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The Control of SMA Actuators through Fuzzy Logic

L. Platino, University of Jaume, Spain

Abstract

The behavior of Shape Memory Alloy (SMA) actuators is based on the transformation of its martensite and austenite phases. The SMA actuation, which depends on heating and cooling process is a non-linear phenomenon and depends on various interrelated factors. Fuzzy logic systems are qualified as models of general nonlinear systems as they are capable of approximating a wide variety of nonlinear functions. Hence the objective of this paper is to realize a trained fuzzy logic controller that reduces the number of fuzzy rules using table look-up scheme for the control of SMA actuators. As the SMA actuation is dependent upon the amount of heat that is added and removed, its heat transfer analysis is presented first. The outcome of the same is considered as the basis for the design of the training algorithm. Simulation has been carried out using MATLAB.

Key words: Set theory, Fuzzy systems, Adaptive control, Non-linear systems, Smart materials, Shape memory alloys.

1. INTRODUCTION

Since Lotfi Zadeh's introductory paper in 1965 [25], the fuzzy set theory and the applications of fuzzy systems have come a long way. The success of the practical application of fuzzy control theory [7,24] in the form of fuzzy controller in modern control engineering is an important factor for its development. Fuzzy controllers, mostly the Mamdani type systems and sometimes the Takagi-Sugeno type systems are well justified for almost any nonlinear modeling problem due to the *Universal Approximation Theorem* [1,6,9,13,16]. These fuzzy controllers are general enough to perform any non-linear control action. Therefore, by carefully choosing the parameters of the fuzzy controllers, it is possible to design an adaptive fuzzy controller [13,22] that is suitable for the control of any non-linear system. The

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capability of the fuzzy logic systems to incorporate linguistic information in a natural and systematic way, is an unique advantage of the fuzzy logic systems which is not shared by other types of universal approximators, including polynomials, neural networks, wavelets or splines.

Conventional fuzzy logic systems together with suitable training algorithms make the system more powerful to act in the fuzzy environment that consists of linguistic and numerical information. If the fuzzy logic system is constructed from linguistic information, by using a set of fuzzy IF-THEN rules formulated with the help of human experts having a priori knowledge of the system, then the training algorithm adjusts the parameters or structures of the fuzzy system based on the numerical information provided by the sensor feedback system. Training algorithms also make fuzzy logic system more adaptive to the environment in which the fuzzy rules are automatically generated. The final fuzzy logic system is, therefore, constructed based on both numerical and linguistic information.

The speed of response of SMA actuators can be enhanced by selecting a suitable heating and cooling method with suitable controllers which could add and remove the heat at a faster rate. Light weight, high power and work density, high thermal conductivity, large recoverable strains, high tensile strength, possessing two phases with different resistivities and good damping properties, silent operation etc., are some of the salient features which make SMA actuators suitable for robotic applications. But the major problem encountered with SMA actuators is the control difficulty as the actuation is non-linear, possessing large hysteresis, stress dependant phase transformation, change in Young's modulus etc and hence classical control methods are not capable [15] of producing accurate results. As the fuzzy controller does the job of mapping all the inputs and outputs, it can be well adopted for controlling SMA actuators provided the controller is suitably trained.

The structure of this paper consists of the following sections. Components of a classical fuzzy controller and the way in which its inputs and output are considered towards SMA actuation and the justification for using the fuzzy controller are explained in Section 2. The inputs for the fuzzy controller are obtained based on the output derived from heat transfer analysis and thus is presented in Section 3. The table look-up scheme as training algorithm for the controller is available in Section 4. The results and discussion are presented in Section 5, followed by conclusion in Section 6 and subsequently the references.

2. THE CLASSICAL FUZZY CONTROLLER

Most of the engineering systems consist of two important information sources, namely sensors and human experts. Human experts provide linguistic information like high, low, medium etc., about the system whereas sensors provide numerical inputs, represented by numbers. Classical engineering approach only makes use of numerical information. But fuzzy systems are capable of integrating both numerical and linguistic inputs, which is the major advantage and also a convenient way of describing the system behavior. Basic information about the fuzzy set theory from which the fuzzy logic control (FLC) system developed is not covered here and the interested reader can refer various textbooks like [2,5,8,10,12] and the related literatures.

Fuzzy controllers are most suitable for systems which are complex, non-linear, or do not have mathematical models. The FLC system, shown in Fig.1 [20], is an approximate reasoning based controller, that does not require exact analytical or

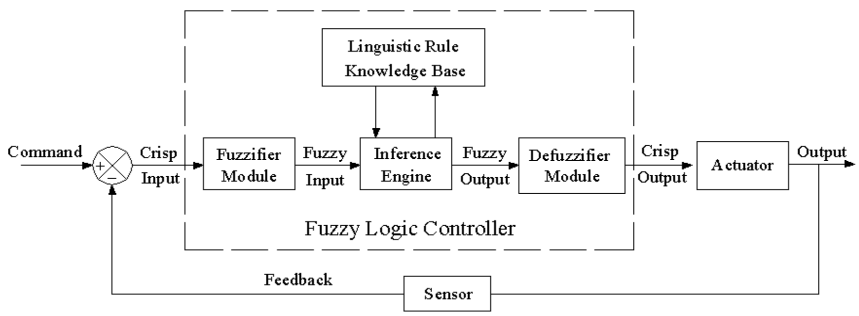

Fig. 1: Classical Fuzzy Logic Controller

mathematical model. In classical control systems, control action is reached through an algorithm, based on multiplication by constant (proportional control), taking a derivative (derivative control), integration or a combination of two or all the three (PID control). Where as in fuzzy control, mapping of all input and output variables are done with linguistic rules, based on the priori knowledge and executed through the inference mechanism. The FLC system mainly consists of three modules, fuzzification, and defuzzification and in between these two is the inference engine, which are described below.

