag{11}$$

Where $\beta = \frac{\tau_0}{\tau_w}$

Whenever the shear stress does not exceed the yield stress, plug flow exists.

Now putting the $r = R_p\beta = \beta R$ in equation (11),

We have the plug flow velocity, as:

$$V_p = \frac{1}{2}\frac{\tau_w R}{\eta}(1-\beta)^2 \tag{12}$$

The volume flow rate Q is given as:

$$Q = Q_1 + Q_2 \tag{13}$$

$$Q_1 = \int_0^{R_p} 2\pi V_p r dr \tag{14}$$

$$Q_2 = \int_{R_P}^{R} 2\pi V r dr \tag{15}$$

Now using equations (11) and (12) in equations (14) and (15) respectively,

We have:

$$Q_1 = \frac{\pi}{2\eta}\tau_w R_0^3 \beta^2 (1-\beta)^2 (\frac{R}{R_0})^3 \tag{16}$$

$$Q_2 = \frac{\pi}{2\eta}\tau_w R_0^3 (\frac{R}{R_0})^3 (\frac{1}{2} - \frac{2}{3}\beta - \beta^2 + 2\beta^2 - \frac{5}{6}\beta^4) \tag{17}$$

Now using equations (16) and (17) in (13) we get

$$Q = \frac{\pi}{2\eta}\tau_w R_0^3 (1 - \frac{4}{3}\beta)\ (\frac{R}{R_0})^3 \tag{18}$$

The non-dimensional flow rate is given as:

$$\overline{Q} = \frac{Q}{Q_0}(1 - \frac{4}{3}\beta)\ (\frac{R}{R_0})^3 \tag{19}$$

Where

$$Q_0 = \frac{\pi}{4\eta}\tau_w R_0^3 \tag{20}$$

At maximum height of stenosis; the volume flow rate is given as:

$$\overline{Q} = (1 - \frac{3\sigma}{R_0})(1 - \frac{4}{3}\beta) \tag{21}$$

The apparent fluidity is given as:

$$\phi_a = \frac{4Q}{\pi \tau_w R_0^3} \tag{22}$$

Using equation (18) in above equation, we have

$$\phi_a = \frac{1}{\eta}(1 - \frac{4}{3}\beta)\ (\frac{R}{R_0})^3 \tag{23}$$

At maximum height of stenosis, $z = d + \frac{L_0}{2}$ we get

$$\phi_a = \frac{1}{\eta}(1 - \frac{4}{3}\beta)\ (1 - \frac{3\sigma}{R_0}) \tag{24}$$

The wall shear stress is given as:

$$\tau_w = \frac{4Q\eta}{\pi R_0^3}(1 - \frac{4}{3}\beta)\ (\frac{R}{R_0})^3 \tag{25}$$

The wall shear stress at maximum height of stenosis is given as:

$$\tau_w = \frac{4Q\eta}{\pi R_0^3}(1 + \frac{4}{3}\beta)\ (1 + \frac{3\sigma}{R_0})^3 \qquad (.26)$$

In non-dimensional scheme,the shear stress at the wall is given as:

$$\bar{\tau}_w = (1 + \frac{4}{3}\beta)\ (1 + \frac{3\sigma}{R_0}) \tag{27}$$

Where $\bar{\tau}_w = \frac{\tau_w}{\tau_1}$ and $\tau_1 = \frac{4Q\eta}{\pi R_0^3}$ (28)

τ_1 is the wall shear stress for no stenosis case.

4 RESULTS AND DISCUSSION:

Here is a polished version suitable for the Results and Discussion section of a research paper:

RESULTS AND DISCUSSION

The results have been obtained numerically for various combinations of the parameters involved in the mathematical model. Analytical expressions for the volumetric flow rate, apparent fluidity, and wall shear stress at the location of maximum stenosis height have been derived. The effects of the governing parameters on these hemodynamic quantities are presented and discussed with the help of Figures 1–6.

