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Fuzzy Rough Systems used for Knowledge Acquisition

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Abstract

Several fuzzy systems work with max and min operators for union (disjunction) and intersection (conjunction) respectively. But, in any fuzzy system based on learning process, max and min operators cannot be effective, as they don't allow acquisition. This paper describes a tool for fuzzy rough approximation using Lukasiewicz T and S norms.

1. INTRODUCTION

Learning denotes changes in the system that are adaptive in the sense that they enable the system to do the same task or tasks drawn from the same population more effectively and more efficiently the next time. Learning involves skill refinement and knowledge acquisition. Knowledge acquisition is the process of adding new knowledge to a knowledge base and refining or otherwise improving knowledge that was previously acquired. Acquired knowledge may consist of facts, rules, concepts, procedures, heuristics, formulae, relationships, statistics or other useful information. To be effective, the newly acquired knowledge should be integrated with existing knowledge in some meaningful way so that nontrivial inferences can be drawn from the resultant body of the knowledge. The knowledge should be accurate, non-redundant, consistent and fairly complete in the sense that it is possible to reliably reason about many of the important conclusions for which the system was intended. The theory of rough sets and fuzzy sets are involved in the process of knowledge acquisition.

In learning process, for knowledge acquisition, it is necessary to repeat the same process several times. Hence, for knowledge acquisition, the usual T-norm and S-norm for fuzzy sets do not give importance. As by Lukasiewicz T and S norms, it is possible to differentiate A and AA for any event A, this paper

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took Lukasiewicz T and S norms as the tools for constructing fuzzy rough approximations.

In this paper, Lukasiewicz T-norm and S-conorm are used for the construction of the fuzzy partition, whose span gives the bags of fuzzy sets. Using the bags of fuzzy sets, the approximation for any fuzzy input is found.

Here, section two gives the required mathematical preliminaries for this paper. It describes bag theory in brief. Section three describes rough and fuzzy concepts required for the construction of fuzzy rough bags. Section four describes the construction and algebra of fuzzy partition and section five gives the idea of constructing fuzzy rough bags.

2. MATHEMATICAL PRELIMINARIES

Let U = $\{x_1, x_2, \ldots, x_n\}$ be the finite universe of discourse in which the elements are order variant. As this paper is meant for application in information system and in any information system the records are order variant, we assumed that U is order variant. Then any subset A can be represented using the characteristic function χ_A by $(\chi_A(x_1), \chi_A(x_2), \ldots, \chi_A(x_n))$. Here, the power of A is given by $\sum_{x_i \in U} \chi_A(x_i)$ and is denoted by pow(A).

The bags [4] are similar to a finite crisp set whereas; here repetition of the elements is allowed. Mathematically, any bag A can be characterized as follows:

Denote the number of repetitions of the most frequently appearing element in A by n. Partition the elements of the universe of discourse U such that A_0 contains all the elements those do not occur in A; A_1 contains all the elements occur exactly once in A; A_2 contains all the elements occur twice in A an so on. Then A can be represented by a simple function $f_A(x_j) = \sum i.\chi_{A_i}(x_j)$. Here A can be represented as $(f_A(x_0), f_A(x_1), \ldots, f_A(x_n))$. Hence, the power of A is defined by $\sum f_A(x_i)$ and is denoted by pow(A).

Example 2.1: Consider a bag A = {a, a, a, b, b, c, c, d} from the universe of discourse U = {a, b, c, d, e}. Define A_0={e}, A_1={d}, A_2={b,c}, A_3={a}. Then, $f_A(a) = 3$, $f_A(b) = 2$, $f_A(c) = 2$, $f_A(d) = 1$, $f_A(e) = 0$. Hence, A can be written as A = (3, 2, 2, 1, 0).

3. ROUGH SETS AND FUZZY SETS

This section deals with the basic definitions of rough sets and fuzzy sets which are necessary for the construction of fuzzy rough bags. First, we describe theory of rough sets.

