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Symmetric Coordinate System in Narrow Route by Transformation Method

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Abstract: In this paper we studied the spherically symmetric (s.s.) coordinate system in V_5 by transformation method. We obtain

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2$$

from the s.s. line element

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2 + 2Edrdt + 2Fdrdu + 2Gdtdu$$

by transformation method. Further we shall obtain the Christoffel symbol, curvature tensor and scalar curvature for the s.s. line element which is called the spherically symmetric coordinate system in a Narrow sense in V_5.

Keywords: Spherically symmetric, Narrow sense, curvature tensor.

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1. INTRODUCTION

According to Takeno [3], the space-time V_4 with metric

$$ds^2 = g_{ij}dx^i dx^j \tag{1.1}$$

is s. s. if

$$L_\xi(g_{ij}) = 0 \tag{1.2}$$

where L_ξ denotes the Lie derivative with respect to Killing vector ξ^i. Takeno has obtained the most general form of the s.s. line element in spherical polar coordinate (r, θ, ϕ, t) as

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 + 2Ddrdt \tag{1.3}$$

Takeno has further reduced line element equation (1.3) in the s.s. coordinate system in a narrow sense

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 \tag{1.4}$$

by the transformation of coordinate T given by

$$T:\begin{cases} \bar{r}=\bar{h}(r,t), \quad \bar{t}=\bar{k}(r,t), \quad \left(\frac{\partial(\bar{r},\bar{t})}{\partial(r,t)}\neq 0\right) \\ r=h(\bar{r},\bar{t}), \quad t=k(\bar{r},\bar{t}), \quad \left(\frac{\partial(r,t)}{\partial(\bar{r},\bar{t})}\neq 0\right) \end{cases} \text{or}$$

Thomas *et al*. [1] have obtained the most general form of the s.s. line element in $V5$ as

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2 + 2Edrdt + 2Fdrdu + 2Gdtdu \tag{1.5}$$

where A, B, C, D, E, F and G are the functions of r, t and u, and $x^i \equiv (r, \theta, \phi, t, u)$.

Pokley *et al.* [4] have obtained various christoffel symbols, curvature tensors, Ricci tensors of the s.s. space-time V_5 with respect to the line element (1.5).

In this paper we shall reduce the line element (1.5) into the form

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2 \tag{1.6}$$

by transformation method and obtain the christoffel symbol, curvature tensors, Ricci tensors for the same.

2. TRANSFORMATION OF COORDINATE *T*

Theorem 2.1: The s.s. line element

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2 + 2Edrdt + 2Fdrdu + 2Gdtdu \tag{2.1}$$

can be reduced into the form

$$ds^2 = -Adr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + Cdt^2 - Ddu^2 \tag{2.2}$$

by suitable transformation of coordinate,

where $x^i = (r, \theta, \phi, t, u)$ and A, B, C, D, E, F and G are the functions of r, t and u respectively.

Proof: The line element equation (2.1) defines a s.s. space-time V_5. The tangential space at each point of this V_5 is composed of two subspaces whose metrics are defined respectively by

$$\left.\begin{aligned} V_3(r,t,u): ds_1^2 &= -Adr^2 + Cdt^2 - Ddu^2 \\ &\quad +2Edrdt + 2Fdrdu + 2Gdtdu, \end{aligned}\right\} \tag{2.3a}$$

$$V_2(\theta, \phi): ds_2^2 = -B(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{2.3b}$$

The three dimensional space $V_3(r, t, u)$ is of indefinite metric and the two dimensional space $V_2(\theta, \phi)$ is of negative definite metric. Here $V_2(\theta, \phi)$ is a space of constant curvature $(-1/B)$ and its scalar curvature K is $2/B$.

When B is independent of r, t and u, i.e. when B is constant, the space-time V_5 is a direct sum of $V_3(r, t, u)$ and $V_2(\theta, \phi)$.

Let us consider transformation of coordinates T [2]

$$T:\begin{cases} \bar{r}=\bar{h}(r,t,u), \qquad \bar{t}=\bar{k}(r,t,u), \\ \bar{u}=\bar{m}(r,t,u), \qquad \left(\dfrac{\partial(\bar{r},\bar{t},\bar{u})}{\partial(r,t,u)}\right)\neq 0 \\ \qquad\qquad\qquad \text{or} \\ r=h(\bar{r},\bar{t},\bar{u}), \qquad t=k(\bar{r},\bar{t},\bar{u}), \\ u=m(\bar{r},\bar{t},\bar{u}), \qquad \left(\dfrac{\partial(r,t,u)}{\partial(\bar{r},\bar{t},\bar{u})}\right)\neq 0 \end{cases} \tag{2.4}$$

for the s.s. space-time (2.1).

Then

$$dr=\left(\frac{\partial r}{\partial \bar{r}}\right)d\bar{r}+\left(\frac{\partial r}{\partial \bar{t}}\right)d\bar{t}+\left(\frac{\partial r}{\partial \bar{u}}\right)d\bar{u} \tag{2.5a}$$

$$dt=\left(\frac{\partial t}{\partial \bar{r}}\right)d\bar{r}+\left(\frac{\partial t}{\partial \bar{t}}\right)d\bar{t}+\left(\frac{\partial t}{\partial \bar{u}}\right)d\bar{u} \tag{2.5b}$$

$$du=\left(\frac{\partial u}{\partial \bar{r}}\right)d\bar{r}+\left(\frac{\partial u}{\partial \bar{t}}\right)d\bar{t}+\left(\frac{\partial u}{\partial \bar{u}}\right)d\bar{u} \tag{2.5c}$$