Figure 1 illustrates the variation of the volumetric flow rate with stenosis height for different values of yield stress. It is observed that the volumetric flow rate decreases as the stenosis height increases. This reduction in flow rate is attributed to the progressive narrowing of the arterial lumen, which increases the resistance to blood flow.

Figure 2 shows the variation of the volumetric flow rate with yield stress for different values of stenosis height. The results indicate that the flow rate decreases gradually with an increase in yield stress. The reduction is more pronounced for larger values of stenosis height, indicating the combined influence of blood rheology and arterial narrowing on the flow characteristics.

Figure 3 depicts the variation of apparent fluidity with stenosis height for different values of yield stress. It is observed that the apparent fluidity decreases approximately uniformly as the stenosis height increases. Thus, an increase in the severity of stenosis results in a reduction in the effective fluidity of blood through the artery.

Figure 4 presents the variation of apparent fluidity with yield stress for different values of stenosis height. The results show that the apparent fluidity decreases gradually with increasing yield stress. This behavior indicates that an increase in yield stress enhances the resistance to blood flow and consequently reduces the apparent fluidity.

Figure 5 illustrates the variation of wall shear stress with stenosis height for different values of yield stress. It is observed that the wall shear stress increases progressively as the stenosis height increases. The increased shear stress is associated with the reduction in the effective cross-sectional area of the artery and the corresponding alteration of the velocity gradient near the arterial wall.

Figure 6 shows the variation of wall shear stress with yield stress for different values of stenosis height. The results indicate that the wall shear stress increases gradually with increasing yield stress. The increase is also influenced by the severity of stenosis, with higher stenosis heights producing comparatively larger wall shear stresses.

Overall, the numerical results demonstrate that increasing stenosis severity adversely affects blood-flow characteristics by reducing the volumetric flow rate and apparent fluidity while increasing the wall shear stress. Similarly, an increase in yield stress tends to reduce the flow rate and apparent fluidity and increase the wall shear stress. These observations are consistent with the expected hemodynamic behavior of blood flowing through a progressively narrowed arterial lumen.

Table (1). Variation of volume flow rate with respect to stenosis height for different values of yield stress.

$\frac{\sigma}{R_0}$	$\overline{Q}$			
	$\beta = 0.05$	$\beta = 0.01$	$\beta = 0.15$	$\beta = 0.20$
0.05	0.7936	0.7367	0.6807	0.6236
0.10	0.6535	0.60676	0.5606	0.5136
0.15	0.5135	0.4768	0.44001	0.4032
0.20	0.3735	0.3468	0.3205	0.2936
0.25	0.2167	0.2168	0.2001	0.1844