3.1. Rough Sets

In 1982, Z. Pawlak [14] developed theory of rough sets. This theory provides a tool for solving the problems of pattern recognition, knowledge representation and knowledge acquisition.

3.1.1. Knowledge Base

Consider the finite universe of discourse U and an equivalence relation R on U. Consider the collection U/R of equivalence classes of R. They are referred as categories or concepts of R or granules and $[x]_R$ denotes a category in R containing an element $x \in U$.

A knowledge base is defined as a relational system K=(U,Ŕ), where U is nonempty and Ŕ is a family of equivalence relations over U. For any subset P of Ŕ, the intersection of all elements of P is also an equivalence relation and is denoted by IND(P) and is called an indiscernibility relation over P. Moreover, $[x]_{IND(P)} = \bigcap_{R \in P} [x]_R$.

For example, consider the universe of discourse U={a,b,c,d,e} with the equivalence relations R and S which produce the equivalence classes {{a,b,c},{d},{e}} and {{a,b,d}, {c,e}} respectively. If P={R,S} then the indiscernibility relation IND(P) partitions U as {{a,b},{c},{d},{e}}.

Thus, U/IND(P) denotes the knowledge associated with the family of equivalence relations P, called P-basic knowledge about U in K.

IND(K) denote the minimal set of equivalence relations containing all elementary relations of K. i.e., IND(K) = $\cap$\{IND(P)/ P$\subseteq$ Ŕ\}

3.1.2. Exact and Rough Sets in K

Let X be any subset of U. X is said to be R-definable [15] if X is the union of some R-basic categories; otherwise X is R-indefinable. The R-definable sets are also termed as R-exact sets and R-indefinable sets are termed as R-inexact or R-rough.

The set X in U is called exact in K, if there exists an equivalence relation R in IND (K) such that X is R-exact and X is said to be rough in K, if X is R-rough for every R in IND(K).

As some of the sets are rough, it can be explicitly expressed by using K. So, it is necessary to approximate them using the elements of K.

3.1.3. Approximations of a Set

Let K=(U, Ŕ) be a knowledge base and R∈IND(K). Then for any subset X of U, define

$$\underline{R}X = \cup\{Y\in U/ R: Y\subseteq X\}$$
$$\overline{R}X = \cup\{Y\in U/ R: Y\cap X \neq \Phi\}$$

Here $\underline{R}X$ and $\overline{R}X$ are said to be R-lower and R-upper approximations of X and ($\underline{R}X$, $\overline{R}X$) is called R-rough set. If X is R-definable then $\underline{R}X = \overline{R}X$ otherwise X is R-Rough.

The boundary $BN_R(X)$ is defined as $BN_R(X) = \overline{R}X - \underline{R}X$. Hence, if X is R-definable, then $BN_R(X) = \Phi$. Any object in $\underline{R}X$ gives the certainty of the object in $\overline{R}X$ with respect to R. Any object in X gives the possibility of the object in X with respect to R. Hence, $\underline{R}X$ is called R-positive region of X and U-$\overline{R}X$ is called the R-negative region of X.

Example 3.1.3.1: Consider the universe of discourse U = {a,b,c,d,e,f} and R be any equivalence relation in IND(K) which partitions U into T={{a,b,d},{c,f},{e}}. Then for any subset X={a,b,c,d} of U, $\underline{R}X$={a,b,d} and $\overline{R}X$ = {a,b,c,d,f}. Hence, $BN_R(X)$={c,f}. Hence, the R-positive region of X is {a,b,d} and the R-negative region of X is {e}.

On the other hand, consider a subset Y={c,e,f}. Here, $\underline{R}Y$={c,e,f} and $\overline{R}Y$={c,e,f}. Therefore, $BN_R(Y) = \Phi$. Hence, Y is said to be R-definable. However, this paper does not concentrate on the choice of the indiscernibility relation R, for convenience we denote $\underline{R}X$ and by $\underline{X}$ and $\overline{X}$ respectively.

