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Behavior Analysis of Gemagnetic Storm Using Time variation Technique

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Abstract

'Fractal Dimension' can be best studied to understand the complexity of a dynamical system. Fractal geometry allows the description of natural patterns by simple numbers to facilitate their comparison and to establish and test models of formation. The purpose of this paper is to quantify the intensity of the great geomagnetic storm in terms of 'Fractal dimension'. The attracter associated with the storm diverging at the Polar Regions and converging at the lower and middle latitudes has been revealed in phase space reconstructions.

1. INTRODUCTION

Our present knowledge of the space environment about the Earth depends upon both satellite discoveries and the interpretation of characteristic surface magnetic field changes. An accurate interpretation of solar-terrestrial disturbance behavior provided by surface observatories plays an important role in shielding the satellite [3]. Spectral characteristics of magnetic storm-induced F-region [4], Upper Mantle electrical conductivity distribution beneath the Indian Subcontinent using Geomagnetic Storm time variations [5], Geomagnetic storm effects on ionosphere total electron content and evidence of equatorial electro jet control [9], Multi Dimensional Scaling technique for analysis of magnetic storms [15], the extreme magnetic storm of 1-2 September 1859 [16]. etc. explains the physics of some of the geomagnetic storms in the past. Although no two storms are identical most storms have certain features in common. [14]. The most powerful G5 Geomagnetic storm during 29, 30 and 31st of October 2003 has the mean value of the 3-hour Aa indices as 298, 230,180 and the corresponding three Kp values as 58, 56 and 50 respectively. This is the only three consecutive days having highest values in recent times. This great geomagnetic storm knocked out airline communications and forces the Japanese space agency to temporarily shutdown their satellites when the contact to the satellite Midoriz was lost. In this paper an attempt has been made to study

Key Words: Geomagnetic Storm, Fractal Dimension, Phase space reconstruction.

the intensity of the most powerful storm in terms of fractal dimension and the disorder of the severe storm represented at each instant of time by a point which traces out a trajectory during this time defined by the state of the variables in phase space reconstructions.

2. DATA ANALYSIS

Data of one minute variations of the sensitive horizontal (H or X) force of the geomagnetic component during the storm on 29-31st of October 2003, 4320 values in each of the 72 hours for the six observatories – Scot base, Crozet, Alibag, Kakioka, Memambetsu, and Alert, having considerable distance from South Pole to North Pole has been obtained from the Kyoto University-World Data Centre for Geomagnetism. Locations of the observatories are described in the table 1. As for the Alibag Observatory is concerned there is no difference between the values of F (total) and H (horizontal) variations considered for this analysis. Storm time ranges, the difference between the maximum and minimum of one minute values and their corresponding fractal dimensions are provided in the table 1.

Table 1

Station Name	*Geographic*		*Geomagnetic*		*Storm time*	*Fractal*
	Latitude	*Longitude*	*Latitude*	*Longitude*	*range nT*	*Dimension*
1. Scott Base	-77.85	166.78	-78.93	291.01	2047	0.41203
2. Crozet	-46.43	51.86	-51.39	112.81	1590	0.44112
3. Alibag	18.64	72.87	10.04	145.97	507	0.43323
4. Kakioka	36.23	140.19	27.18	208.51	395	0.41274
5. Memambetsu	43.91	144.19	35.16	211.00	509	0.4049
6. Alert	82.50	297.50	86.73	158.85	3647	0.32309

3. FRACTAL DIMENSION

A fractal is a recently discovered kind of geometrical object and it is furthermore one which describes the nature much better than Euclidean objects, which are based on straight lines, circles etc. A mathematical fractal is defined as any series for which the Hausdorff dimension exceeds the discrete topological dimension [7]. To each subset of the Euclidean space $\Re^m$ there is assigned a topological dimension d which is an integer satisfying the property $0 \leq d \leq m$. Mandelbrot [3] defined a fractal as a set with dimension D and topological dimension d satisfying $d<D$. We have implemented the technique developed by Higuchi [8] and followed by Kabin and Papitashvilli [10].