If we use these in (2.1) then we have

$$d\bar{s}^2=-\bar{A}d\bar{r}^2-\bar{B}(d\theta^2+\sin^2\theta\, d\phi^2)+\bar{C}d\bar{t}^2-\bar{D}\, d\bar{u}^2$$
$$+2\bar{E}d\bar{r}d\bar{t}+2\bar{F}d\bar{r}d\bar{u}+2\bar{G}d\bar{t}d\bar{u} \tag{2.6}$$

where

$$\left.\begin{aligned}\bar{B}&=\bar{B}(\bar{r},\bar{t},\bar{u})=B(r,t,u)\\ &=B\{h(\bar{r},\bar{t},\bar{u}),k(\bar{r},\bar{t},\bar{u}),m(\bar{r},\bar{t},\bar{u})\},\end{aligned}\right\} \tag{2.7a}$$

$$
\left.\begin{aligned}
-\bar{A} = & -A\left(\frac{\partial r}{\partial \bar{r}}\right)^2 + C\left(\frac{\partial t}{\partial \bar{r}}\right)^2 - D\left(\frac{\partial u}{\partial \bar{r}}\right)^2 + 2E\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial t}{\partial \bar{r}}\right) \\
& +2F\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right) + 2G\left(\frac{\partial t}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right),
\end{aligned}\right\} \tag{2.7b}
$$

$$
\left.\begin{aligned}
\bar{C} = & -A\left(\frac{\partial r}{\partial \bar{t}}\right)^2 + C\left(\frac{\partial t}{\partial \bar{t}}\right)^2 - D\left(\frac{\partial u}{\partial \bar{t}}\right)^2 + 2E\left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial t}{\partial \bar{t}}\right) \\
& +2F\left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right) + 2G\left(\frac{\partial t}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right),
\end{aligned}\right\} \tag{2.7c}
$$

$$
\left.\begin{aligned}
-\bar{D} = & -A\left(\frac{\partial r}{\partial \bar{u}}\right)^2 + C\left(\frac{\partial t}{\partial \bar{u}}\right)^2 - D\left(\frac{\partial u}{\partial \bar{u}}\right)^2 + 2E\left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial t}{\partial \bar{u}}\right) \\
& +2F\left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + 2G\left(\frac{\partial t}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right),
\end{aligned}\right\} \tag{2.7d}
$$

$$
\left.\begin{aligned}
\bar{E} = & -A\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial r}{\partial \bar{t}}\right) + E\left[\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial t}{\partial \bar{t}}\right) + \left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial t}{\partial \bar{r}}\right)\right] \\
& +C\left(\frac{\partial t}{\partial \bar{r}}\right)\left(\frac{\partial t}{\partial \bar{t}}\right) + F\left[\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right) + \left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right)\right] \\
& -D\left(\frac{\partial u}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right) + G\left[\left(\frac{\partial t}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right) + \left(\frac{\partial t}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right)\right],
\end{aligned}\right\} \tag{2.7e}
$$

$$
\left.\begin{aligned}
\bar{F} = & -A\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial r}{\partial \bar{u}}\right) + E\left[\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial t}{\partial \bar{u}}\right) + \left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial t}{\partial \bar{r}}\right)\right] \\
& +C\left(\frac{\partial t}{\partial \bar{r}}\right)\left(\frac{\partial t}{\partial \bar{u}}\right) + F\left[\left(\frac{\partial r}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + \left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right)\right] \\
& -D\left(\frac{\partial u}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + G\left[\left(\frac{\partial t}{\partial \bar{r}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + \left(\frac{\partial t}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{r}}\right)\right]
\end{aligned}\right\} \tag{2.7f}
$$

and

$$\left.\begin{aligned}\bar{G} = & -A\left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial r}{\partial \bar{u}}\right) + E\left[\left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial t}{\partial \bar{u}}\right) + \left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial t}{\partial \bar{t}}\right)\right] \\ & + C\left(\frac{\partial t}{\partial \bar{t}}\right)\left(\frac{\partial t}{\partial \bar{u}}\right) + F\left[\left(\frac{\partial r}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + \left(\frac{\partial r}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right)\right] \\ & - D\left(\frac{\partial u}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + G\left[\left(\frac{\partial t}{\partial \bar{t}}\right)\left(\frac{\partial u}{\partial \bar{u}}\right) + \left(\frac{\partial t}{\partial \bar{u}}\right)\left(\frac{\partial u}{\partial \bar{t}}\right)\right]\end{aligned}\right\} \quad (2.7g)$$

where A, B, C, D, E, F & G are function of $\bar{r}$, $\bar{t}$ and $\bar{u}$.

From equation (2.1)

$$g = \det(g_{ij}) = MB^2 \sin^2 \theta, \quad (2.8a)$$

where

$$M \equiv ACD + 2EFG + AG^2 + DE^2 - CF^2 > 0 \quad (2.8b)$$

and the non-vanishing independent components of g^{ij} are

$$\left.\begin{aligned} & g^{11} = -\frac{(CD+G^2)}{M}, \; g^{22} = -\frac{1}{B}, \; g^{33} = -\frac{1}{B\sin^2\theta} \\ & g^{44} = \frac{(AD-F^2)}{M}, \; g^{55} = -\frac{(AC+E^2)}{M}, \; g^{14} = -\frac{(ED+FG)}{M} \\ & g^{15} = \frac{(EG-CF)}{M}, \; g^{45} = -\frac{(AG+EF)}{M}\end{aligned}\right\} \quad (2.9)$$