The definition of rough sets has been generalized by different ways; such as generalizing the approximation space [11]; generalizing the partitions into overlapping granules [12] etc.

3.1.4 Modified Definitions of Rough Sets

Let $U=\{x_1,x_2,\ldots,x_n\}$ be a finite universe of discourse and $\mathcal{B} =\{B_1,B_2,\ldots,B_t\}$ a partition on U under some equivalence relation.

Consider the topology T($\mathcal{B}$) on U, by taking all possible unions of the elements of $\mathcal{B}$ including the null set. Denote T($\mathcal{B}$)={C_1,C_2,…,C_p}. Here, $\mathcal{B}$ is an open base for T($\mathcal{B}$). Now, it is necessary to note the theorems in [6].

Theorem 3.1.4.1: Suppose that $\mathcal{B}$ ={ B_1, B_2,…,B_t} is any partition of U and F is any subset of U, then $\cup\{B_j / B_j \cap F \neq \Phi\} = \cap \{C_j / F \subseteq C_j\}$

Theorem 3.1.4.2: $\cup\{B_j / B_j \subseteq A\} = \cup \{C_j / C_j \subseteq A\}$

By theorems 3.1.4.1 and 3.1.4.2, the definitions of lower and upper rough approximations can be redefined as follows:

$\underline{A} = \cup\{C_j / C_j \subseteq A\}$ and

$\overline{A} = \cap\{C_j / C_j \supseteq A\}$ respectively.

Example 3.1.4.3: Let U={a,b,c,d} and $\mathcal{B}$ ={{a},{b,c},{d}}. Here, $\mathcal{B}$ is a partition of U. Consider a set R={a,b}. Here, T($\mathcal{B}$) = {Φ, {a}, {b,c}, {d}, {a,d}, {a,b,c}, {b,c,d},U}. Therefore, $\underline{R} = \Phi \cup \{a\} = \{a\}$ and $\overline{R} = \{a,b,c\} \cap U = \{a,b,c\}$

As the finite intersection and union of open sets are again open, the lower and upper rough approximations are the elements of T($\mathcal{B}$) satisfying the following properties

$\underline{A} = C_j$, where $C_j \in$ T($\mathcal{B}$); $C_j \subseteq A$ and $|A-C_j|$ is minimum and

$\overline{A} = C_j$, where $C_j \in$T($\mathcal{B}$); $C_j \supseteq A$ and $|C_j-A|$ is minimum respectively.

Example 3.1.4.4: Let U={a,b,c,d} and $\mathcal{B}$ ={{a},{b,c},{d}}. Here, $\mathcal{B}$ is a partition of U. Consider a set R={a,b}. Here, T($\mathcal{B}$) = {Φ,{a},{b,c},{d},{a,d},{a,b,c},{b,c,d},U}. The sets A= Φ and B={a} are contained in R. Now |R-A|=2 and |R-B|=1. Therefore, B is the lower approximation of R. The sets C={a,b,c} and D=U contain R. Also, |C-R|=1 and |D-R|=2. Therefore, C is the upper approximation of R.

Now, we describe fuzzy sets and S, T norms on them.

3.2. Fuzzy Sets

In 1965, Zadeh introduced fuzzy sets [13]. In crisp sets, the codomain of characteristic function is {0,1}. Fuzzy sets are obtained by replacing this codomain {0,1} with [0,1]. Here, the function, which is defined, is called as membership function and the value assumed by the membership function is called the grade of membership in the given fuzzy set A. It can be defined as follows:

Consider the universe of discourse U = $\{x_1, x_2, \ldots, x_n\}$. Then any fuzzy subset A can be defined as $\left\{\frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \ldots + \frac{\mu_A(x_n)}{x_n}\right\}$ where μ_A is the membership function defined from U to [0,1].

In any information system the records are order variant and this work focuses on the information system, for computational purpose, the fuzzy set A is defined as $A=(\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n))$.