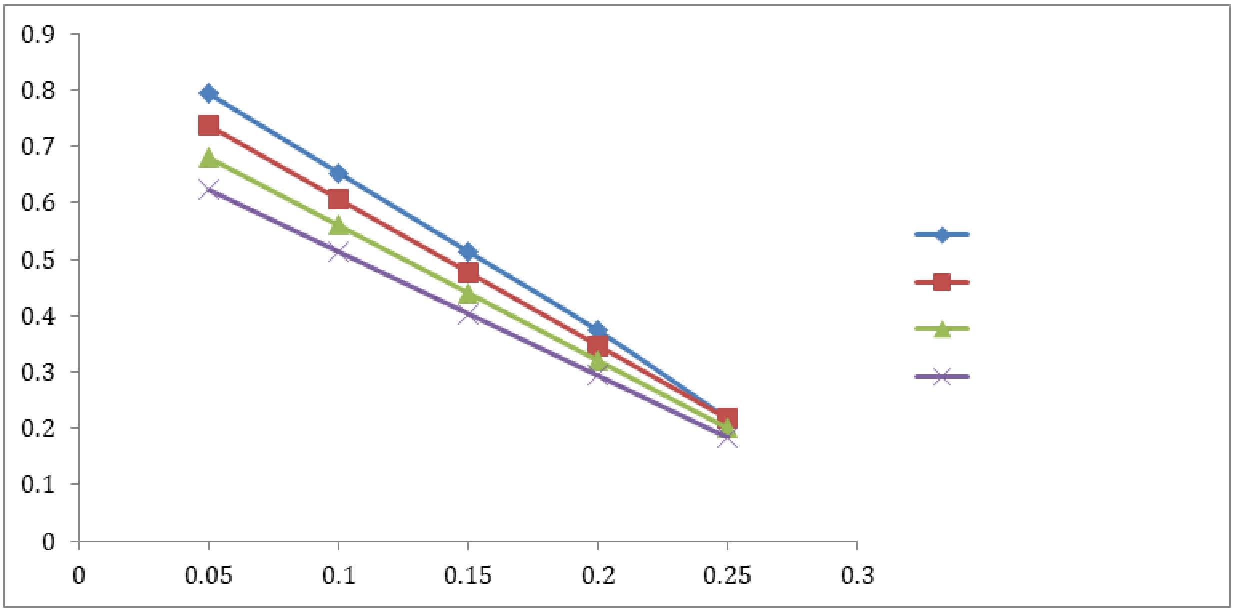

Figure (1). Variation of volume flow rate with respect to stenosis height for different values of yield stress.

Table (2). Variation of volume flow rate with respect to yield stress for different values of stenosis height .

β	$\overline{Q}$		
	$\frac{\sigma}{R_0}=0.20$	$\frac{\sigma}{R_0}=0.15$	$\frac{\sigma}{R_0}=0.10$
0.05	0.3733	0.5133	0.6533
0.10	0.3467	0.4767	0.6067
0.15	0.3200	0.4400	0.5600
0.20	0.2933	0.4033	0.5133