By using the above contravariant components g^{ij}, equation (2.7) can be written as

$$\bar{B} = B, \quad (2.10a)$$

$$\begin{aligned}-\frac{(\bar{C}\bar{D}+\bar{G}^2)}{\bar{M}} = \frac{1}{M}\Bigg[& -(CD+G^2)\left(\frac{\partial \bar{r}}{\partial r}\right)^2 - 2(ED+FG)\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial t}\right) \\ & + (AD-F^2)\left(\frac{\partial \bar{r}}{\partial t}\right)^2 + 2(EG-CF)\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial u}\right) \\ & - (AC+E^2)\left(\frac{\partial \bar{r}}{\partial u}\right)^2 - 2(AG+EF)\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)\Bigg],\end{aligned} \quad (2.10b)$$

$$\frac{(\bar{A}\bar{D}-\bar{F}^2)}{\bar{M}}=\frac{1}{M}\left[-(CD+G^2)\left(\frac{\partial\bar{t}}{\partial r}\right)^2-2(ED+FG)\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial t}\right)\right.$$

$$+(AD-F^2)\left(\frac{\partial\bar{t}}{\partial t}\right)^2+2(EG-CF)\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial u}\right) \qquad (2.10c)$$

$$\left.-(AC+E^2)\left(\frac{\partial\bar{t}}{\partial u}\right)^2-2(AG+EF)\left(\frac{\partial\bar{t}}{\partial t}\right)\left(\frac{\partial\bar{t}}{\partial u}\right)\right],$$

$$-\frac{(\bar{A}\bar{C}+\bar{E}^2)}{\bar{M}}=\frac{1}{M}\left[-(CD+G^2)\left(\frac{\partial\bar{u}}{\partial r}\right)^2-2(ED+FG)\left(\frac{\partial\bar{u}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial t}\right)\right.$$

$$+(AD-F^2)\left(\frac{\partial\bar{u}}{\partial t}\right)^2+2(EG-CF)\left(\frac{\partial\bar{u}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial u}\right) \qquad (2.10d)$$

$$\left.-(AC+E^2)\left(\frac{\partial\bar{u}}{\partial u}\right)^2-2(AG+EF)\left(\frac{\partial\bar{u}}{\partial t}\right)\left(\frac{\partial\bar{u}}{\partial u}\right)\right],$$

$$-\frac{(\bar{E}\bar{D}+\bar{F}\bar{G})}{\bar{M}}=\frac{1}{M}\left[-(CD+G^2)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial r}\right)+(ED-FG)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial t}\right)\right.$$

$$+(AD-F^2)\left(\frac{\partial\bar{r}}{\partial t}\right)\left(\frac{\partial\bar{t}}{\partial t}\right)+(EG-CF)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial u}\right)$$

$$-(AC+E^2)\left(\frac{\partial\bar{r}}{\partial u}\right)\left(\frac{\partial\bar{t}}{\partial u}\right)-(AG+EF)\left(\frac{\partial\bar{r}}{\partial t}\right)\left(\frac{\partial\bar{t}}{\partial u}\right) \qquad (2.10e)$$

$$+(ED-FG)\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{r}}{\partial t}\right)+(EG-CF)\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{r}}{\partial u}\right)$$

$$\left.-(AG+EF)\left(\frac{\partial\bar{t}}{\partial t}\right)\left(\frac{\partial\bar{r}}{\partial u}\right)\right],$$

$$\frac{(\bar{E}\bar{G}-\bar{C}\bar{F})}{\bar{M}}=\frac{1}{M}\left[-(CD+G^2)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial r}\right)-(ED+FG)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial t}\right)\right.$$

$$+(AD-F^2)\left(\frac{\partial\bar{r}}{\partial t}\right)\left(\frac{\partial\bar{u}}{\partial t}\right)+(EG-CF)\left(\frac{\partial\bar{r}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial u}\right)$$

$$-(AC+E^2)\left(\frac{\partial \bar{r}}{\partial u}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)-(AG+EF)\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{u}}{\partial u}\right) \quad (2.10f)$$

$$-(ED+FG)\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial t}\right)+(EG-CF)\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)$$

$$-(AG+EF)\left(\frac{\partial \bar{u}}{\partial t}\right)\left(\frac{l\bar{r}}{\partial u}\right)\Bigg],$$

$$-\frac{(\bar{A}\bar{G}+\bar{E}\bar{F})}{\bar{M}}=\frac{1}{M}\Bigg[-(CD+G^2)\left(\frac{\partial \bar{t}}{\partial r}\right)\left(\frac{\partial \bar{u}}{\partial r}\right)-(ED+FG)\left(\frac{\partial \bar{t}}{\partial r}\right)\left(\frac{l\bar{u}}{\partial t}\right)$$

$$+(AD-F^2)\left(\frac{\partial \bar{t}}{\partial t}\right)\left(\frac{\partial \bar{u}}{\partial t}\right)+(EG-CF)\left(\frac{\partial \bar{t}}{\partial r}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)$$

$$-(AC+E^2)\left(\frac{\partial \bar{t}}{\partial u}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)-(AG+EF)\left(\frac{\partial \bar{t}}{\partial t}\right)\left(\frac{\partial \bar{u}}{\partial u}\right) \quad (2.10g)$$

$$-(ED+FG)\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{t}}{\partial t}\right)+(EG-CF)\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{t}}{\partial u}\right)$$

$$-(AG+EF)\left(\frac{\partial \bar{u}}{\partial t}\right)\left(\frac{\partial \bar{t}}{\partial u}\right)\Bigg],$$

where

$$\bar{M}\equiv\bar{A}\bar{C}\bar{D}+2\bar{E}\bar{F}\bar{G}+\overline{A}\overline{G}^2+\bar{D}\overline{E}^2-\bar{C}\overline{F}^2 \quad (2.10h)$$