For any two fuzzy sets A and B, union and intersection of them can be obtained by using the max and min operators say S-norms and T-norms. In [7], they are defined as

$\mu_{A\cup B}(x_i)=\max(\mu_A(x_i), \mu_B(x_i))$ and

$\mu_{A\cap B}(x_i)=\min(\mu_A(x_i), \mu_B(x_i))$ respectively.

As in the case of crisp sets, here the union and intersection of two fuzzy sets are not unique; i.e., S norms and T norms are not unique.

The basic definitions of S-norm and T-norm given [1] are defined below.

Definition 3.2.1: A T-conorm or S-norm denoted by S, is a binary operation from $I \times I \rightarrow I$ where I=[0,1] such that for all a,b,c in I, the following properties hold.

(a) S(a,1)=1

(b) S(a,0)=a

(c) S(a,b)=S(b,a)

(d) S(a,S(b,c))=S(S(a,b),c)

(e) S(a,b) is monotonic in both variable

(f) S is continuous

Definition 3.2.2: A T-norm T is a binary operation from $I \times I \rightarrow I$ where I=[0,1] such that for all a,b,c in I, the following properties hold.

(a) T(a,1)=a

(b) T(a,0)=0

(c) T(a,b)=T(b,a)

(d) T(a,T(b,c))=T(T(a,b),c)

(e) T(a,b) is monotonic in both variable

(f) T is continuous

There are several T and S norms are meant for numerous applications. In this paper, we use T and S norms given by Lukasiewicz.

For any two fuzzy sets $A=(\mu_A(x_1), \mu_A(x_2),\ldots, \mu_A(x_n))$ and $B=(\mu_B(x_1), \mu_B(x_2),\ldots, \mu_B(x_n))$, using Lukasiewicz norms union and intersection are given by the membership functions

$\mu_{A\cup B}(x_j) = \min(1, \mu_A(x_j) + \mu_B(x_j))$ and
$\mu_{A\cap B}(x_j) = \max(0, \mu_A(x_j) + \mu_B(x_j) - 1)$ respectively [3,7,9].

Now, we describe the construction of fuzzy bags and the algebra of fuzzy partition.

4. ALGEBRA OF FUZZY PARTITION AND FUZZY BAGS

First we discuss fuzzy partition given by Lukasiewicz S and T norms and the algebra spanned by it.

4.1. Fuzzy Partition and Algebra of Fuzzy Partition

The set $X=\{A_1, A_2,\ldots, A_m\}$ is said to be a fuzzy partition of U where $A_j = (p_{j1}, p_{j2},\ldots, p_{jn})$, j=1,2,…,m, if $\cup A_i = U$ and $A_i \cap A_j = \Phi$ whenever $i\neq j$ where p_{ij} represents the membership value of x_i in A_j.

Hereafter, for convenience, we denote $A=(p_1, p_2,\ldots, p_n)$ by $A = (p_t)$

Lemma 4.1.1: $A\cup(B\cup C)=(A\cup B)\cup C$

Proof: Let $A=(p_{it})$; $B=(p_{jt})$; $C=(p_{kt})$; $A\cup B=(a_t)$; $B\cup C=(b_t)$; $A\cup(B\cup C) = (c_t)$ and $(A\cup B)\cup C=(d_t)$. Then $a_t=\min(1, p_{it}+ p_{jt})$; $b_t=\min(1,p_{jt}+p_{kt})$; $c_t=\min(1,p_{it}+b_t)= \min(1, p_{it}+ \min(1, p_{jt}+ p_{kt})) = \min(1, 1+p_{it}, p_{it}+p_{jt}+p_{kt})= \min(1, p_{it}+ p_{jt}+ p_{kt})$.

Similarly, it can be proved that $d_t = \min(1, p_{it}+ p_{jt}+ p_{kt})$.

Hence, $A\cup(B\cup C)=(A\cup B)\cup C$.