2.1 Fuzzification

Fuzzification is converting the numerical inputs into fuzzy sets using suitable functions, with a membership value between 0 and 1, so that they could be interpreted by the fuzzy control system. Different kinds of membership functions are used for the fuzzification of both inputs and outputs. Functions like triangular, trapezoidal, bell shaped, sigmoid, beta functions [1] etc, are widely used. The widths of the functions need not be uniform as the narrow regions provide tight control and the wide regions provide looser control.

The fuzzifier performs a mapping from a crisp point, $\underline{x} = [x_1, x_2, \ldots\ldots, x_n]^T \in U$, into a fuzzy set $\tilde{A}\ in\ U$. Mostly singleton and nonsingleton fuzzifiers are used for the mapping of input crisp values into fuzzy sets. Singleton fuzzification is that, if $\tilde{A}$ is a fuzzy singleton with *support* $\underline{x}$, then $\mu_{\tilde{A}}(\tilde{\underline{x}}) = 1\ for\ \tilde{\underline{x}} = \underline{x}\ and\ \mu_{\tilde{A}}(\tilde{\underline{x}}) = 0$ for all other $\tilde{\underline{x}} \in U\ with\ \tilde{\underline{x}} \neq \underline{x}$. Whereas for nonsingleton fuzzifier, $\mu_{\tilde{A}}(\underline{x}) = 1$ and $\mu_{\tilde{A}}(\tilde{\underline{x}})$ decreases from 1 as $\tilde{\underline{x}}$ moves away from $\underline{x}$ and finally reaches zero. If Gaussian membership functions are considered, as used here, then the function is of the form

$$\mu_j^i(x_j) = exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right) \tag{1}$$

where i = 1, 2,….R (number of rules) and j = 1,2,…n(number of inputs), c_j^i = centre of the membership of the ‘i’th rule for ‘j’th universe of discourse.

2.2 Fuzzy Rule Base and Inference Engine

A fuzzy rule base consists of a collection of fuzzy IF-THEN rules in the following form

$$R^i : IF\ x_1\ is\ F_1^i\ and\ \ldots\ and\ x_n\ is\ F_n^i,\ THEN\ y\ is\ G^i \tag{2}$$

where F_j^i and G^i are fuzzy sets in $U_j \subset R$ and $V \subset R$ respectively and the linguistic variables are

$$\underline{x} = (x_1, x_2, \ldots\ldots x_n)^T \in U_1 \times U_2 \times \ldots\ldots \times U_n\ \ and\ \ y \in V \tag{3}$$

The number of rules used in controlling the system using fuzzy control is represented by, R=(m)^n , where m-number of membership functions in each input (Fuzzy sets), n-number of input variables. For SMA actuation, after having a priori-knowledge of the system, a general fuzzy system [20,21] can be formulated with temperature and the heating time as two inputs and power required for heating the SMA as output. Each input variable can be divided into seven membership functions as Extremely Low (EL), Very Low (VL), Low (LO), Medium (ME), High (HI), Very High (VH), Extremely High (EH). Hence n=2, m=7 and R=49. Therefore 49 *Fuzzy IF...THEN… Rules* can be formed as under.

R^1: IF *temperature* is 'EL' and *heating time* is 'EL', THEN *power required* is 'EH'

R^2: IF *temperature* is 'EL' and *heating time* is 'VL', THEN *power required* is 'EH'

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R^{49}: IF *temperature* is 'EH' and heating *time* is 'EH', THEN *power required* is 'EL'

These 49 fuzzy rules are presented in the form of 'Rule-Base Matrix Array' as shown in Table 1. The '*' marked values are just to identify the above rules with the matrix and also to help formulating the other rules.

Table 1
Fuzzy Rule Matrix for 49 Rules

Power (Output)		*Heating Time (Input 2)*						
		EL	*VL*	*LO*	*ME*	*HI*	*VH*	*EH*
Temperature (Input 1)	EL	EH*	EH*	EH	EH	VH	VH	VH
	VL	VH	VH	VH	VH	HI	HI	HI
	LO	HI	HI	HI	ME	ME	ME	ME
	ME	ME	ME	ME	ME	LO	LO	LO
	HI	LO	LO	LO	LO	LO	VL	VL
	VH	VL	VL	VL	VL	VL	VL	VL
	EH	EL	EL	EL	EL	EL	EL	EL*