If we have the observational time series of values $X(t_i)$ where time intervals are supposed to be equal, then the increments of X can be defined as $\Delta(t_j - t_i) = X(t_j)\text{-}X(t_i)$, j>i. The non-normalized apparent length of the time series curve is defined as:

$$L_k = \Sigma i\ |\Delta(t_{i+k} - t_i)|.$$

If X (t) is a fractal function, then the graph ln (L_k) versus ln (k) should be a straight line with a slope 1–d ('d' is the fractal dimension) for small enough $\Delta_k t$. For large values of $\Delta_k t$ the graph ln(L_k) versus ln(k) can obviously deviate from the straight line approximation because in this case it is derived for just a few data points chosen from the entire data set. In this range there is not enough statistical information to approximate the fractal properties of the curve.

Accordingly the fractal dimensions of the geomagnetic storm time variations of highly sensitive horizontal (H or X) component of the six stations, using their successive difference of 1 minute values are calculated. Figure 1 shows the nature of the storm, figure 2 shows the fractals of the individual observatories and figure 3 shows the variations in dimensions of the observatories.

4. PHASE SPACE ANALYSIS

Phase space is defined as a coordinate space defined by the state of variables of dynamical system [6]. In experiments, we usually do not have the luxury of working with the actual vectors of phase space variables. Normally only the time series of a single variable is available to characterize the behavior of each system. Therefore to be able to analyze experimental as well as numerical data we will rely upon the phase space reconstruction method [1]. A dynamical system is one whose state changes with time (t). Two main types of dynamical system are the time variable is discrete $t \in$ Z or N (integers or natural numbers) and those for which it is continuous $t \in$ R (real numbers). Discrete dynamical system can be represented as the iteration of a function, i.e

$$X_{t+1} = f\ (X_t) \qquad t \in \text{N or Z}$$

When t is continuous, the dynamics are usually described by a differential equation, $dx/dt = x' = X(x)$, x represents the state of the system [2] and takes value which gives a geometrical description of the solutions in the phase space. If the scattered final states exhibit sensitive dependence on the incident condition, then the process is called chaotic [11]. Accordingly, the time delay embedding technique is used here to construct the phase space analysis. The idea of time delay embedding technique is as follows:

