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Linear Memory Theorem with Suspension Bridge on Global Attractors

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Abstract

The model with linear memory arise in the case of a generalized Kirchhoot viscoelastic bar, where a bending-moment relation with memory is considered. In this paper, after defining a new variable we discuss the existence of the global attractors for the model (1.1) with non-smooth semi-linear term u^+ and linear memory using the new semigroup approach.

1. Introduction

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary Γ, we are concerned with the following equations associated with the oscillation of the suspension bridge:

$$\begin{cases} u_{tt} + \delta u_t + \phi(0)\Delta^2 u + \int_0^\infty \phi'(s)\Delta^2 u(t-s)ds + ku^+ + g(u) = h, \ in\ \Omega \times \mathbb{R}^+, \\ u(x,t) = \Delta u(x,t) = 0, \quad x \in \Gamma, \ t \in \mathbb{R}, \\ u(x,t) = u_0(x,t), \quad x \in \Omega, \ t \le 0, \end{cases} \tag{1.1}$$

where $\phi'(s)$ denotes the memory kernel, $\phi(0)$, $\phi(\infty) > 0$ and $\phi'(s) \le 0$ for $\forall\ s \in \mathbb{R}^+$. $\delta > 0$ is the viscous damping and k indicates the spring constant. If $\delta = k \equiv 0$, then (1.1) is attributed to a general viscous elastic beam model when the bending-moment relation is considered[5]. In addition, if $\phi' \equiv 0$, it is obvious that (1.1) reduces to the suspension bridge equations, where g represents some displacement-dependent body force density and the suspension bridge equations were presented by Lazer and McKenna as the new problems in fields of nonlinear analysis[1], they were obtained by a onesided Hooke's law. If $k = 0$, there are many classical results to study existence of global attractors, please refer to [2,4,5]. However, once $k > 0$,

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due to the non-smooth semi-linear term u^+ appears in the equations and $\frac{\partial(u^+)}{\partial t} \neq \left(\frac{\partial u}{\partial t}\right)^+$, there are some difficulties in the process of proving the existence of global attractors. In this paper, after defining a new variable we obtain the existence of global attractors for equation (1.1) using the new semigroup methods. Concerning else literatures about attractors please the reader to see [3, 6-10], and therein references. Analogous to discuss of [4], we define

$$\eta^t(x, s) = u(x, t) - u(x, t - s). \tag{1.2}$$

We set for simplicity $\mu(s) = -\phi'(s)$ and $\phi(\infty) = 1$. In view of (1.2), adding and subtracting the term $\Delta^2 u$, equation (1.1) transform into the system

$$\begin{cases} u_{tt} + \delta u_t + \Delta^2 u + \int_0^\infty \mu(s)\Delta^2 \eta^t(s)ds + ku^+ + g(u) = h, \\ \eta_t = -\eta_s + u_t, \end{cases} \tag{1.3}$$

where the second equation is obtained by dierentiating (1.2). Initial-boundary value conditions are then given by

$$\begin{cases} u(x,t) = \Delta u(x,t) = 0, \ x \in \Gamma, \ t \geq 0, \\ \eta^t(x,s) = \Delta\eta^t(x,s) = 0, \ x \in \Gamma, \ t \geq 0, \ s \in \mathbb{R}^+, \\ u(x,0) = u_1(x), \ x \in \Omega, \\ u_t(x,0) = u_2(x), \ x \in \Omega, \\ \eta^0(x,s) = \eta_0(x,s), \ (x,s) \in \Omega \times \mathbb{R}^+, \end{cases} \tag{1.4}$$

here

$$\begin{cases} u_1(x) = u_0(x,0), \\ u_2(x) = \partial_t u_0(x,t)|_{t=0}, \\ \eta_0(x,s) = u_0(x,0) - u_0(x,-s). \end{cases}$$

Assume that the nonlinear function $g \in C^2(\mathbb{R}, \mathbb{R})$ satisfying the following conditions:

(g1) $\liminf_{|s|\to\infty} \frac{G(s)}{s^2} \geq 0, \quad here \ G(s) = \int_0^s g(\tau)d\tau;$

(g2) $\limsup_{|s|\to\infty} \frac{|g'(s)|}{|s|^{\gamma}} = 0, \quad \forall\, 0 \le \gamma < \infty;$

(g3) There exists $C_1 > 0$, such that $\liminf_{|s|\to\infty} \frac{sg(s) - C_1 G(s)}{s^2} \ge 0.$

For simplicity, we denote $\phi(u) = \int_\Omega G(u(x))dx.$

The memory kernel μ is required to satisfy the following assumptions:

(h1) $\mu \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+)$, $\mu(s) \ge 0$, $\mu'(s) \le 0$, $\forall\, s \in \mathbb{R}^+$;

(h2) $\int_0^\infty \mu(s)ds = M > 0;$

(h3) $\mu'(s) + \alpha\mu(s) \le 0$, $\forall$ s $\in \mathbb{R}^+$, for some $\alpha > 0$.

We write $H = L^2(\Omega)$, $V = H_0^2(\Omega)$, the scalar product and the norm on H and V are denoted by $(\cdot,\ \cdot)$, $|\cdot|$ and $((\cdot,\ \cdot))$, $\|\cdot\|$ respectively, where

$$(u,v) = \int_\Omega u(x)v(x)dx, \quad ((u,v)) = \int_\Omega \Delta u(x)\Delta v(x)dx.$$

Define $D(A) = \{v \in V, Av \in H\}$, here $A = \Delta^2$. For the operator A, we assume that

$$A: \begin{array}{l} V \to V^*, \\ D(A) \to H \end{array}$$

are isomorphism, and there exists $\alpha > 0$ such that

$$\langle Au, u\rangle \ge \alpha\|u\|^2, \ \forall\, u \in V,$$

here $\langle\cdot\ ,\ \cdot\rangle$ denotes the dual inner product. We also define the power A^s of A for $s \in \mathbb{R}$ which operate on the spaces $D(A^s)$, and we write $V_{2s} = D(A^s)$, $s \in \mathbb{R}$. This is a Hilbert space for the scalar product and the norm as follows

$$(u,v)_{2s} = (A^s u, A^s v)_H,\ \|u\|_{2s} = ((u,u)_{2s})^{\frac{1}{2}},\ \forall\, u, v \in D(A^s),$$

and A^r is an isomorphism from $D(A^s)$ onto $D(A^{s-r})$, $\forall\, s, r \in \mathbb{R}$. It is clearly that $D(A^0) = H, D\left(A^{\frac{1}{2}}\right) = V, D\left(A^{-\frac{1}{2}}\right) = V^*$ and $D(A) \subset H = H^* \subset V \subset V^*$, here H^*, V^*

are the dual of H, V respectively, and each space is dense in the following one and the injections are continuous.

Let λ_1 denote the first eigenvalue of $A^{\frac{1}{2}}$, clearly, λ_1^2 is the first eigenvalue of A, namely,

$$\lambda_1^2 = \inf_{v \in V, v \neq 0} \frac{\| v \|^2}{| v |^2}.$$

In view of (h1), let $L^2_\mu(\mathbb{R}^+, H_0^2)$ be the Hilbert space of H_0^2-valued functions on $\mathbb{R}^+$, endowed with the following inner product

$$(\varphi, \psi)_{\mu,V} = \int_0^\infty \mu(s)(\Delta\varphi(s), \Delta\psi(s))ds$$

and

$$|\varphi|^2_{\mu,V} = (\varphi, \varphi)_{\mu,V} = \int_0^\infty \mu(s) \| \varphi \|^2 \, ds.$$

We denote $\mathcal{H} = V \times H \times L^2_\mu(\mathbb{R}^+, V)$.