If $\mu_i(x)$ is the certainty of the *premise (IF part)* of the 'i'th rule or the membership function value, then it can be expressed as

$$\mu_i(x) = \prod_{j=1}^{n} \mu_j^i(x) \tag{4}$$

In the fuzzy inference mechanism the fuzzy IF-THEN rules are combined to map the fuzzy sets in $U = U_1 \times U_2 \times ...... \times U_n$ to fuzzy sets in V which is based on the interpretation of the rules using sup-star composition. The interpretations are generally based on the fuzzy implications, mini-operation rule, product-operation rule, arithmetic rule, max-min rule, Boolean rule and Goguen's rule [13]. However mini-operation rule, following from the fuzzy conjunction by using the fuzzy intersection operator, is considered here as it is widely used. These are expressed by

$$Sup - Star\ Composition: \ \mu_{PoQ}(u,w) = sup_{v \in V}[\mu_P(u,v) * \mu_Q(v,w)] \quad (5)$$

$$Mini - Operation\, Rule: \ \mu_{A \to B}(\underline{x}, y) = min[\mu_A(\underline{x}), \mu_B(y)] \quad (6)$$

Here *P* and *Q* are fuzzy relations in *U*×*V* and *V*×*U* where *U* and *V* are two universe of discourses. The sup-star composition *P* and *Q* is the fuzzy relation denoted by *P* o *Q*, $u \in U$, $w \in W$, '*' is the 'min' operator in the class of t-norm [13].

2.3 Defuzzification

It is the process of converting a fuzzified value into a numerical (crisp) value. It is required for generating the real world output. The defuzzifier performs a mapping from fuzzy sets in *V* to a crisp point v. At least seven methods are popular for defuzzifying the fuzzy output functions (membership functions). They are, Max membership principle; Centroid method (also called as "centre of gravity-COG" or "centre of area-COA" methods); Weighted average method; Mean-max membership (also called as "middle of maxima method"); Center of sums; Center of largest area; First (or last) of maxima method. Center of sums method that is also called as centre average defuzzifier, can be expressed as

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i(x)}{\sum_{i=1}^{R} \mu_i(x)} \quad (7)$$

where b_i is the centre of the output membership function due to the ith rule. For characterizing the shape of the membership function, like making the width unequal or making it nonsymmetrical, a parameter d^i [13] can be introduced. Now the above expression can be modified as

$$y = \frac{\sum_{i=1}^{R} b_i \left(\mu_i(x) / \delta^i\right)}{\sum_{i=1}^{R} \left(\mu_i(x) / \delta^i\right)} \quad (8)$$

The modified centre average defuzzifier implies that the sharper the shape of the membership function, output y will be nearer to the center of the membership function and hence better control.

2.4 The Total Controller

The overall fuzzy control system with center average defuzzification with product-inference rule, singleton fuzzifier and Gaussian membership function [10], considered for our application, can be expressed by

$$y = f(\underline{x}) = \frac{\sum_{i=1}^{R} b_i \left[\prod_{j=1}^{n} a_j^i exp \left(-\frac{1}{2} \left(\frac{x_j - \bar{x}_j^i}{\sigma_j^l} \right)^2 \right) \right]}{\sum_{i=1}^{R} \left[\prod_{j=1}^{n} a_j^i exp \left(-\frac{1}{2} \left(\frac{x_j - \bar{x}_j^i}{\sigma_j^i} \right)^2 \right) \right]} \tag{9}$$

If the system is used with modified average defuzzifier, then the expression will be

$$y = f(\underline{x}) = \frac{\sum_{i=1}^{R} b_i \left[\prod_{j=1}^{n} a_j^i exp \left(-\frac{1}{2} \left(\frac{x_j - \bar{x}_j^i}{\sigma_j^l} \right)^2 \right) \right] / \delta^i}{\sum_{i=1}^{R} \left[\prod_{j=1}^{n} a_j^i exp \left(-\frac{1}{2} \left(\frac{x_j - \bar{x}_j^i}{\sigma_j^i} \right)^2 \right) \right] / \delta^i} \tag{10}$$

The parameters b_i, a_j^i, $\bar{x}_j^i$ and σ_j^i are with the constraints [9,13,16] $b_i \in$V, $a_j^i \in (0,1)$, $\bar{x}_j^i \in U_j$ and $\sigma_j^i > 0$ respectively.

2.5 Universal Approximation Theorem

The fuzzy logic system as expressed in Eq. 9 or 10, is capable of uniformly approximating any nonlinear function over *U* to any degree of accuracy if *U* is compact based on *Universal Approximation Theorem*. It states that for any given real function 'g' on a compact set $U \subset R^n$ and any arbitrary value e < 0, there exists a fuzzy logic system 'f' in the form, Eq. 9 or 10, such that

$$sup_{\underline{x} \in U} \left| f(\underline{x}) - g(\underline{x}) \right| < \varepsilon \tag{11}$$

And a corollary is that for any $g \in L_2(U)$ and arbitrary value e<0, there exists a fuzzy logic system f in the form, Eq. 9 or 10 such that

$$\left(\int_U \left| f(\underline{x}) - g(\underline{x}) \right|^2 d\,\underline{x} \right)^2 < \varepsilon \quad and \tag{12}$$

$$L_2(U) = [g : R \left| \int_U |g(\underline{x})|^2 d\,\underline{x} < \infty \right. \tag{13}$$

The integrals are in the Lebesgue sense. Based on this universal approximation theorem, SMA actuation system can be controlled by a fuzzy controller with an arbitrary accuracy if the controller is in the form as expressed in the Eq. 9 or 10. Proof of this theorem and corollary are available in [13], based on Stone-Weierstrass Theorem [6,16].