Figure 1: Geomagnetic Storm on 29-31 October 2003

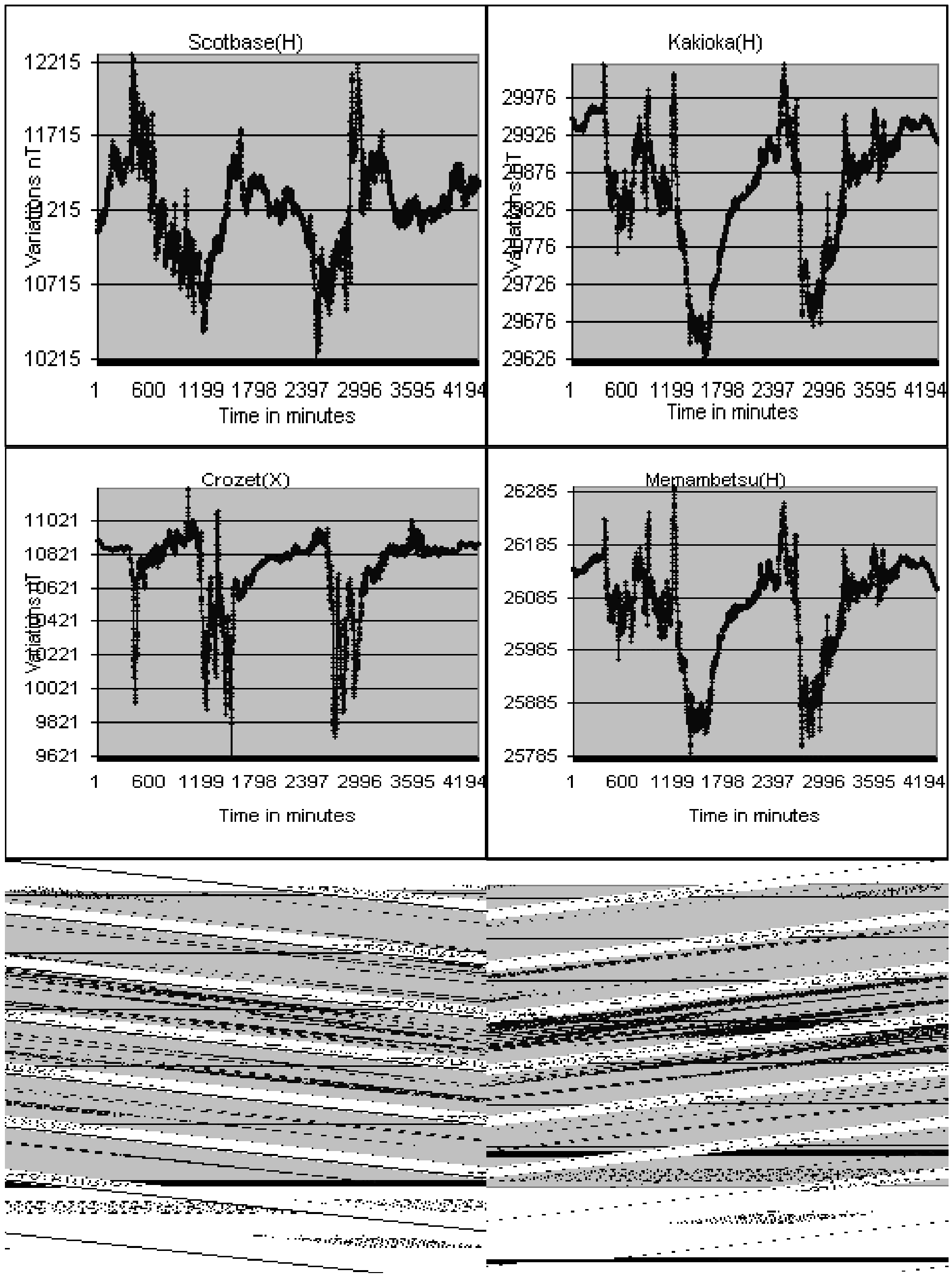


Figure 2: Fractal nature of Geomagnetic Storm on 29-31 October 2003