From lemma 4.1.1, for any three sets $A=(p_{it})$, $B=(p_{jt})$ and $C=(p_{kt})$, $A\cup B\cup C$ is given by $\min(1, p_{it}+ p_{jt}+ p_{kt})$ and it can be generalized for any finite number of sets by iteration. Hence, $A_1\cup A_2\cup\ldots\cup A_m= \min(1,p_{1t}+p_{2t}+\ldots+p_{mt})$. As X is the partition, it is necessary that $p_{1t}+p_{2t}+\ldots+p_{mt}\geq 1$.

Also, by the definition of T-norm, $p_{it}+p_{jt}\leq 1$ if $i\neq j$. Hence, by using above two conditions any fuzzy partition can be constructed. However, these fuzzy partitions can be optimized using fuzzy c means.

Similarly, as in lemma 4.1.1, it can be proved that $A_1 \cap A_2 \cap \ldots \cap A_m = \max(0, p_{1t}+ p_{2t}+ \ldots +p_{mt}-m)$.

Now, we construct an algebra Σ spanned by X as follows:

(a) $A\in X \Rightarrow A\in\Sigma$

(b) $A,B\in\Sigma \Rightarrow A\cup B\in\Sigma$

(c) $A\in S \Rightarrow A^c\in\Sigma$

(d) $\Phi\in\Sigma$

Here, Σ is constructed based on the definition given by Royden on algebra of sets [10]. In order to show that all possible intersections of finite number of sets in Σ is in Σ, it is necessary to prove De Morgan's Laws.

Lemma 4.1.2: For any two fuzzy sets A and B, $A^c\cup B^c=(A\cap B)^c$ and $A^c\cap B^c=(A\cup B)^c$

Proof: Let $A=(a_t)$; $B=(b_t)$; $A\cap B=(c_t)$. Then $A^c=(1-c_t)$ and $B^c=(1-b_t)$. Hence, $A^c\cup B^c= (\min((1,(1-a_t)+(1-b_t)))=(\min(1-2-a_t-b_t))$————————(4.1.2.1)

Now, $A\cap B=(\max(0,a_t+b_t-1))\Rightarrow c_t\geq 0$; $c_t\geq a_t+b_t-1$ and either $c_t=0$ or $c_t=a_t+b_t-1$, which is maximum $\Rightarrow 1-c_t\leq 1$; $1-c_t\leq 2-a_t-b_t$ and either $1-c_t=1$ or $1-c_t=2-a_t-b_t$ which is minimum.

Hence, $(A\cap B)^c=(1-c_t)=(\min(1,2-a_t-b_t))$ .....(4.1.2.2)

From (4.1.2.1) and (4.1.2.2), the result follows.

Similarly, it can be proved that $A^c\cap B^c=(A\cup B)^c$

Theorem 4.1.3: If $A,B\in\Sigma$ then $A\cap B\in\Sigma$ and $A-B\in\Sigma$

Proof: $A,B\in\Sigma \Rightarrow A^c,B^c\in\Sigma \Rightarrow A^c\cup B^c\in\Sigma \Rightarrow (A\cap B)^c\in\Sigma \Rightarrow A\cap B\in\Sigma$

Now, $A,B\in\Sigma \Rightarrow A,B^c\in\Sigma \Rightarrow A\cap B^c \in \Sigma \Rightarrow A-B\in\Sigma$.

Hence, the algebra of fuzzy sets contains all possible unions, intersections, differences and complements.

4.2. Bags of Fuzzy Sets

Let $X=\{A_1, A_2,\ldots, A_m\}$ be the collection of fuzzy sets defined on a finite universe of discourse U. Let f be any set function defined on X to the set of all nonnegative integers W. Using the function f, in the section, three types of bags are used. They are (a) U Bag (b) I Bag and (c) M Bag.

In the following notations, the notation set(A) is used for set representation of A which is the tuple of the membership values of the elements of the universe of discourse in A.

