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ASSESSING THE ROLE OF COINTEGRATION ON PORTFOLIO EFFICIENCY: A MODERN PORTFOLIO THEORY PERSPECTIVE

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ABSTRACT

The purpose of this article is to identify whether or not the cointegration test enhances the efficiency of modern portfolio theory. Critics have raised concerns regarding the reliance of this theory on the short-term measure known as correlation. In order to address this critique, the present study conducted a cointegration test on a group of assets, including the stock index, agriculture futures, energy futures, metals futures, and cryptocurrency, using daily data spanning from 2018 to 2023. Following that, portfolios are constructed based on modern portfolio theory using two distinct approaches: one that considers the cointegration among assets and another that does not. The empirical analysis shows that the risk-return characteristics of the portfolios are essentially the same regardless of the use of the cointegration test. Portfolios purely based on modern portfolio theory, on the other hand, do a little better, but not enough to be statistically significant.

Keywords: Modern portfolio theory, Correlation, Cointegration

JEL Classification: G11, C58

INTRODUCTION

A portfolio is a collection of various financial assets held by an investor. Portfolio optimization is a decision- making process with the objective of maximising return and minimising risks. These competing objectives are solved in three phases: asset selection, asset allocation, and asset management. Asset selection is a process of choosing a group of assets, which may belong to the same or other asset classes. The practise of asset allocation assists investors in finding the optimal distribution of funds across various assets, with the aim of minimising risk and maximising returns. Asset management, the last phase, aids investors in assessing their portfolios and formulating plans for the acquisition, divestment, or retention of assets (Jothimani, Shankar, and Yadav 2017). To build an optimal portfolio, there are many different strategies, like the Markowitz model portfolio, the Sharpe single index model, and factor models (Senthilkumar, Namboothiri, and Rajeev 2022).

The portfolio design based on the Markowitz model is the most popular (Massahi, Mahootchi, and Arshadi Khamseh 2020). Harry Markowitz laid the groundwork for what is now known as modern portfolio theory in his seminal work (Markowitz 1952). The main objective is to maximise the expected (mean) return while at the same time minimising the risk (variance) of the portfolio. Markowitz's modern portfolio theory provides an easy-to-implement methodology for portfolio selections, but at the same time, many researchers claim that his mean–variance model performs poorly in practise and is faced with many criticisms. For example, Gupta and Guidi (2012) pointed out that Modern portfolio theory uses a short-term measure called correlation as a measure of asset co-movements as input into the portfolio optimization issue, Frankfurter et al. (1971) found that the Markowitz optimal portfolio is located below the true efficient frontier a large proportion of the time, Jobson and Korkie (1981) pointed out that sometimes the Markowitz optimal portfolio is outperformed by an equally weighted portfolio. Jorion (1985) found that Markowitz's optimal portfolio is not well-diversified. Particularly when short sales are forbidden, Markowitz's optimal portfolios may invest in too few assets. This result is also illustrated by Egozcue et al. (2011) using the rankings of completely and partially diversified portfolios.

However, this current study is mainly focused on the criticism of the correlation measure. The previous studies suggested cointegration instead of correlation. Cointegration and correlation are related but different concepts. Correlation reflects co-movements in returns, which are liable to great instabilities over time. It is fundamentally a short-run measure, and correlation-based portfolio management strategies commonly require rebalancing. Cointegration, on the other hand, measures long-run co-movements in prices, which may occur even during periods when static correlations appear low. Hence, asset selection based on the cointegration test may be more effective for long-term diversification (Syriopoulos 2004). Even though most of the studies agree on the efficiency of cointegration measures in the investment decision process over correlation measures, some authors are arguing that cointegration is primarily based on the concept that first differencing data will result in the loss of trend information and therefore provide incorrect diversification advice (Aroskar and Ogden 2011).

The previous studies have different opinions about the efficacy of correlation and cointegration during the investment decision making process. This will cause confusion among long-term investors when making investment decisions. Since modern portfolio theory uses correlation, this study makes an attempt to examine whether or not using the cointegration test enhances the efficiency of modern portfolio theory. The integration of cointegration findings with modern portfolio theory offers potential insights into the potential of cointegration to address the aforementioned objective. To solve this, this study first conducts a cointegration analysis among the chosen assets and then builds portfolios after eliminating the cointegrated assets. To know the efficiency of the cointegration test, this study also builds the portfolios with the whole sample, regardless of the cointegration of the assets. Following that, a risk adjusted evaluation was undertaken between portfolios with and without a cointegration test.

The rest of the paper is organized as follows: The second section discusses the literature review and the research gap. Third section outlines the data and methodological design of the study. Section 4 presents the results and empirical findings. Finally, the fifth section summarises the study and concludes.