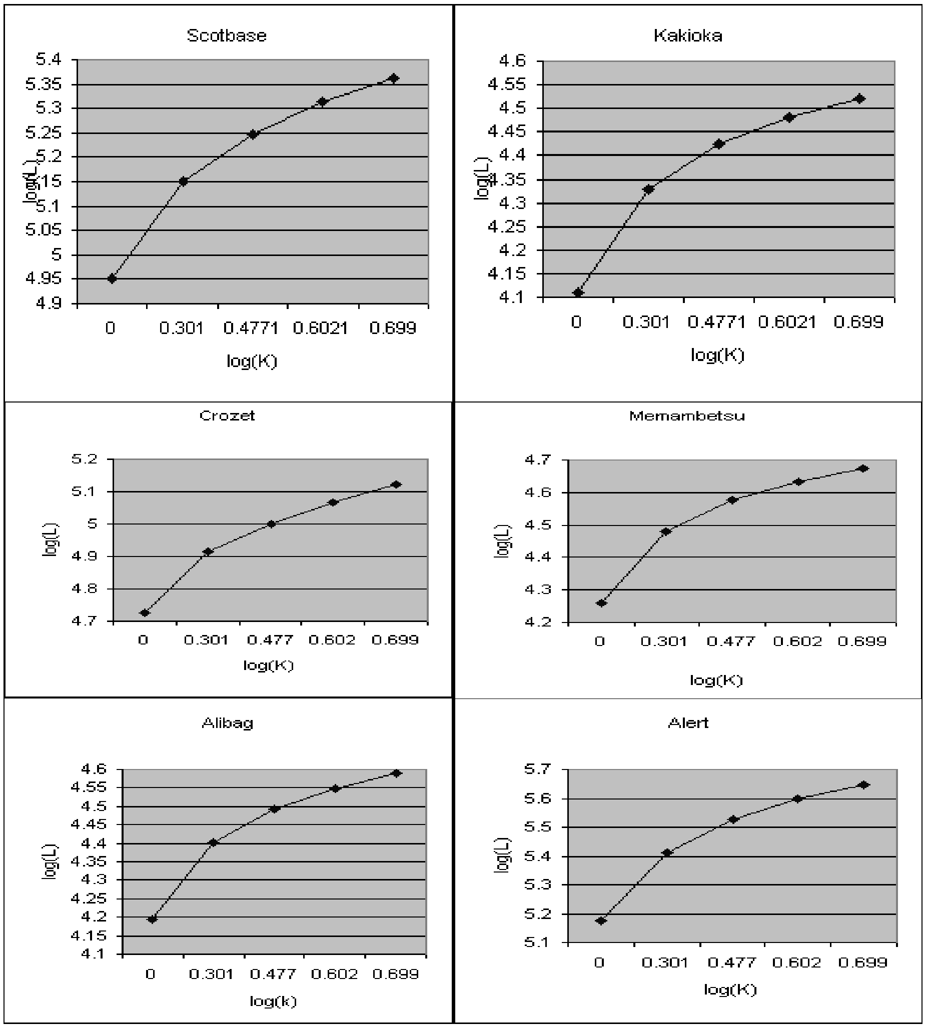


Figure 3

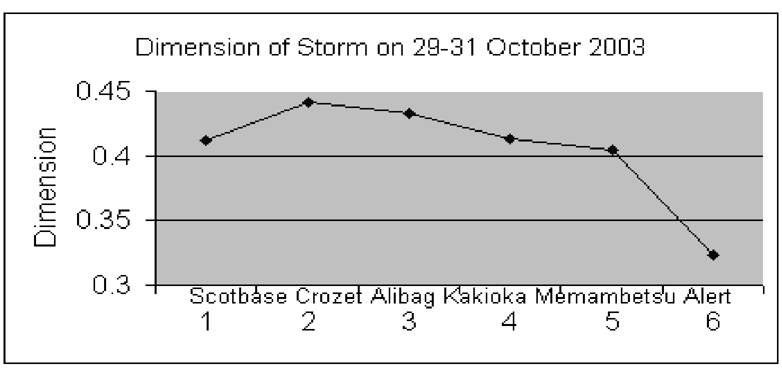
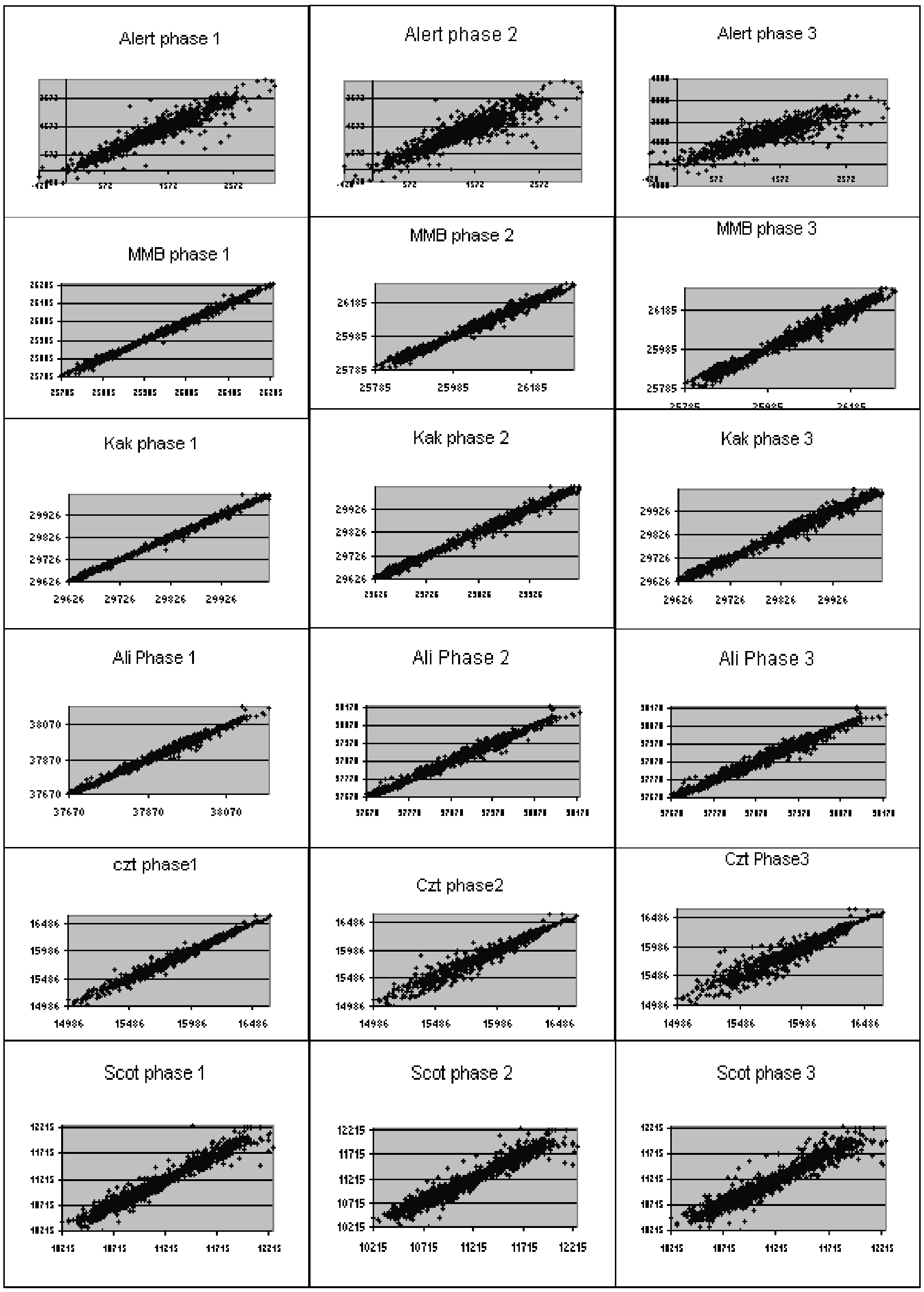