3. HEAT TRANSFER ANALYSIS

The major disadvantage of using SMA actuator is its slow response, which is generally in the range of 2Hz. It mainly depends on how fast the heat is added and removed. Of the various heating and cooling methods, joule heating combined with forced convective cooling results in higher frequency of operation [21]. The following fundamental heat transfer expressions are considered for obtaining the input and output variables for the proposed fuzzy controller [4,15,20].

$$mc_p\left(\frac{dT}{dt}\right) = i^2 R - hA(T - T_a) \text{ ...for heating} \tag{14}$$

$$mc_p\left(\frac{dT}{dt}\right) = -hA(T - T_a) \text{ ...for cooling} \tag{15}$$

where, m-mass of the SMA wire, c_p-specific heat constant, T-instantaneous temperature, T_a-ambient temperature, i-current required, R-resistance of the wire, h –convective heat transfer coefficient, A – surface area of the wire, and dT/dt-rate of change of temperature with respect to time t.

Fast SMA actuation basically depends upon the rate of addition and removal of heat. Resistive heating is considered for heat input and natural convective cooling is considered for the removal of heat. As the temperature of the wire increases from T_a, Eq.14 can be solved as under in order to obtain the temperature of the wire, T at any time, t.

$$T e^{(\frac{hA}{mc_p})t} = (\frac{i^2 R}{hA} + T_a) e^{(\frac{hA}{mc_p})t} + c \tag{16}$$

$$(T - T_a) = \frac{i^2 R}{hA} + c e^{-(\frac{hA}{mc_p})t} \tag{17}$$

In the above equation, c is the constant of integration which can be obtained using the boundary condition, when t=0, then T=T_2, and hence:

$$c = (T_2 - T_a) - \frac{i^2 R}{hA} \tag{18}$$

Using Eq. 17 and 18, the general equation of heat transfer during the heating phase is

$$(T - T_a) = (T_2 - T_a) e^{-(\frac{hA}{mc_p})t} + \frac{i^2 R}{hA}(1 - e^{-(\frac{hA}{mc_p})t}) \tag{19}$$

If the time constant of the system is denoted as t and using another boundary condition, when $t = \tau$, then $T = T_{1,}$ the general expression is modified as

$$(T_1 - T_a) = (T_2 - T_a) e^{-(\frac{hA}{mc_p})\tau} + \frac{i^2 R}{hA}(1 - e^{-(\frac{hA}{mc_p})\tau}) \tag{20}$$

In Eq. 20, T_1 is the maximum temperature of the wire, which can be just above the austenite finish (A_f) temperature in order to ensure complete transformation. T_a is the minimum temperature of the wire that is the ambient temperature. T_2 is the temperature at any time t, which determines the power requirement in order to increase the temperature of the wire to $T_{1,}$ for achieving the complete phase transformation. The factor hA/mc_p represents the frequency component. Further, Eq.14 can be modified as under which simplifies the expression, Eq. 20 to Eq. 23.

$$\alpha \Delta T + \beta \frac{dT}{dt} = i^2 \tag{21}$$

$$\alpha = hA/R, \quad \beta = mc_p/R, \quad and \quad \Delta T = T - T_a$$

$$(T_1 - T_a) = (T_2 - T_a)e^{-(\alpha/\beta)\tau} + (i^2/\alpha)(1 - e^{-(\alpha/\beta)\tau}) \tag{22}$$

$$i^2 = \alpha \frac{(T_1 - T_a) - (T_2 - T_a)e^{-(\alpha/\beta)\tau}}{(1 - e^{-(\alpha/\beta)\tau})} \tag{23}$$

For simulation, the following values [11,17,18] are considered. SMA wire diameter=0.25mm, length=50mm, h=110 W/m^2 °C, R=27.5 W/m, A_f=90°C [3], $T_1 = A_f$ + 10°C = 100°C, T_a = 25°C, c_p = 837.2 J/kg °C, Hence the calculated values of a and b are 4.050671x10^{-6} and 4.07412x10^{-4} respectively. In this application, τ values of 0.1, 0.5 and 1.0 are considered with the restriction that, i<1.0. Fig. 2 shows the relationship between T_2 and i for τ values of 0.01, 0.1, 0.5 and 1.0 and the value of 0.5 or 1.0 are better as the power required for heating the SMA wire is in the reasonable range. However the value of 0.5 is best compared to 1.0 in terms of speed of response.

4. TRAINING WITH TABLE-LOOKUP SCHEME

When the design of a fuzzy controller is undertaken, many design parameters have to be considered such as scaling factors, membership functions, control rules etc. Adjustment of these parameters is called as 'tuning' and is required in order to obtain optimum results and to make the system adaptive to the environment. Back propagation, orthogonal least squares [14], table look-up scheme, nearest neighborhood clustering and gradient methods are the generally used training algorithms. The first two training algorithms are not very simple and their computational requirements are intense for complex problems. Training using table look-up scheme is simple to construct, it just performs a one-pass operation on the training data and hence considered for this application. Moreover, the training scheme simplifies the control algorithm by reducing the number of rules and hence the memory requirements, maintaining the required accuracy of the control system.