2. Preliminaries

Using the standard Faedo-Galerkin methods [3-4] it's easy to obtain the existence, uniqueness of solution for (1.1) and the continuous dependence to the initial value, so we omit it and only give the following theorem:

Theorem 2.1 [3-4] Let (g1)-(g3) and (h1)-(h3) hold. Then given any time interval I, problem (1.3)-(1.4) has a solution (u, u_t, η^t) in $I = [0, T]$ with initial data $(u_1, u_2, \eta_0) \in \mathcal{H}$, and the mapping

$$\{u_1, u_2, \eta_0\} \to \{u(t), u_t(t), \eta^t(s)\}$$

is continuous in $\mathcal{H}$.

Thus, it admits to define a C^0 semigroup

$$S(t) : \{u_1, u_2, \eta_0\} \to \{u(t), u_t(t), \eta^t(s)\}, \ t \in \mathbb{R}^+,$$

and it maps $\mathcal{H}$ into itself.

In order to prove our main results, we also need the following abstract results.

Definition 2.1[6] A C^0 semigroup $\{S(t)\}_{t\geq 0}$ in a Banach space X is said to satisfy condition (C), if for any $\varepsilon > 0$ and for any bounded set B of X, there exists $t(B) > 0$ and a finite dimensional subspace X_1 of X, such that $\{\| PS(t)x \| : t \geq t(B), x \in B\}$ is bounded and

$$\{ \|(I - P)\, S(t)x\| \} < \varepsilon, \quad \text{for} \quad t \geq t(B),\, x \in B,$$

where $P : X \to X_1$ is a bounded projector.

Theorem 2.2 [6] Let $\{S(t)\}_{t\geq 0}$ be *a* C^0 semigroup in a Hilbert space X. Then $\{S(t)\}_{t\geq 0}$ has a global attractor if and only if

(1) $\{S(t)\}_{t\geq 0}$ satisfies condition (C);

(2) there exists a bounded absorbing subset B of X.

3. Bounded absorbing set in $\mathcal{H}$

Choose $0 < \sigma < 1$, and take the scalar product of the first equation of (1.3) with $v = u_t + \sigma u$ in H, after computation, we conclude

$$\frac{1}{2}\frac{d}{dt}(\| u \|^2 + | v |^2) + \sigma \| u \|^2 + (\delta - \sigma) | v |^2 \tag{3.1}$$

$$-\sigma\,(\delta - \sigma)\,(u, v) + (\eta, v)_{\mu,V} + (ku^+, v) + (g(u), v) = (h, v).$$

Combining with (h1) and the second equation of (1.3), we have

$$(\eta, v)_{\mu,V} = (\eta, u_t)_{\mu,V} + \sigma(\eta, u)_{\mu,V} = (\eta, \eta_t + \eta_s)_{\mu,V} + \sigma(\eta, u)_{\mu,V}$$

$$= \frac{1}{2}\frac{d}{dt}| \eta |^2_{\mu,V} + (\eta, \eta_s)_{\mu,V} + \sigma(\eta, u)_{\mu,V},$$

by (h2) (h3) entails

$$(\eta, \eta_s)_{\mu,V} = -\frac{1}{2}\int_0^\infty \mu'(s)\, | \Delta \eta^t(s) |^2\, ds \tag{3.3}$$

$$\geq \frac{\alpha}{2} | \eta |^2_{\mu,V}$$

and by Young and Hölder inequalities, this lead to

$$\sigma(\eta, u)_{\mu,V} = \sigma\int_0^\infty \mu(s)\, (\Delta\eta(s),\, \Delta u)ds$$

$$\geq -\frac{\alpha}{4}\int_0^\infty \mu(s)\,|\Delta\eta^t(s)|^2\,ds - \frac{\sigma^2}{\alpha}\int_0^\infty \mu(s)\,|\Delta u|^2\,ds$$

$$\geq -\frac{\alpha}{4}|\eta|^2_{\mu,V} - \frac{M\sigma^2}{\alpha}\|u\|^2 . \tag{3.4}$$