Figure 4
Phase Space Reconstructions

Abbreviations: MMB–Memambetsu, Kak–Kakioka, Ali–Alibag, Czt-Crozet, Scot-Scot base



Let us suppose that the given time series pertains to the measurement of a variable x (t) among the describing system. The first step in the embedding technique is to construct an m-component delay or state vector X_i at time $t = t_i$ as,

$$X_i = [x_1(t_i), x_2(t_i), \ldots\ldots\ldots\ldots x_m(t_i)], \text{ where } x_k(t_i) = x\,(t_i + (k\text{-}1)t),$$

τ is an appropriate time delay. The time delay τ should be chosen in such a way that it is small enough to resolve the physical process of interest. Now the m-dimensional reconstructed phase portrait will have the same properties of fractal dimension as one constructed from the measurement of N independent variables. The points in the phase space are chaotic in the sense that two nearby points diverge rapidly with time before escaping to infinity [12]. The figures 4 shows the phase space reconstructions of the geomagnetic storm time variations for the values $\tau = 1$, $\tau = 2$ and $\tau = 3$ (minutes). The increase in value of τ corresponds to the divergence in the phase space for all observatories can be seen in figure 4.

5. CONCLUDING REMARKS

The studies on fractal dimensions of geomagnetic storm at different observatories will provide information of its topological behavior, which may lead to significant theories as well as applications. The recognition that the dynamics of a system can be measured by fractal analysis provides a new exciting and rigorous framework to understand and to predict its pattern. So far, there is no strict theory to testify what pre requirements and conditions are needed for fractal calculation. Hence we are restricting our comments on this topic. The electromagnetic response of the earth during magnetic storms determined by the response function for a period range over a few hours is a diagnostic tool to illuminate the underlying dynamics. In phase space analysis, the increase in values of t corresponds to the divergence in the scattering indicates the chaotic behavior of the dynamical system [12]. Here, the increase in the value of t corresponds to the uniform divergence in all the latitudes. A feature common to the chaotic systems is, that, as some external parameters are varied, the dynamical behavior of the system changes. One aim of chaos theory is to describe transitions from simple to complicated motion from a universal point of view [17]. The dimension of the storm in different locations and the phase space reconstruction method has been detailed in this paper. However, studies on fractal dimension of many such geomagnetic storms and their correlation with their geographic locations are very essential for better understanding of this complex nature.

As a result of this study it is expected that, the ideas and methodologies on 'fractal dimension' explained in this paper can be used as a measure for scaling the intensity of geomagnetic storms at a particular location and can be a useful tool for the observatory data analysis for exploring some new results in Geomagnetism.

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