The basic idea here is to generate the rules based on the training data and then combine the generated rules with the rules from human experts into the final fuzzy logic system.

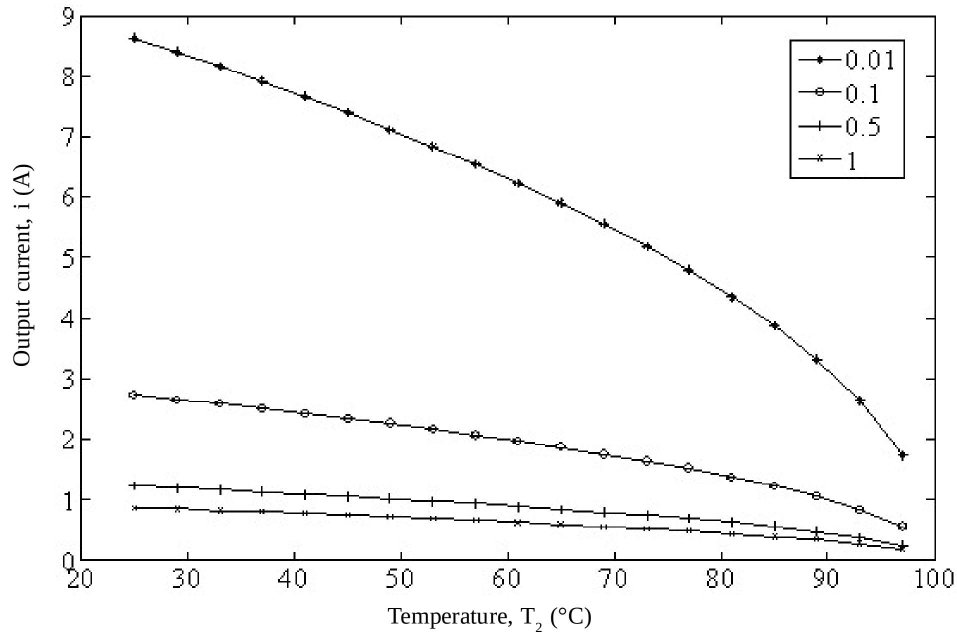


Fig. 2: Relationship between T_2 and i for various τ

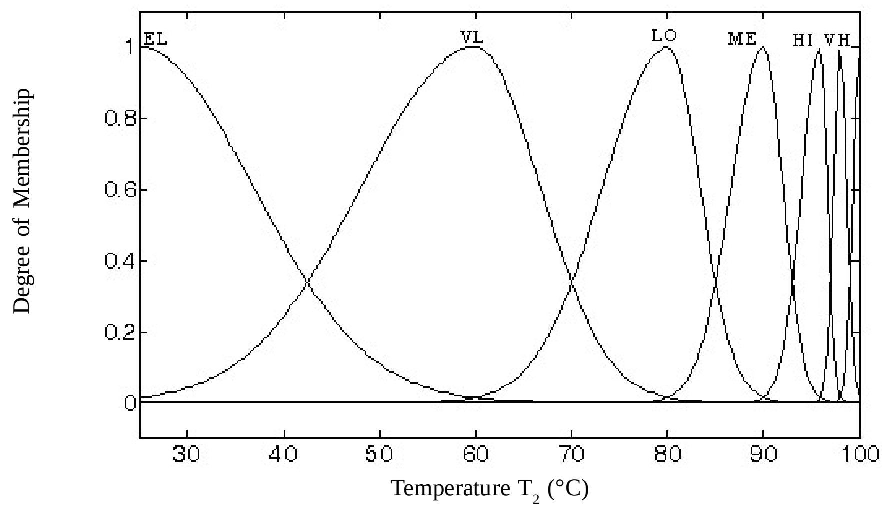
For the two input (x) and one output (y) system, the data pairs are in the following form.

$$(x_1^{(1)}, x_2^{(1)}; y^{(1)}), (x_1^{(2)}, x_2^{(2)}; y^{(2)}), ..., (x_1^{(n)}, x_2^{(n)}; y^{(n)}) \tag{24}$$