Combining with (3.3), (3.4), from (3.2) we obtain

$$(\eta,v)_{\mu,V} \geq \frac{1}{2}\frac{d}{dt}|\eta|^2_{\mu,V} + \frac{\alpha}{4}|\eta|^2_{\mu,V} - \frac{M\sigma^2}{\alpha}\|u\|^2 . \tag{3.5}$$

In addition, it's easy to have

$$(ku^+,v) = \frac{1}{2}\frac{d}{dt}k\,|u^+|^2 + \sigma k\,|u^+|^2 . \tag{3.6}$$

Exploiting (g1), (g3), there exists constants K_1, $K_2 > 0$ only depending on u, such that

$$\phi(u) + \frac{1}{8}\|u\|^2 \geq -K_1, \quad \forall \quad u \in V, \tag{3.7}$$

$$(u, g(u)) - C_1\phi(u) + \frac{1}{4}\|u\|^2 \geq -K_2, \quad \forall \quad u \in V. \tag{3.8}$$

Therefore

$$(g(u),\, v) = \frac{d}{dt}\int_\Omega G(u)dx + \sigma\int_\Omega g(u)udx$$

$$\geq \frac{d}{dt}\phi(u) + \sigma(C_1\phi(u) - \frac{1}{4}\|u\|^2 - K_2). \tag{3.9}$$

Integrating with (3.5), (3.6) and (3.9), from (3.1) we get

$$\frac{1}{2}\frac{d}{dt}(\|u\|^2 + |v|^2 + 2\phi(u) + k\,|u^+|^2 + |\eta|^2_{\mu,V}) + (\delta-\sigma)|v|^2 - \sigma(\delta-\sigma)(u,v)$$

$$+\ \sigma\left(\frac{3}{4} - \frac{M\sigma}{\alpha}\right)\|u\|^2 + \frac{\alpha}{4}|\eta|^2_{\mu,V} + \sigma k\,|u^+|^2 + \sigma C_1\phi(u) - \sigma K_2$$

$$\leq \frac{|h|^2}{\delta} + \frac{\delta}{4}|v|^2 . \tag{3.10}$$

Take σ small enough, such that

$$\frac{3\delta}{4} - \sigma \geq \frac{\delta}{2}, \quad \frac{3}{4} - \frac{\sigma\delta}{\lambda_1^2} - \frac{M\sigma}{\alpha} \geq \frac{1}{2}.$$

Thus, we have

$$\begin{aligned}
&(\delta-\sigma)|v|^2 - \sigma(\delta-\sigma)(u,v) + \sigma\left(\frac{3}{4} - \frac{M\sigma}{\alpha}\right)\|u\|^2 \\
&\geq (\delta-\sigma)|v|^2 - \frac{\sigma\delta}{\lambda_1}\|u\|.|v| + \sigma\left(\frac{3}{4} - \frac{M\sigma}{\alpha}\right)\|u\|^2 \\
&\geq (\delta-\sigma)|v|^2 - \frac{\sigma^2\delta}{\lambda_1^2}\|u\|^2 - \frac{\delta}{4}|v|^2 + \sigma\left(\frac{3}{4} - \frac{M\sigma}{\alpha}\right)\|u\|^2 \qquad (3.11) \\
&\geq \frac{\delta}{2}|v|^2 + \frac{\sigma}{2}\|u\|^2 ,
\end{aligned}$$

then combining with (3.10) we conclude

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}(\|u\|^2 + |v|^2 + 2\phi(u) + k|u^+|^2 + |\eta|^2_{\mu,V}) + \frac{\delta}{4}|v|^2 \\
&+ \frac{\sigma}{2}\|u\|^2 + \frac{\alpha}{4}|\eta|^2_{\mu,V} + \sigma k|u^+|^2 + \sigma C_1\phi(u) \leq \frac{|h|^2}{\delta} + \sigma K_2
\end{aligned}$$