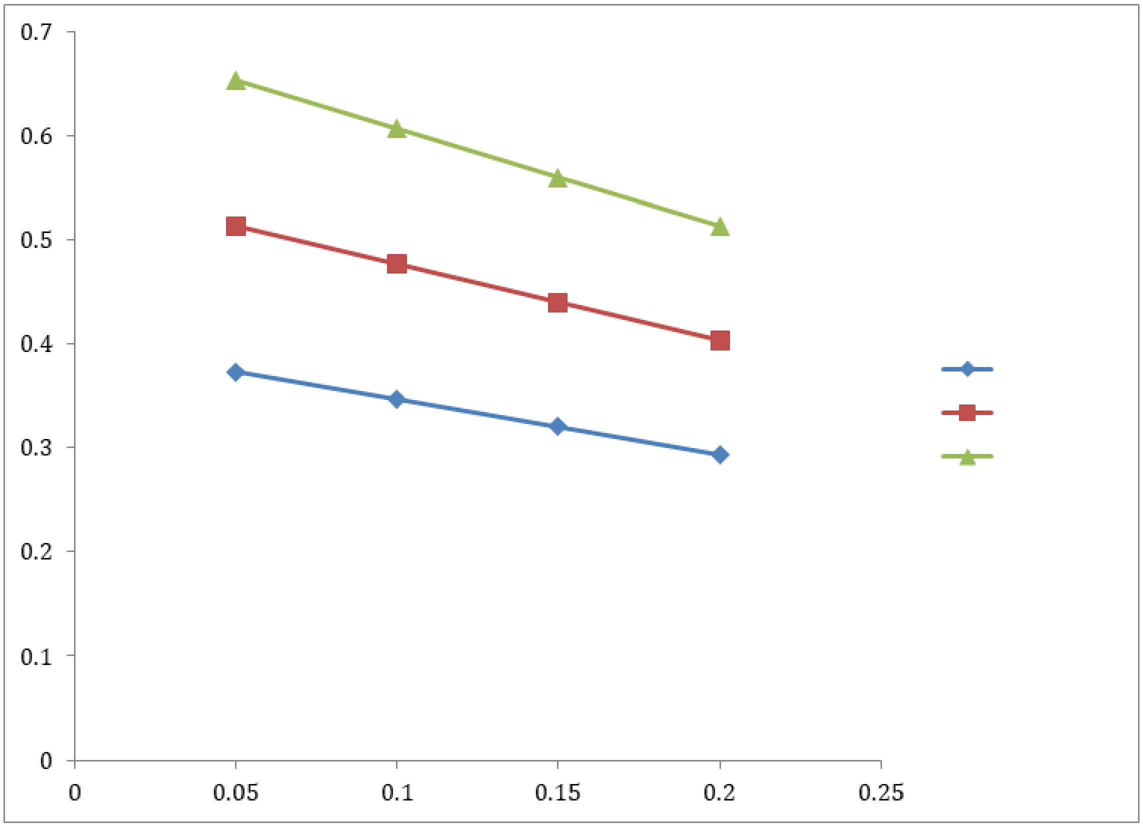

Figure (2). Variation of volume flow rate with respect to yield stress for different values of stenosis height .

Table (3). Variation of apparent fluidity with respect to stenosis height for different values of yield stress.

$\frac{\sigma}{R_0}$	ϕ_a				
	$\beta = 0.25$	$\beta = 0.20$	$\beta = 0.15$	$\beta = 0.10$	$\beta = 0.05$
0.05	0.2518	0.2770	0.3022	0.3274	0.3526
0.10	0.2074	0.2281	0.2489	0.2696	0.2904
0.15	0.1630	0.1792	0.1955	0.2118	0.2281
0.25	0.0741	0.8184	0.0889	0.0963	0.1037

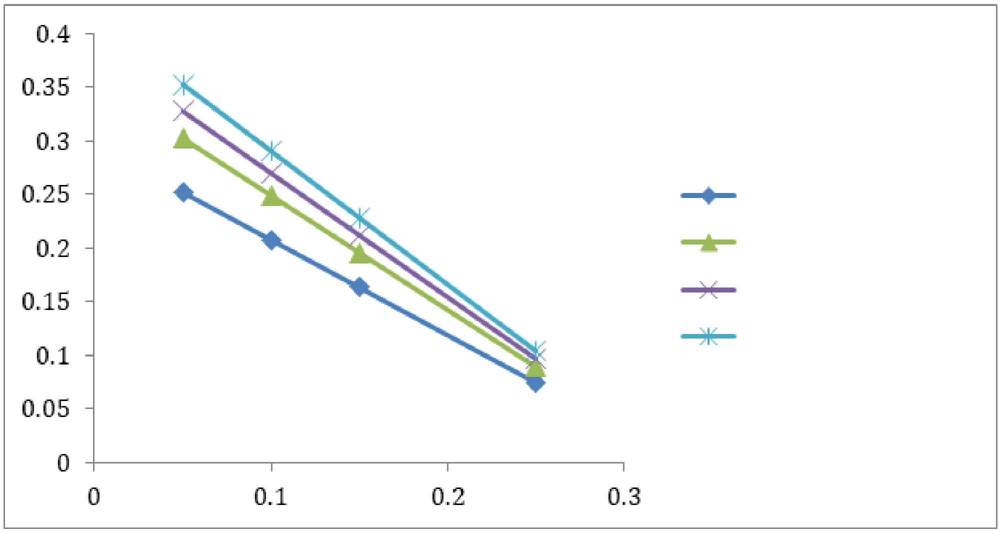

Figure (3). Variation of apparent fluidity with respect to stenosis height for different values of yield stress.

Table (4). Variation of apparent fluidity with respect to yield stress for different values of stenosis height .