(a) **U Bag**: The bag of fuzzy sets taken from X, whose set representation is the union of all members of the bag is called a U bag and is represented using ( ). If A,B and C are the fuzzy sets taken from U for 2 times, 3 times and 4 times respectively, for a U bag, the corresponding set representation is given by A∪A∪B∪B∪B∪C∪ C∪ C∪ C and in general, it is denoted by (2A,3B,4C).

The bag $T=(f(A_1),f(A_2),\ldots,f(A_m))$ is said to be a U-Bag if the corresponding set representation of T is

$$Set(T) = \bigcup_{j=1}^{m} \bigcup_{i=1}^{f(A_j)} Set(A_j)$$

(b) **I Bag:** The bag T consists of U bags and the elements of X, which has the set representation as the intersection of all the set representations of all the elements of T, is called an I bag and is enclosed by [ ]. Alternatively, it can be defined as follows:

The bag $T=[f(B_1), f(B_2),\ldots, f(B_s)]$ is said to be a I Bag where each B_j is one of the A_i^{s}' or an U Bag, if the corresponding set representation of T is

$$Set(T) = \bigcap_{j=1}^{s} \bigcap_{i=1}^{f(B_j)} Set(B_j)$$

(c) **M Bag:** A bag consists of I bags and U bags is called as M bag (multiple bag) and is enclosed with 〈 〉. Consider a M bag, $T=\langle f(B_1),f(B_2),\ldots,f(B_t)\rangle$ where each B_j is either an U bag or I bag. The corresponding set representation of T is given by. $Set(T) = \bigcup_{j=1}^{t} \bigcup_{i=1}^{f(B_j)} Set(B_j)$.

By the definitions of union and intersection of fuzzy sets, it can be observed that the idempotent law does not hold for fuzzy sets and hence each member of the algebra can be represented as a M Bag.

Example 4.2.1: $[(A\cup A)\cup(A\cup B)^c]^c\cup C=[(A\cup A)^c\cap(A\cup B)]\cup C=[A^c\cap A^c\cap(A\cup B)]\cup C$ Hence, the M bag of $[(A\cup A)\cup(A\cup B)^c]^c\cup C$ is $\langle[2A^c, (A,B)],C\rangle$.

The next section describes the construction of fuzzy rough bags. Here, an algorithm is proposed to compute all possible fuzzy rough bags given by fuzzy partition.

5. FUZZY ROUGH BAGS

First, we discuss the theory given by Nakamura, Dubois and Prade for fuzzy rough sets.

5.1. Fuzzy Rough Sets

On replacing crisp equivalence relation into fuzzy similarity relation, the universe of discourse U can be partitioned into a set G, a collection of all fuzzy similarity classes.

For a given fuzzy set F and a family $\Gamma = \{F_1,F_2,\ldots,F_m\}$of fuzzy sets on U the lower and upper approximations of F by Γ are defined by

$$\mu_{\underline{\Gamma}F}(F_j) = \inf_{x} \mu_{Fi}(x)\rightarrow \mu_F(x)$$

$$\mu_{\overline{\Gamma}F}(F_j) = \sup_{x} \mu_{Fi}(x)^* \mu_F(x)$$

where * denotes an operator for which $a*b\leq\min(a,b)$ and $\rightarrow$ is called S –implication [1] operator for which $a\rightarrow b=1-a*(1-b)$.

The lower approximation of F is given by $\underline{\Gamma}F=(\mu_{\underline{\Gamma}F}(F_1), \mu_{\underline{\Gamma}F} (F_2),\ldots, \mu_{\underline{\Gamma}F} (F_m))$ and the upper approximation is given by $\overline{\Gamma}F = (\mu_{\overline{\Gamma}F}(F_1),\mu_{\overline{\Gamma}F}(F_2),\ldots \mu_{\overline{\Gamma}F}(F_m))$.

Here $(\underline{\Gamma}F, \overline{\Gamma}F)$ is called a Γ-fuzzy rough set [2,5,8].