The condition $\bar{g}^{14}=0$, which is equivalent to ($\bar{E}\ \bar{D}+\ \bar{F}\ \bar{G}$) = 0, (2.10e) becomes

$$-(CD+G^2)\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{t}}{\partial r}\right)-(ED+FG)\left[\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{t}}{\partial t}\right)+\left(\frac{\partial \bar{t}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial t}\right)\right]$$

$$+(AD-F^2)\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{t}}{\partial t}\right)+(EG-CF)\left[\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{t}}{\partial u}\right)+\left(\frac{\partial \bar{t}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)\right]$$

$$-(AC+E^2)\left(\frac{\partial \bar{r}}{\partial u}\right)\left(\frac{\partial \bar{t}}{\partial u}\right)-(AG+EF)\left[\left[\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{t}}{\partial u}\right)+\left(\frac{\partial \bar{t}}{\partial t}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)\right]=0 \quad (2.11)$$

as a results of (2.10).

When $(CD + G^2) \neq 0$ and $(EG - CF) \neq 0$, if we put $\bar{r} = r$, then (2.11) becomes

$$-(CD+G^2)\left(\frac{\partial \bar{t}}{\partial r}\right)-(ED+FG)\left(\frac{\partial \bar{t}}{\partial t}\right)+(EG-CF)\left(\frac{\partial \bar{t}}{du}\right)=0, \qquad (2.12a)$$

this equation admits a non-constant solution $\bar{t}$, such that $\left(\frac{\partial \bar{t}}{\partial t}\right) \neq 0$.

And when $(AD - F^2) \neq 0$ and $(AG + EF) \neq 0$, if we put $\bar{t} = t$, then

$$(AD-F^2)\left(\frac{\partial \bar{r}}{\partial t}\right)-(ED+FG)\left(\frac{\partial \bar{r}}{\partial r}\right)-(AG+EF)\left(\frac{\partial \bar{r}}{\partial u}\right)=0. \qquad (2.12b)$$

This equation admits a non-constant solutions $\bar{r}$, such that $\left(\frac{\partial \bar{r}}{\partial r}\right) \neq 0$.

Obviously, T thus determined is non-singular and satisfies the equation (2.11).

Equations (2.12a) and (2.12b) implies $(ED + FG) = 0$.

Similarly, the condition $\bar{g}^{15} = 0$ which is equivalent to $(\bar{E}\bar{G} - \bar{C}\,\bar{F}) = 0$, equation (2.10f) becomes

$$-(CD+G^2)\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{u}}{\partial r}\right)-(ED+FG)\left[\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{u}}{\partial t}\right)+\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial t}\right)\right]$$

$$+(AD-F^2)\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{u}}{\partial t}\right)+(EG-CF)\left[\left(\frac{\partial \bar{r}}{\partial r}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)+\left(\frac{\partial \bar{u}}{\partial r}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)\right]$$

$$-(AC+E^2)\left(\frac{\partial \bar{r}}{\partial u}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)-(AG+EF)\left[\left(\frac{\partial \bar{r}}{\partial t}\right)\left(\frac{\partial \bar{u}}{\partial u}\right)+\left(\frac{\partial \bar{u}}{\partial t}\right)\left(\frac{\partial \bar{r}}{\partial u}\right)\right]=0. \qquad (2.13)$$

When $(CD + G^2) \neq 0$ and $(EG - CF) \neq 0$, if we put $\bar{r} = r$, then (2.13) becomes

$$-(CD+G^2)\left(\frac{\partial \bar{u}}{\partial r}\right)-(ED+FG)\left(\frac{\partial \bar{u}}{\partial t}\right)+(EG-CF)\left(\frac{\partial \bar{u}}{du}\right)=0, \qquad (2.14a)$$

this equation admits a non-constant solution $\bar{u}$, such that $\left(\frac{\partial \bar{u}}{\partial u}\right) \neq 0$.

And when $(AC + E^2) \neq 0$ and $(AG + EF) \neq 0$, if we put $\bar{u} = u$, then

$$-(AC-E^2)\left(\frac{\partial\bar{r}}{\partial u}\right)+(EG-CF)\left(\frac{\partial\bar{r}}{\partial r}\right)-(AG+EF)\left(\frac{\partial\bar{r}}{\partial t}\right)=0 \qquad (2.14b)$$

This equation admits a non-constant solutions $\bar{r}$, such that $\left(\frac{\partial\bar{r}}{\partial r}\right)\neq 0$.

Equations (2.14a) and (2.14b) implies $(EG-CF)=0$.

Similarly, the condition $\bar{g}^{45}=0$ which is equivalent to $(\bar{A}\ \bar{G}-\bar{E}\bar{F})=0$, equation (2.10g) becomes

$$-(CD+G^2)\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial r}\right)-(ED+FG)\left[\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial t}\right)+\left(\frac{\partial\bar{u}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial t}\right)\right]$$

$$+(AD-F^2)\left(\frac{\partial\bar{t}}{\partial t}\right)\left(\frac{\partial\bar{u}}{\partial t}\right)+(EG-CF)\left[\left(\frac{\partial\bar{t}}{\partial r}\right)\left(\frac{\partial\bar{u}}{\partial u}\right)+\left(\frac{\partial\bar{u}}{\partial r}\right)\left(\frac{\partial\bar{t}}{\partial u}\right)\right] \qquad (2.15)$$

$$-(AC+E^2)\left(\frac{\partial\bar{t}}{\partial u}\right)\left(\frac{\partial\bar{u}}{\partial u}\right)-(AG+EF)\left[\left(\frac{\partial\bar{t}}{\partial t}\right)\left(\frac{\partial\bar{u}}{\partial u}\right)+\left(\frac{\partial\bar{u}}{\partial t}\right)\left(\frac{\partial\bar{t}}{\partial u}\right)\right]=0$$