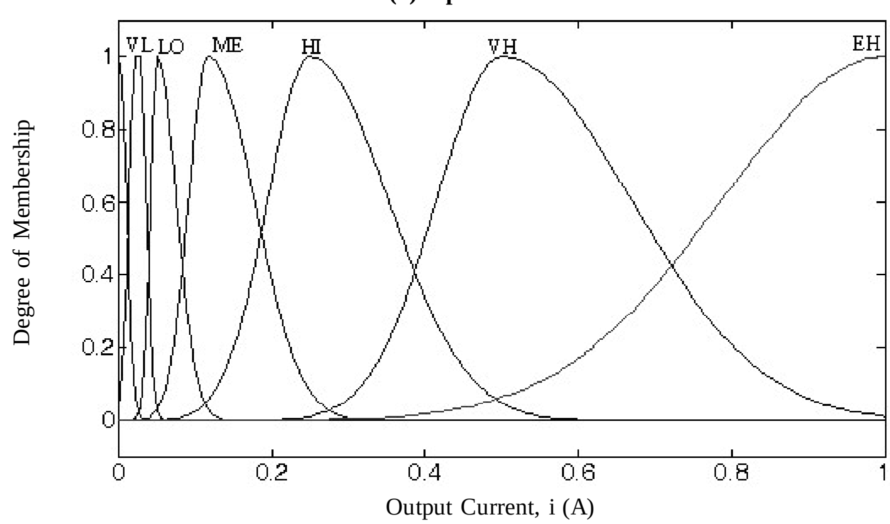
These data pairs can now generate a set of fuzzy IF-THEN rules in order to find the fuzzy logic system, which is of the form $f:(x_1, x_2) \rightarrow y$. The data pairs for the SMA actuator are presented in Table 2. These are generated with the help of Eq. 23. T is the temperature that is the same as T_2 in the expression. The data pairs are considered with the condition that the current i is always less than 1.0 so that, the overall power requirement for heating the SMA become less. Training of the controller is accomplished with the help of the following steps.

4.1 Dividing the Input and Output Spaces into Fuzzy Regions

The domain interval of each variable, two inputs (Temperature, T and Time constant, t) and one output (current supplied, i) can be divided into 7 regions as explained in Sec.2.2. The width of the regions is not same, as close control is possible where the width is low and vice versa. For example, if the temperature of the SMA wire is approaching its austenite finish temperature (A_f), minimum current has to be applied in order to avoid over heating or overshoot and hence the width of these regions are low compared to lower temperature regions. Reduction in width ultimately increases the sensitivity of the system in that region as it increases the slope of the membership function. The width of the membership functions are reduced or increased by a factor, δ^i =1.5, from the preceding membership function. These are in the form of Gaussian as they have smooth transition property [13]. Fig. 3a and b shows the fuzzification of inputs using 'gaus2mf' [23], which is applicable for nonsymmetric or skewed type Gaussian function. Fuzzification of output (current) is similar to the input (time constant) and hence not shown.



(a) Input variable 1



(b) Input variable 2

Fig. 3: Fuzzification

4.2. Generating Fuzzy IF-THEN Rules from Given Data Pairs

For the temperature $25 \leq T < 100$, with the increment of 1 and τ values 0.1, 0.5 and 1.0, Eq. 23 generates 225 data pairs. By putting the restriction that $i < 1.0$, there are 139 data pairs and hence 139 rules which are very high in number from the implementation point of view. In order to reduce the data pairs further, the following strategies are adopted again with the restriction, $i < 1.0$.

Strategy-I:

$90 \leq T < 100 : \tau = 0.1$; $65 < T < 90 : \tau = 0.5$; $25 \leq T \leq 65 : t = 1.0$

This means that when the temperature is on the higher side, the power to be applied for a short time and vice versa. This strategy reduces the number of rules to 75. Further reduction is also possible by selecting the data pairs as shown in the next strategy. Here the pairs with increments are selected when the temperature is in the lower range where severe control is not required.

Strategy-II:

$25 \leq T \leq 65$: Increment of 3°C ; $65 < T \leq 89$: Increment of 2°C ; $89 \leq T \leq 99$: Increment of 1°C

Now the data pairs are reduced to 36, which are shown in Table 2, and hence a total of 36 rules are being generated. A general fuzzy controller can be constructed using the procedure mentioned in Section 2 with the help of 49 rules by having priori knowledge about the heating system of the SMA. The general rule matrix for this controller is shown in Table 1 earlier, which is based on dividing the inputs into seven membership functions. But the disadvantage of this control system is that in certain areas the control is not required.

4.3 Assigning a Degree to each Rule

Considering only the maximum membership value for the given data, the respective linguistic terms are identified. For example, for $T=25 \rightarrow EL$, $\tau=1.0 \rightarrow EH$, $i=0.86 \rightarrow EH$ and for the next pair $T=30 \rightarrow EL$, $\tau=1.0 \rightarrow EH$, $i=0.83 \rightarrow EH$. It is to be noted that for some data pairs the IF and THEN part are same and for some data pairs, IF part alone same and sometimes THEN part alone same. Hence this kind of conflicting rules can be identified and the rules with maximum degree are accepted. Accordingly the rules are reduced to 9 as shown in the following rule matrix, Table 3.

Table 2
Input and Output data pairs in SMA actuation

T	τ	i	T	τ	i
25	1.0	0.86	74	0.5	0.72
28	1.0	0.85	76	0.5	0.69
31	1.0	0.83	78	0.5	0.66
34	1.0	0.81	80	0.5	0.63
37	1.0	0.79	82	0.5	0.60
40	1.0	0.77	84	0.5	0.56
43	1.0	0.75	86	0.5	0.53
46	1.0	0.73	88	0.5	0.49
49	1.0	0.71	90	0.5	0.45
52	1.0	0.69	91	0.1	0.95
55	1.0	0.67	92	0.1	0.89
58	1.0	0.65	93	0.1	0.83
61	1.0	0.62	94	0.1	0.77
64	1.0	0.60	95	0.1	0.71
66	0.5	0.82	96	0.1	0.63
68	0.5	0.80	97	0.1	0.55
70	0.5	0.77	98	0.1	0.45
72	0.5	0.75	99	0.1	0.32

Table 3
Fuzzy Rule Matrix for Selected data pairs

	HI	VH	EH
EL			EH
VL		EH	VH
LO		VH	
ME	EH	VH	
HI	VH		
VH	VH		
EH	HI		

Membership functions EL, VL, LO and ME of Input 2 are not involved and hence can be removed. In order to simplify the design with 9 rules, the range of both input variables are divided into three Gaussian membership functions as LO (Low), ME (Medium) and HI (High). The simplified fuzzification for the inputs and outputs are shown in Fig. 4 and rule matrix in Table 4.