LITERATURE REVIEW AND RESEARCH GAP

In the growing literature, the cointegration test has been used in numerous research papers to investigate the possibility of links between different financial assets, and the majority of studies propose tying the outcomes to investment decision-making. For example, in order to maximise the benefits of portfolio diversification through the use of cointegration tests, Derindere, Glu, and Karagülle (2013) examine the ten different ships from the shipping industry; Caporale et al. (2021) examine stock market integration between the ASEAN stock markets and the US and China; Alagidede et al. (2011) examine African stock markets and the rest of the world; and Gil- Alaña et al. (2020) examines six major cryptocurrencies and their bilateral linkages with six stock market indices.

Even if the previous studies recommended combining the results of the cointegration test at the time of portfolio construction, studies that empirically examine this are extremely rare. As an illustration, Gallo et al. (2013) built two different portfolio types using the cointegration test. The indices in both portfolios are not cointegrated. One is equally weighted, while the other assigns weights to each index based on how independent or dependent it is, as determined by cointegration tests. They place greater weight on independent assets and less on dependent assets when constructing portfolios and compare their risk/return profiles to the conventional MPT (modern Portfolio theory) portfolio. They concluded that the portfolios based on the cointegration test had much higher returns and significantly lower levels of diversifiable risk than the MPT portfolio. Through the pairs trading cointegrated technique, Naccarato et al. (2019) solve the problem of Markowitz portfolio optimization for a long-term horizon investment. This strategy identifies the prices and returns of each stock on the basis of a cointegration relationship estimated by means of the Vector Error Correction Model (VECM), and it provides better results than the Markowitz model portfolio in the case of long-term investments. Dunis et al. (2011) constructed MCP tracking cointegration portfolios, the weights of each constituent currency pair will be determined from the least squares regression and notional cointegration-based portfolios constructed by taking the cointegration equation coefficients within a portfolio and multiplying them by their respective daily exchange rate. The sum of these then gives the portfolio price for each given day, and portfolio returns are generated from the equally weighted average of the returns from the moving averages. These two portfolios are compared with simple benchmark portfolios called equally weighted portfolios and historical return portfolios. The findings on which portfolio optimisation method works best yield mixed results here. The notional cointegration portfolios produce the strongest risk-adjusted returns: the MCP tracking portfolios also perform strongly, giving good risk-adjusted returns for each currency, The equal weighting method performs best for euro portfolios, whereas historical return portfolios perform the worst. In general, cointegration-based optimisation strategies add value, but as with all optimisation techniques, they should be used cautiously. In contrary, By employing Johansen's cointegration methodology to recognise long-term relationships between assets and comparing the performance of optimised portfolios built from samples of country funds and iShares with portfolios from the same samples but not optimised, Aroskar and Ogden (2011) finds that modern portfolio theory, which relies on short-term information from asset returns and correlation, has the ability to provide knowledge to aid long-term investment decisions, and cointegration analysis may suggest long-term integration among assets and will bias investors to exclude all assets in the cointegrating relation. This will lead to the wrong diversification decisions.

The expanding body of literature also reveals a pattern that using different methods to improv the investment results of traditional modern portfolio theory like; Kaszuba (2012) assesses whether correct application of robust estimators in the construction of minimum variance portfolios' (MVP) allows to achieve better investment results in comparison with portfolios defined using classical estimators, Seidl (2012)discusses an adjusted regime switching model in the context of portfolio optimization and compares the attained portfolio weights and the performance to a classical mean-variance set-up as introduced by Markowitz. Due to the poor performance of the mean–variance portfolio model, Dai and Kang (2022) propose some new efficient mean–variance portfolio selection models by considering L1- regularisation in the objective function to obtain a sparse portfolio, using the shrinkage method of Ledoit and Wolf to estimate the covariance matrix, and using the robust optimization method to mitigate the estimation errors of the expected return, Chen et al. (2021) propose a novel approach to portfolio construction by combining a machine learning-based model for stock prediction with the MV model for portfolio selection.

This study attempts to determine whether or not the cointegration test improves the investment results of modern portfolio theory. By reviewing the literature, it is found that most of the studies do not combine the cointegration result into modern portfolio theory; instead, they construct cointegration-based portfolios and MPT portfolios separately and compare their efficiency. But, , Aroskar and Ogden (2011) applied cointegration test to the optimized portfolios. In contrast, this current study conducts the cointegration test first and then optimize the portfolios with and without considering the results of the cointegration test. Then, a thorough investigation of the risk-return characteristics of portfolios with risk adjusted evaluation is undertaken to determine if the investment results of modern portfolio theory have improved or not. Some studies from literature examine optimal asset allocations, while others examine only risk-return characteristics without risk-adjusted evaluation. But no study has examined optimum weights and risk-return characteristics, notably risk-adjusted evaluation, of portfolios while incorporating cointegration test results into modern portfolio theory. In this scenario, the present study has attempted to bridge this gap in the existing body of literature.