When $(AD-F^2)\neq 0$ and $(ED+FG)\neq 0$, if we put $\bar{t}=t$ then (2.15) becomes

$$(AD-F^2)\left(\frac{\partial\bar{u}}{\partial t}\right)-(ED+FG)\left(\frac{\partial\bar{u}}{\partial r}\right)-(AG+EF)\left(\frac{\partial\bar{u}}{\partial u}\right)=0 \qquad (2.16a)$$

such that $\left(\frac{\partial\bar{u}}{\partial u}\right)\neq 0$

And when $(AC+E^2)\neq 0$ and $(EG-CF)\neq 0$, if we put $\bar{u}=u$ then

$$-(AC+E^2)\left(\frac{\partial\bar{t}}{\partial u}\right)+(EG-CF)\left(\frac{\partial\bar{t}}{\partial r}\right)-(AG+EF)\left(\frac{\partial\bar{t}}{\partial t}\right)=0 \qquad (2.16b)$$

such that $\left(\frac{\partial\bar{t}}{\partial t}\right)\neq 0$.

From above implies $(AG+EF)=0$.

Using above in equation (2.6) we get equation (2.2), which is called the s.s. coordinate system in a narrow sense in V_5.

Thus the theorem is proved.

3. THE CURVATURE TENSOR OF S.S. SPACE-TIME V_5 IN A NARROW SENSE

Theorem 3.1: The non-vanishing independent components of the curvature tensor K_{ijkl} of the s.s. space-time in a narrow sense of V_5 with the metric (2.2) are

$$\left.\begin{aligned}
&K_{1212} = \frac{K_{1313}}{\sin^2\theta} = f_1, K_{1414} = f_2, K_{1515} = f_3,\\
&K_{1224} = \frac{K_{1334}}{\sin^2\theta} = f_4, K_{1225} = \frac{K_{1335}}{\sin^2\theta} = f_5, K_{1415} = f_6,\\
&K_{1445} = f_7, K_{1545} = f_8, K_{2424} = \frac{K_{3434}}{\sin^2\theta} = f_9,\\
&K_{2525} = \frac{K_{3535}}{\sin^2\theta} = f_{10}, K_{2425} = \frac{K_{3435}}{\sin^2\theta} = f_{11}, K_{2323} = f_{12},\\
&K_{4545} = f_{13}
\end{aligned}\right\} \tag{3.1}$$

where $f_1, f_2, \cdots f_{13}$ are functions of r, t, u.

Proof: From the line element equation (2.2), we have

$$g_{11} = -A,\ g_{22} = -B,\ g_{33} = -B\sin^2\theta,\ g_{44} = C,\ g_{55} = -D \tag{3.2}$$

then

$$g = \det(g_{ij}) = MB^2\sin^2\theta, \text{ where } M \equiv ACD. \tag{3.3}$$

And

$$\left.\begin{aligned}
&g^{11} = -\frac{1}{A}, g^{22} = -\frac{1}{B}, g^{33} = -\frac{1}{B\sin^2\theta}, g^{44} = \frac{1}{C}, g^{55} = -\frac{1}{D}\\
&\text{the remaining } \ g^{ij} = 0
\end{aligned}\right\} \tag{3.4}$$

Using $\begin{Bmatrix} k \\ ij \end{Bmatrix} = g^{kl}[ij, l]$, the non-vanishing independent components of second kind Christoffel symbol obtained from (2.2) are

$$\left.\begin{array}{l}
\left\{ {1 \atop 11} \right\} = \dfrac{A'}{2A},\ \left\{ {1 \atop 14} \right\} = \dfrac{\dot{A}}{2A},\ \left\{ {1 \atop 15} \right\} = \dfrac{\hat{A}}{2A},\ \left\{ {1 \atop 22} \right\} = -\dfrac{B'}{2A}, \\
\left\{ {1 \atop 33} \right\} = -\dfrac{B'\sin^2\theta}{2A}, \left\{ {1 \atop 44} \right\} = \dfrac{C'}{2A},\ \left\{ {1 \atop 55} \right\} = -\dfrac{D'}{2A}, \\
\left\{ {2 \atop 12} \right\} = \left\{ {3 \atop 13} \right\} = \dfrac{B'}{2B}, \left\{ {2 \atop 24} \right\} = \left\{ {3 \atop 34} \right\} = \dfrac{\dot{B}}{2B}, \\
\left\{ {2 \atop 25} \right\} = \left\{ {3 \atop 35} \right\} = \dfrac{\hat{B}}{2B}, \left\{ {3 \atop 23} \right\} = \cot\theta,\ \left\{ {2 \atop 33} \right\} = -\sin\theta\cos\theta, \\
\left\{ {4 \atop 11} \right\} = \dfrac{\dot{A}}{2C}, \left\{ {4 \atop 14} \right\} = \dfrac{C'}{2C}, \left\{ {4 \atop 22} \right\} = \dfrac{\dot{B}}{2C}, \left\{ {4 \atop 33} \right\} = \dfrac{\dot{B}\sin^2\theta}{2C}, \\
\left\{ {4 \atop 44} \right\} = \dfrac{\dot{C}}{2C}, \left\{ {4 \atop 45} \right\} = \dfrac{\hat{C}}{2C}, \left\{ {4 \atop 55} \right\} = \dfrac{\dot{D}}{2C} \\
\left\{ {5 \atop 11} \right\} = -\dfrac{\hat{A}}{2D},\ \left\{ {5 \atop 15} \right\} = \dfrac{D'}{2D},\ \left\{ {5 \atop 22} \right\} = -\dfrac{\hat{B}}{2D}, \\
\left\{ {5 \atop 33} \right\} = -\dfrac{\hat{B}\sin^2\theta}{2D}, \left\{ {5 \atop 44} \right\} = \dfrac{\hat{C}}{2D},\ \left\{ {5 \atop 45} \right\} = \dfrac{\dot{D}}{2D}, \\
\left\{ {5 \atop 55} \right\} = \dfrac{\hat{D}}{2D}
\end{array}\right\} \tag{3.5}$$