As it is mentioned earlier, this theory does not give impact on the repeated occurrence of a particular event. So, 5.2 introduces the theory of fuzzy rough bags.

5.2. Fuzzy Rough Bags

Consider the algebra S spanned by the given fuzzy partition $X=\{A_1, A_2,\ldots, A_m\}$ of U where $A_j = (p_{j1}, p_{j2},\ldots, p_{jn})$; j = 1,2,…, m

For any fuzzy subset F of U, define $up(F) = \inf_{T \supseteq F}$ and $low(F) = \sup_{T \subseteq F} pow(T)$. The upper and lower approximations of F are defined as

$\underline{F} = T$ where Pow(T)-low(F) is minimum and T⊆F and

$\overline{F} = T$ where up(F)-Pow(T) is minimum and T⊇F respectively.

The M Bags corresponding to $\underline{F}$ and $\overline{F}$ are called the lower and upper fuzzy rough bags of F respectively. They are denoted by lowbag(F) and upbag(F) respectively. As for any fuzzy set, the bag representation is not unique; it can be noticed that for any set, there may be more than one fuzzy rough bags. It is shown in the following example.

Example 5.2.1: Consider a fuzzy partition {(0.2,0.8), (0.8,0.2)} on U={x_1,x_2}. Then the algebra spanned by the partition given by Σ = {(0,0), (0,0.2), (0,0.4), (0,0.6), (0,0.8), (0,1), (0.2,0.2), (0.2,0.4),(0.2,0.6),(0.2,0.8),(0.2,1),(0.4,0), (0.4,0.2), (0.4,0.4), (0.4,0.6), (0.4,0.8), (0.4,1), (0.6,0), (0.6,0.2), (0.6,0.4), (0.6,0.6), (0.6,0.8), (0.6,1), (0.8,0), (0.8,0.2), (0.8,0.4), (0.8,0.6), (0.8,0.8), (0.8,1), (1,0), (1,0.2), (1,0.4), (1,0.6), (1,0.8), (1,1)}

Let F=(0.3,0.4) be any fuzzy set. By definition, up(F)=0.8 and low(F)=0.6. The sets (0.0.8), (0.8,0), (0.2,0.6), (0.6,0.2), (0.4,0.4) have the same power. Among these, (0.4,0.4) contains F. Hence, it is called as the upper fuzzy rough approximation of F and the one of the corresponding upper fuzzy rough bags is given by ⟨2[(2A),A^c]⟩. The sets (0,0.6),(0.2,0.4),(0.4,0.2),(0.6,0) have the same power, in which (0.2,0.4) contained in F. Hence, (0.2,0.4) is called the lower fuzzy rough approximation of F. One of the corresponding fuzzy rough bags is given by ⟨[(2A),A^c],[4B^c]⟩.

The above procedure of computing fuzzy rough bags is illustrated by an algorithm.

5.2.2 Algorithm

Let A be a list of elements of A_1, A_2,…, A_n where A_i = {a_{i1}, a_{i2},….,a_{im}} with n= |A|

1. Copy A to B
2. For each X in B

    Find X^c

    If X^c is not in B then include X^c in B

3. For every X_i,X_j in B, i=1,2,….,n and j=1,2,….n

   Find U=Union(X_i,X_j) and U^C

   I= Intersect(X_i,X_j) and I^C

   If U is not in B include U in B

   If U^c is not in B include U^C in B

   If I is not in B include I in B

   If I^c is not in B include I^c in B

   n=|B|

4. If !issame(A,B)

   Copy B to A

   Goto Step 3

5. Display A

6. End

However, this approach deals with the entire algebra, it increases the time as well as computational complexity. But, the results obtained by this process reduce the ambiguity than the existing methods. So, it is necessary to derive some tools to reduce this complexity.

6. CONCLUSION

In this paper, we discussed three different kinds of fuzzy bags, which were induced by Lukasiewicz T-norm and S-conorm. Using these bags, the lower and upper approximations were derived.

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