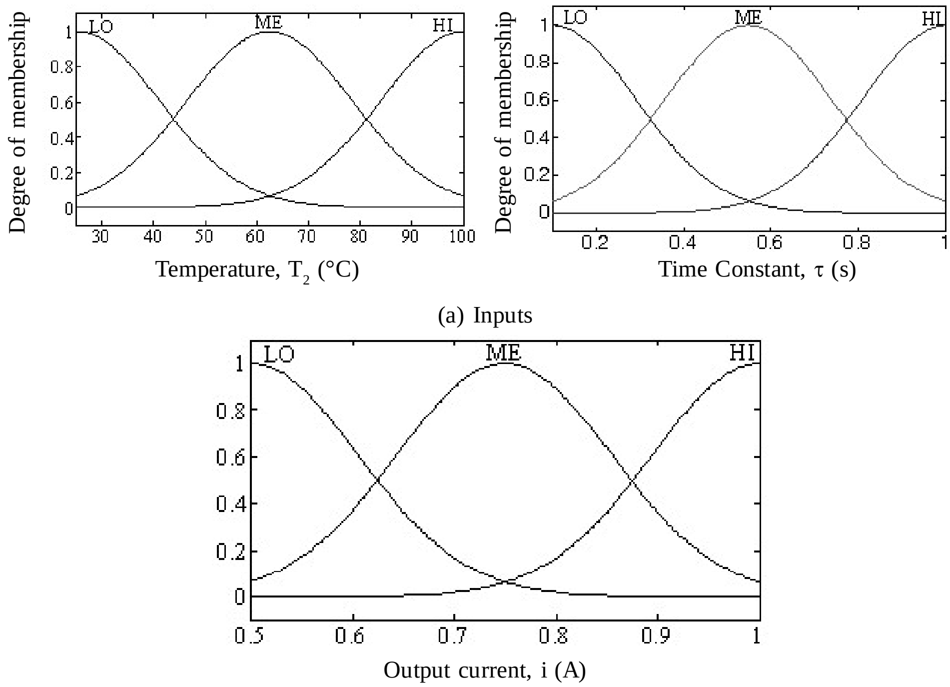


Fig. 4: Fuzzification of Selected data pairs

Table 4
Fuzzy Rule Matrix for 9 Rules

	LO	ME	HI
LO	HI	HI	HI
ME	ME	ME	ME
HI	LO	LO	LO

5. RESULTS AND DISCUSSION

Researchers have explored various control schemes like P, PI, PID, PWM, Variable Structure Control, Sliding Mode Control etc. for the control of SMA actuators. But the fuzzy control scheme has not been implemented effectively as it contains large number of parameters that are required to be tuned for obtaining optimal results. When we tune the fuzzy controller, it changes the shape of the control surface, which in turn affects the behavior of the closed loop control system. However, in [19], authors confirmed that the fuzzy controller was effective in controlling the oscillations of a SMA actuated vibration absorber. The nonlinear relationships between T_2, τ and i, before being considered as inputs to the fuzzy controller, are shown in Fig. 5. It can be inferred that when the time constant is less than 0.1s, the power requirements are very high and hence the minimum value of 0.1s is considered and the maximum value is set at 1.0s with respect to the SMA actuation properties. The simulation results, Fig. 6 and 7, shows the 3D control surface plotted between the inputs and output of the fuzzy controller. Fig. 6 has some similarity with the Fig. 5 as the entire ranges of values are considered with 49 rules for obtaining the surface. The control surface, Fig. 7, is the result of the tuned fuzzy controller with 9 rules that shows that the unwanted control area has been removed. This is also similar to the control surface for a PD controller that is a plane in three dimensions.