here a prime, a dot and a cap mean derivative with respect to r, t and u respectively. Using the relation from (3.2) to (3.5), the non-vanishing independent components of the curvature tensor K_{ijkl} given by

$$\begin{aligned}
K_{ijkl} = g_{ia}K^{a}_{jkl} &= \frac{1}{2}\left(\frac{\partial^2 g_{jk}}{\partial x^i \partial x^l} + \frac{\partial^2 g_{il}}{\partial x^j \partial x^k} - \frac{\partial^2 g_{ik}}{\partial x^j \partial x^l} - \frac{\partial^2 g_{jl}}{\partial x^i \partial x^k} \right) \\
&+ \left\{ {a \atop jk} \right\}[il, a] - [ik, a]\left\{ {a \atop jl} \right\}
\end{aligned}$$

are

$$K_{1212} = \frac{K_{1313}}{\sin^2\theta} = \frac{B''}{2} - \frac{B'^2}{4B} - \frac{1}{4}\left(\frac{A'B'}{A} + \frac{\dot{A}\dot{B}}{C} - \frac{\hat{A}\hat{B}}{D} \right) = f_1 \tag{3.6}$$

$$K_{1414} = \left(\frac{\ddot{A} - C''}{2} \right) - \frac{1}{4}\left[\frac{\dot{A}^2}{A} - \frac{C'^2}{C} - \frac{A'C'}{A} + \frac{\dot{A}\dot{C}}{C} + \frac{\hat{A}\hat{C}}{D} \right] = f_2 \tag{3.7}$$

$$K_{1515}=\left(\frac{\hat{\hat{A}}+D''}{2}\right)-\frac{1}{4}\left[\frac{\hat{A}^2}{A}+\frac{D'^2}{D}+\frac{A'D'}{A}+\frac{\dot{A}\dot{D}}{C}+\frac{\hat{A}\hat{D}}{D}\right]=f_3 \tag{3.8}$$

$$K_{1224}=\frac{K_{1334}}{\sin^2\theta}=-\frac{\dot{B}'}{2}+\frac{1}{4}\left[\frac{\dot{A}B'}{A}+\frac{\dot{B}C'}{C}+\frac{B'\dot{B}}{B}\right]=f_4 \tag{3.9}$$

$$K_{1225}=\frac{K_{1335}}{\sin^2\theta}=-\frac{\hat{B}'}{2}+\frac{1}{4}\left[\frac{\hat{A}B'}{A}+\frac{\hat{B}D'}{D}+\frac{B'\hat{B}}{B}\right]=f_5 \tag{3.10}$$

$$K_{1415}=\frac{\hat{\dot{A}}}{2}-\frac{1}{4}\left[\frac{\dot{A}\hat{A}}{A}+\frac{\dot{A}\hat{C}}{C}+\frac{\hat{A}\dot{D}}{D}\right]=f_6 \tag{3.11}$$

$$K_{1445}=\frac{\hat{C}'}{2}-\frac{1}{4}\left[\frac{\hat{A}C'}{A}+\frac{\hat{C}D'}{D}+\frac{C'\hat{C}}{C}\right]=f_7 \tag{3.12}$$

$$K_{1545}=\frac{\dot{D}'}{2}-\frac{1}{4}\left[\frac{\dot{D}D'}{D}+\frac{\dot{A}D'}{A}+\frac{C'\dot{D}}{c}\right]=f_8 \tag{3.13}$$

$$K_{2424}=\frac{K_{3434}}{\sin^2\theta}=\frac{\ddot{B}}{2}-\frac{1}{4}\left[\frac{\dot{B}^2}{B}+\frac{B'C'}{A}+\frac{\dot{B}\dot{C}}{C}+\frac{\hat{B}\hat{C}}{D}\right]=f_9 \tag{3.14}$$

$$K_{2525}=\frac{K_{3535}}{\sin^2\theta}=\frac{\hat{\hat{B}}}{2}-\frac{1}{4}\left[\frac{\hat{B}^2}{B}-\frac{B'D'}{A}+\frac{\dot{B}\dot{D}}{C}+\frac{\hat{B}\hat{D}}{D}\right]=f_{10} \tag{3.15}$$

$$K_{2425}=\frac{K_{3435}}{\sin^2\theta}=\frac{\hat{\dot{B}}}{2}-\frac{1}{4}\left[\frac{\dot{B}\hat{B}}{B}+\frac{\dot{B}\hat{C}}{C}+\frac{\hat{B}\dot{D}}{D}\right]=f_{11} \tag{3.16}$$

$$K_{2323}=\sin^2\theta\left[-B+\frac{1}{4}\left(\frac{B'^2}{A}-\frac{\dot{B}^2}{C}+\frac{\hat{B}^2}{D}\right)\right]=\sin^2\theta\,.f_{12} \tag{3.17}$$

$$K_{4545}=\left(\frac{\ddot{D}-\hat{\hat{C}}}{2}\right)+\frac{1}{4}\left[\frac{\hat{C}^2}{C}-\frac{\dot{D}^2}{D}-\frac{C'D'}{A}-\frac{\dot{C}\dot{D}}{C}+\frac{\hat{C}\hat{D}}{D}\right]=f_{13} \tag{3.18}$$

Thus, the above equations prove the theorem.