β	ϕ_a		
	$\frac{\sigma}{R_0}=0.20$	$\frac{\sigma}{R_0}=0.15$	$\frac{\sigma}{R_0}=0.10$
0.05	0.1660	0.2281	0.2903
0.10	0.1541	0.2118	0.2696
0.15	0.1422	0.1955	0.2489
0.20	0.1304	0.1792	0.2281
o.25	0.1185	0.1630	0.2074

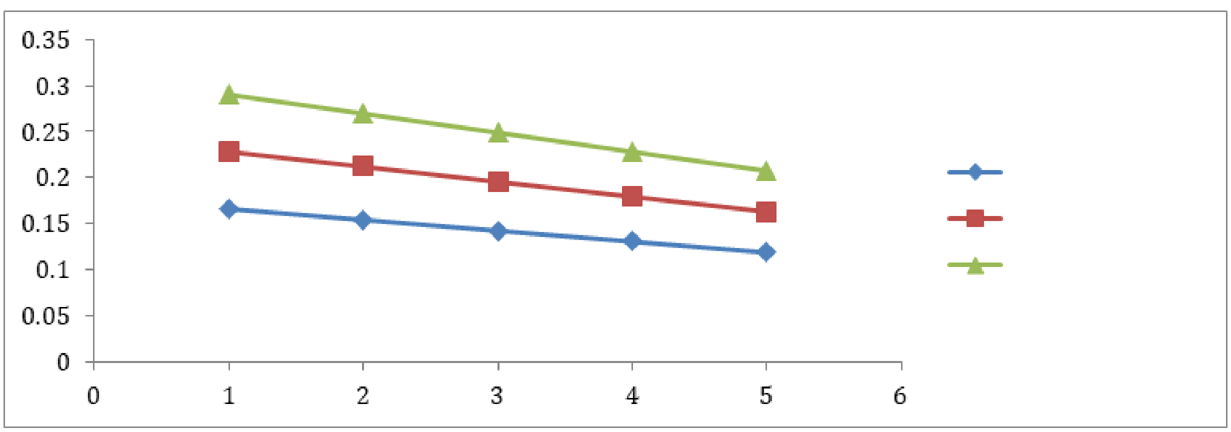

Figure (4). Variation of apparent fluidity with respect to yield stress for different values of stenosis height .

Table (5). Variation of wall shear stress with respect to stenosis height for different values of yield stress.

$\frac{\sigma}{R_0}$	$\overline{\tau_w}$				
	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.15$	$\beta = 0.20$	$\beta = 0.25$
0.05	1.2267	1.3303	1.3800	1.4567	1.5333
0.10	1.3867	1.4733	1.5600	1.6467	1.7333
0.15	1.5467	1.6433	1.7400	1.8367	1.9333
0.20	1.7067	1.8133	1.9200	2.0267	2.1333
0.25	1.8667	1.9833	2.1000	2.2167	2.3333

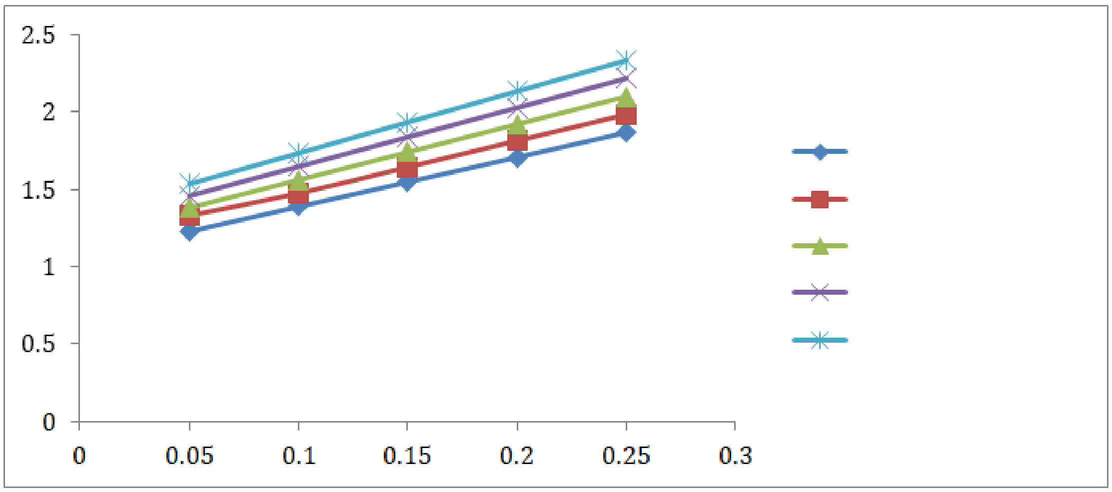

Figure (5). Variation of wall shear stress with respect to stenosis height for different values of yield stress.