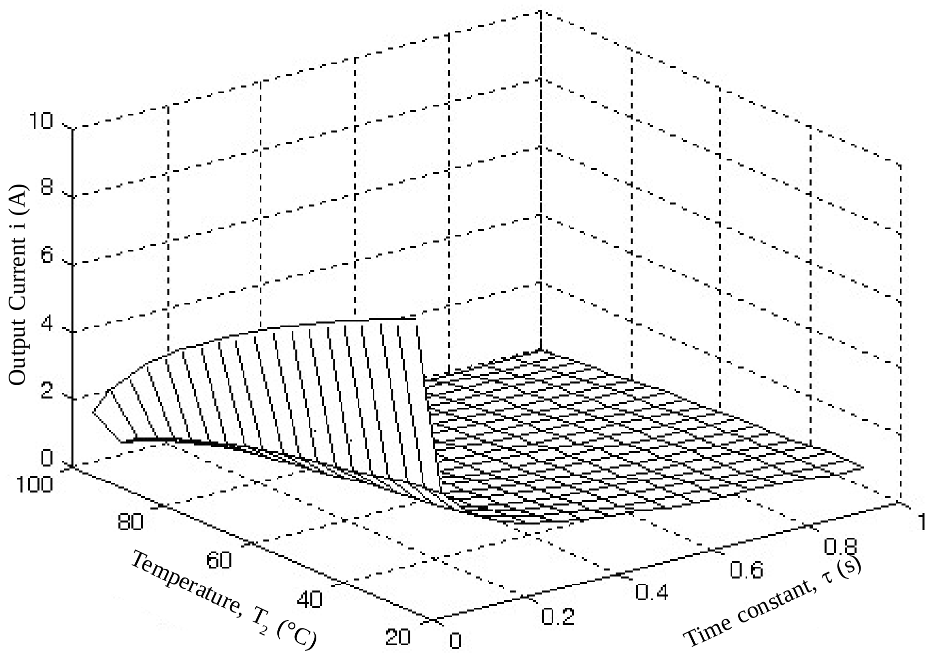


Fig. 5: The Nonlinear relationship between T_2, τ and i

6. CONCLUSIONS

This paper presented the design of a MISO (Multiple Inputs and Single Output), Mamdani type fuzzy controller with table-lookup scheme as its training algorithm for the control of shape memory alloy actuators. Fuzzy controllers are most effective to map the nonlinearity present in any nonlinear control problems as justified by the universal approximation theorem. The inputs and output for the controllers are obtained from the heat transfer model and arranged in a table. The various rules

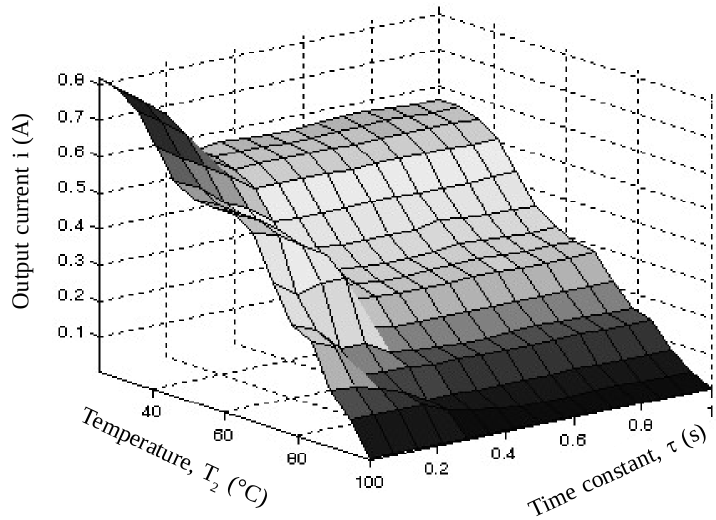


Fig. 6: Control Surface for 49 Rules

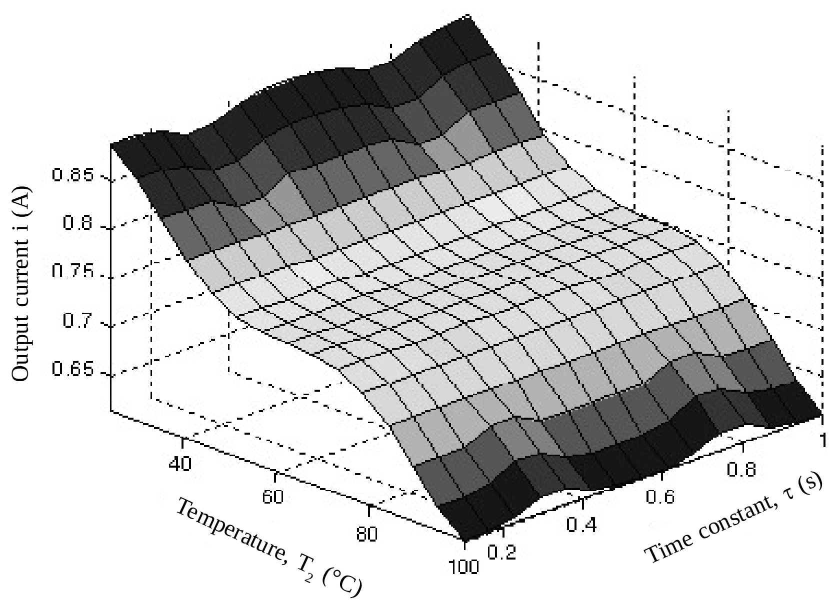


Fig. 7: Control Surface for 9 Rules

formed with the help of the table were studied and only the required rules were considered after dropping the rules from the unwanted control area. The simulation results proved that the trained fuzzy controller is most effective, simple to construct and adaptive to the environmental inputs. The training scheme is capable of reducing the number of rules from 225 to just 9 and hence 9 data pairs are well enough to represent and map the 225 data pairs. The study is made only for the heating system but it can also be applied for a cooling system or both. Though the final control surface is smooth, due to the Gaussian membership functions, further tuning is required for making the surface similar to the classical PD controller.

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