Let $\sigma_0 = \min\ \{\sigma, \frac{\delta}{2}, \frac{\alpha}{2}, \sigma C_1\}$, we have

$$\begin{aligned}
&\frac{d}{dt}(\|u\|^2 + |v|^2 + 2\phi(u) + k|u^+|^2 + |\eta|^2_{\mu,V} + 2K_1) \\
&+ \sigma_0(\|u\|^2 + |v|^2 + 2\phi(u) + k|u^+|^2 + |\eta|^2_{\mu,V} + 2K_1) \qquad (3.12) \\
&\leq \frac{2|h|^2}{\delta} + 2\sigma K_2 + 2\sigma_0 K_1.
\end{aligned}$$

By (3.7), (3.12) we denote

$$W(t) = \|u\|^2 + |v|^2 + 2\phi(u) + k|u^+|^2 + |\eta|^2_{\mu,V} + 2K_1 > 0, \tag{3.13}$$

then

$$\frac{d}{dt}W(t)+\sigma_0 W(t)\leq C,$$

where $C = \dfrac{2|h|^2}{\delta} + 2\sigma K_2 + 2\sigma_0 K_1$. By the Gronwall lemma, we have

$$W(t) \leq W(0)\exp(-\sigma_0 t) + \frac{C}{\sigma_0}(1 - \exp(-\sigma_0 t)), \ \forall\, t \leq 0.$$

In line with ($g2$) and Sobolev embedding theorem, if $\|u(0)\|^2$, $|u_t(0)|^2$, $|\eta(0)|^2_{\mu,V}$ are bounded, then $\phi(u(0))$ is bounded, too, therefore $W(0)$ is bounded, and

$$\limsup_{t\to\infty} W(t)\leq \rho_0^2, \tag{3.14}$$

where $\rho_0^2 = \dfrac{C}{\sigma_0}\cdot$ Thus, we have the following theorem:

Theorem 3.1 Suppose that $k > 0$, ($g1$) - ($g3$) and ($h1$) - ($h3$) are hold. The ball $B_0 = B_{\mathcal{H}}(0, \rho_0)$ of $\mathcal{H}$, centered at 0 of radius 0, is a bounded absorbing set in $\mathcal{H}$ for the semigroup $\{S(t)\}_{t\geq 0}$. Namely, for any bounded subset B of $\mathcal{H}$, there exists $t_0 = t_0(B) > 0$, such that $S(t)B \subset B_0$ for $t \geq t_0$.

4. Global attractor in $\mathcal{H}$

In order to obtain our main results, we first need the following lemma of compactness property about the nonlinear term g.

Lemma 4.1 [9] Let g be C^2 function from $\mathbb{R}$ into $\mathbb{R}$ satisfying (g_2). Then g : $H_0^2(\Omega)\to H^{1,p}(\Omega),\ \forall\ p>1$ is continuously compact.

Theorem 4.2 Suppose that $k > 0$, the conditions ($g1$)-($g3$) and ($h1$)-($h3$) are hold. Then the semigroup $\{S(t)\}_{t\geq 0}$ associated with the initial boundary value problem (1.3)-(1.4) possesses a global attractor $\mathcal{A}$ in $\mathcal{H}$ which attracts all bounded subsets of $\mathcal{H}$ in the norm of $\mathcal{H}$.

Proof Applying theorem 2.2 and theorem 2.3, it is sufficient to prove that $\{S(t)\}_{t\geq 0}$ satisfies the condition (C) in $\mathcal{H}$.