Table (6). Variation of apparent fluidity with respect to yield stress for different values of stenosis height .

β	$\overline{\tau_w}$		
	$\frac{\sigma}{R_0} = 0.20$	$\frac{\sigma}{R_0} = 0.15$	$\frac{\sigma}{R_0} = 0.10$
0.05	1.3867	1.5467	1.7067
0.10	1.4733	1.6433	1.8133
0.15	1.5600	1.7400	1.9200
0.20	1.6467	1.8367	2.0267
o.25	1.7333	1.9333	2.133

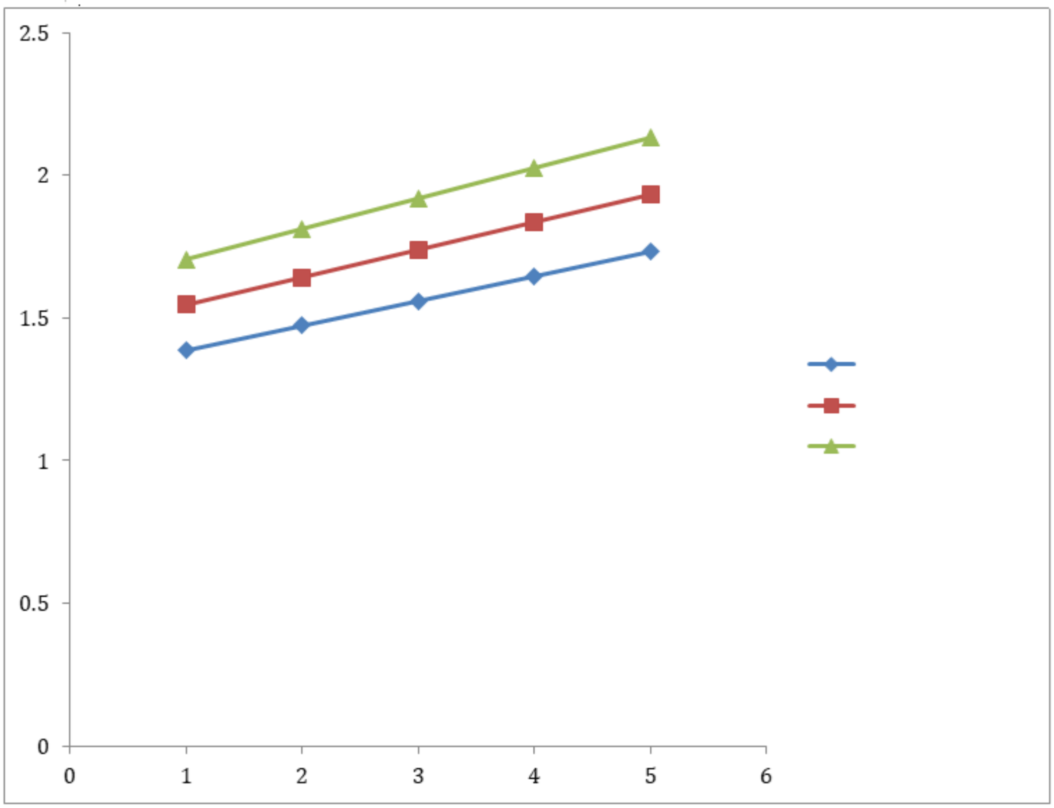

Figure (6). Variation of apparent fluidity with respect to yield stress for different values of stenosis height.

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