Remark 3.1: Here we observed that there exist thirteen non-vanishing independent components of the curvature tensor K_{ijkl} of the s.s. space time V_5 in a narrow sense with the metric (2.2) as compared to five non-vanishing independent components of the s.s. space time V_4 obtained by Takeno.

4. THE SCALAR CURVATURE FOR THE S.S. SPACE TIME V_5 IN A NARROW SENSE

Theorem 4.1. The Scalar Curvature $K = g^{ij}K_{ij}$ of the s.s. space time V_5 in narrow sense with the metric (2.2) are given by

$$K = -\frac{4}{AB}f_1 + \frac{2}{AC}f_2 - \frac{2}{AD}f_3 + \frac{4}{BC}f_9 - \frac{4}{BD}f_{10} - \frac{2}{B^2}f_{12} + \frac{2}{CD}f_{13} \tag{4.1}$$

Proof: Consider the Ricci tensor

$$K_{ij} = K_{kij}^{\cdot\cdot\cdot h}, (h = k)$$

$$= \frac{\partial}{\partial x^j}\begin{Bmatrix} h \\ ik \end{Bmatrix} - \frac{\partial}{\partial x^k}\begin{Bmatrix} h \\ ij \end{Bmatrix} + \begin{Bmatrix} h \\ lj \end{Bmatrix}\begin{Bmatrix} l \\ ik \end{Bmatrix} - \begin{Bmatrix} h \\ lk \end{Bmatrix}\begin{Bmatrix} l \\ ij \end{Bmatrix}$$

Now using equations from (3.5) to (3.18), the non-vanishing components of K_{ij} are

$$K_{11} = \frac{2}{B}f_1 - \frac{1}{C}f_2 + \frac{1}{D}f_3 \tag{4.2}$$

$$K_{22} = \frac{K_{33}}{\sin^2\theta} = \frac{1}{A}f_1 - \frac{1}{C}f_9 + \frac{1}{D}f_{10} + \frac{1}{B}f_{12} \tag{4.3}$$

$$K_{44} = \frac{1}{A}f_2 + \frac{1}{D}f_{13} + \frac{2}{B}f_9 \tag{4.4}$$

$$K_{55} = \frac{1}{A}f_3 - \frac{1}{C}f_{13} + \frac{2}{B}f_{10} \tag{4.5}$$

$$K_{14} = \frac{1}{D}f_8 - \frac{2}{B}f_4 \tag{4.6}$$

$$K_{15} = \frac{1}{C}f_7 - \frac{2}{B}f_5 \tag{4.7}$$

$$K_{45} = \frac{1}{A}f_6 - \frac{2}{B}f_{11} \tag{4.8}$$

Using (3.4) and equations from (4.2) to (4.8), the scalar curvature $K \equiv g^{ij}K_{ij}$ is

$$K = g^{11}K_{11} + g^{22}K_{22} + g^{33}K_{33} + g^{44}K_{44}$$
$$+ g^{55}K_{55} + 2g^{14}K_{14} + 2g^{15}K_{15} + 2g^{45}K_{45}$$

i.e.

$$K = -\frac{4}{AB}f_1 + \frac{2}{AC}f_2 - \frac{2}{AD}f_3 + \frac{4}{BC}f_9 - \frac{4}{BD}f_{10} - \frac{2}{B^2}f_{12} + \frac{2}{CD}f_{13}.$$

Remark 4.1: Here we obtained an analogous result with the work of Takeno. We further observed that if the space-time is s.s. then the scalar curvature K of the s.s. space-time is independent of θ and ϕ and function of x^i, $i \neq 2, 3$.

5. THE SCALAR CURVATURE $K = K_i^{\cdot i}$

Theorem 5.1: The scalar curvature $K = K_i^i$ of the s.s. space-time V_5 in a narrow sense with the metric (2.2) are given by

$$K = -2(2\alpha_1 + 2\alpha_2 + 2\alpha_6 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_{19})$$

where $\alpha_1, \alpha_2, \cdots, \alpha_{19}$ are the mixed components of $K_{ij}^{\cdot\cdot kl}$ of the space time V_5 in a narrow sense.

Proof: Consider $K_{ij}^{\cdot\cdot kl} = g^{k\alpha}g^{l\beta}K_{ij\alpha\beta}$.