Let $\{\omega_k\}_{k=1}^{\infty}$ be an orthonormal basis of V which consists of eigenvectors of A, the corresponding eigenvalues are denoted by

$$0 < v_1 < v_2 \leq v_3 \leq \cdots, v_k \to \infty, \text{ as } k \to \infty.$$

We write $H_m = \text{span}\{\omega_1, \cdots, \omega_m\}$. Since $h \in H$ and $g : V \to H^{1,p(\Omega)}$, $\forall\, p > 1$ is compact operators verified in Lemma 4.1, therefore, for any $\varepsilon > 0$, there exists some m such that

$$|(I - P_m)h|_H \leq \frac{\varepsilon}{4}, \tag{4.1}$$

$$|(I - P_m)g(u)|_H \leq \frac{\varepsilon}{4}, \ \forall u \in B_V(0, \rho_0), \tag{4.2}$$

where $P_m : H \to H_m$ is an orthogonal projector, and ρ_0 is given by theorem 3.1. For any $(u, u_t, \eta) \in \mathcal{H}$, we write $(u, u_t, \eta) = (u_1, u_{1t}, \eta_1) + (u_2, u_{2t}, \eta_2)$, here $(u_1, u_{1t}, \eta_1) = (P_m u, P_m u_t, P_m \eta)$.

Choose $0 < \sigma < 1$, taking the scalar product in H of the first equation of (1.3) with $v_2 = u_{2t} + \sigma u_2$, combining the second equation of (1.3), we obtain

$$\frac{1}{2}\frac{d}{dt}(\|u_2\|^2 + |v_2|^2 + |\eta_2|^2_{\mu,V}) + \sigma\|u_2\|^2 + (\delta - \sigma)|v_2|^2 \tag{4.3}$$

$$-\sigma(\delta - \sigma)(u_2, v_2) + (k(u^+)_2, v_2) + \frac{\alpha}{4}|\eta_2|^2_{\mu,V} + (g(u), v_2) \leq (h, v_2).$$

Take σ small enough, like (3.11), we have

$$\sigma\|u_2\|^2 + (\delta - \sigma)|v_2|^2 - \sigma(\delta - \sigma)(u_2, v_2) \geq \frac{\sigma}{2}\|u_2\|^2 + \frac{\delta}{2}|v_2|^2. \tag{4.4}$$

Thanks to u is uniformly bounded in V and exploiting $|u^+| \leq |u|$, by the Sobolev embedding theorem, for above any $\varepsilon > 0$, we obtain $|(u^+)_2| < \varepsilon$. Therefore, we conclude that

$$(k(u^+)_2, v_2) \leq k|(u^+)_2|.|v_2| \leq \varepsilon k|v_2| \leq \frac{\delta}{8}|v_2|^2 + \frac{2k^2\varepsilon^2}{\delta} \tag{4.5}$$

Combining with (4.1)-(4.5), we find

$$\frac{d}{dt}(\| u_2 \|^2 + | v_2 |^2 + | \eta_2 |^2_{\mu,V}) + \sigma \| u_2 \|^2 + \frac{3\delta}{8} | v_2 |^2 + \frac{\alpha}{2} | \eta_2 |^2_{\mu,V}$$

$$\leq \frac{\varepsilon^2}{2\delta} + \frac{2k^2\varepsilon^2}{\delta}, \quad t \geq t_0.$$

Let

$$V(t) = \|u_2\|^2 + |v_2|^2 + |\eta_2|^2_{\mu,V}, \quad t \geq t_0.$$

Take σ small enough, such that

$$\frac{d}{dt}V(t) + \sigma V(t) \leq C\varepsilon^2, \quad t \geq t_0.$$

where $C = \frac{1}{2\delta} + \frac{2k^2}{\delta}$. By the Gronwall lemma, we have

$$V(t) \leq V(t_0) \exp(-\sigma(t - t_0)) + \frac{C\varepsilon^2}{\sigma}(1 - \exp(-\sigma(t - t_0))).$$

Take $t_1 - t_0 = \frac{1}{\sigma} \log \frac{\rho_0^2}{\varepsilon^2}$, then we conclude

$$V(t) \leq \left(1 + \frac{C}{\sigma}\right)\varepsilon^2, \quad t \geq t_1.$$

Thus, the semigroup $\{S(t)\}_{t\geq 0}$ satisfies Condition (C).

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