Using the equation (3.4) and equations from (3.6) to (3.18), the non-vanishing mixed components of $K_{ij}^{\cdot\cdot kl}$ are given by

$$\alpha_1 = K_{12}^{\cdot\cdot 12} = K_{13}^{\cdot\cdot 13} = \frac{1}{AB}f_1 \quad (5.1)$$

$$\alpha_2 = K_{24}^{\cdot\cdot 24} = K_{34}^{\cdot\cdot 34} = -\frac{1}{BC}f_9 \quad (5.2)$$

$$\alpha_3 = K_{14}^{\cdot\cdot 14} = -\frac{1}{AC}f_2 \quad (5.3)$$

$$\alpha_4 = K_{15}^{\cdot\cdot 15} = \frac{1}{AD}f_3 \quad (5.4)$$

$$\alpha_5 = K_{23}^{\cdot\cdot 23} = \frac{1}{B^2}f_{12} \quad (5.5)$$

$$\alpha_6 = K_{25}^{\cdot\cdot 25} = K_{35}^{\cdot\cdot 35} = \frac{1}{BD}f_{10} \quad (5.6)$$

$$\left.\begin{aligned} &\alpha_7 = K_{12}^{\cdot\cdot 24} = K_{13}^{\cdot\cdot 34} = -\frac{1}{BC}f_4 \\ &\alpha_8 = K_{24}^{\cdot\cdot 12} = K_{34}^{\cdot\cdot 13} = \frac{1}{AB}f_4 \\ &\text{i.e. } \alpha_7 = -\frac{A}{C}\alpha_8 \end{aligned}\right\} \quad (5.7)$$

$$\left.\begin{aligned} &\alpha_9 = K_{12}^{\cdot\cdot 25} = K_{13}^{\cdot\cdot 35} = \frac{1}{BD} f_5 \\ &\alpha_{10} = K_{25}^{\cdot\cdot 12} = K_{35}^{\cdot\cdot 13} = \frac{1}{AB} f_5 \\ &\text{i.e. } \alpha_9 = \frac{A}{D}\alpha_{10} \end{aligned}\right\} \tag{5.8}$$

$$\left.\begin{aligned} &\alpha_{11} = K_{24}^{\cdot\cdot 25} = K_{34}^{\cdot\cdot 35} = \frac{1}{BD} f_{11} \\ &\alpha_{12} = K_{25}^{\cdot\cdot 24} = K_{35}^{\cdot\cdot 34} = -\frac{1}{BC} f_{11} \\ &\text{i.e. } \alpha_{11} = -\frac{C}{D}\alpha_{12} \end{aligned}\right\} \tag{5.9}$$

$$\left.\begin{aligned} &\alpha_{13} = K_{14}^{\cdot\cdot 15} = \frac{1}{AD} f_6 \\ &\alpha_{14} = K_{15}^{\cdot\cdot 14} = -\frac{1}{AC} f_6 \\ &\text{i.e. } \alpha_{13} = -\frac{C}{D}\alpha_{14} \end{aligned}\right\} \tag{5.10}$$

$$\left.\begin{aligned} &\alpha_{15} = K_{15}^{\cdot\cdot 45} = -\frac{1}{CD} f_8 \\ &\alpha_{16} = K_{45}^{\cdot\cdot 15} = \frac{1}{AD} f_8 \\ &\text{i.e. } \alpha_{15} = -\frac{A}{C}\alpha_{16} \end{aligned}\right\} \tag{5.11}$$

$$\left.\begin{aligned} &\alpha_{17} = K_{14}^{\cdot\cdot 45} = -\frac{1}{CD} f_7 \\ &\alpha_{18} = K_{45}^{\cdot\cdot 14} = -\frac{1}{AC} f_7 \\ &\text{i.e. } \alpha_{17} = \frac{A}{D}\alpha_{18} \end{aligned}\right\} \tag{5.12}$$

and

$$\alpha_{19} = K_{45}^{\cdot\cdot 45} = -\frac{1}{CD} f_{13} \tag{5.13}$$

Using above, the Ricci tensor $K_i^i = g^{ij}K_{ij}$ can be expressed in terms of $\alpha_1, \alpha_2, \cdots, \alpha_{19}$ as

$$K_1^{\cdot 1} = -(2\alpha_1 + \alpha_3 + \alpha_4) \quad (5.14)$$

$$K_2^{\cdot 2} = K_3^{\cdot 3} = -(\alpha_1 + \alpha_2 + \alpha_5 + \alpha_6) \quad (5.15)$$

$$K_4^{\cdot 4} = -(2\alpha_2 + \alpha_3 + \alpha_{19}) \quad (5.16)$$

$$K_5^{\cdot 5} = -(2\alpha_6 + \alpha_4 + \alpha_{19}) \quad (5.17)$$

$$K_1^{\cdot 4} = (2\alpha_7 - \alpha_{15}) \quad (5.18)$$

$$K_4^{\cdot 1} = (2\alpha_8 - \alpha_{16}) \quad (5.19)$$

$$K_{\cdot 1}^{\cdot 5} = (2\alpha_9 + \alpha_{17}) \quad (5.20)$$

$$K_{\cdot 5}^{\cdot 1} = (2\alpha_{10} + \alpha_{18}) \quad (5.21)$$

$$K_{\cdot 4}^{\cdot 5} = -(2\alpha_{11} + \alpha_{13}) \quad (5.22)$$

$$K_5^{\cdot 4} = -(2\alpha_{12} + \alpha_{14}) \quad (5.23)$$

Consider the scalar curvature $K = K_i^{\cdot i}$ i.e. $K = K_1^{\cdot 1} + K_2^{\cdot 2} + K_3^{\cdot 3} + K_4^{\cdot 4} + K_5^{\cdot 5}$. Now substituting the values of $K_1^{\cdot 1}, K_2^{\cdot 2}, K_3^{\cdot 3}, K_4^{\cdot 4}, K_5^{\cdot 5}$ from the equations (5.14) to (5.23) we obtain the Scalar Curvature

$$K = -2(2\alpha_1 + 2\alpha_2 + 2\alpha_6 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_{19}).$$

6. CONCLUSION

The curvature tensor and scalar curvature of the s.s. space time V_5 in a narrow sense with the metric (2.2) are independent θ and ϕ i.e. (x^2, x^3) and is a function of r, t, u i.e. (x^1, x^4, x^5). This result can be extended to the spherically symmetric space-time V_n.

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