METHODOLOGY

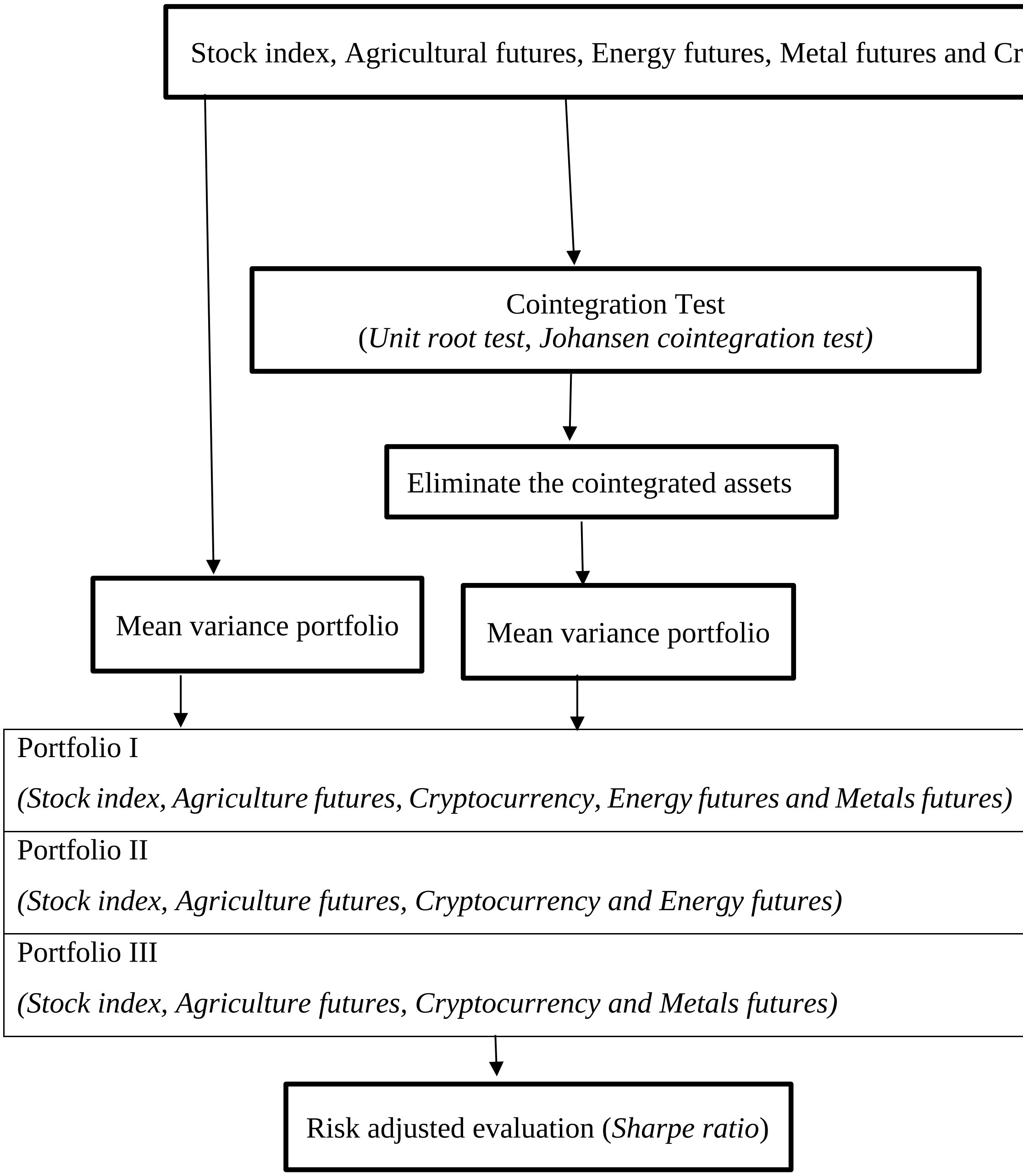

Figure 1: Conceptual Framework of the study

The conceptual framework for the investigation is depicted in Fig. 1. There are two ways to construct portfolios: one is built directly using all of the assets in accordance with modern portfolio theory, and the other is built by conducting a cointegration test, eliminating the cointegrated assets, and then constructing portfolios in accordance with modern portfolio theory. There are three types of portfolios, with different combinations of assets constructed in each type of portfolios. The Sharpe ratio is then used to evaluate the portfolio. Since stocks are the favourite investment choice of investors (Tai Wu, et al. 2021), S&P 500 is used as the benchmark index of all stock indices and bivariate cointegration test is executed between the S&P 500 and other assets. The assets that are cointegrated with the S&P 500 were eliminated for the portfolio construction.

In brief, asset selection is conducted in this study with and without consideration of the cointegration test; asset allocation is based on modern portfolio theory; and portfolio assessment is based on the Sharpe ratio.

Modern Portfolio Theory

The Markowitz model of portfolio optimization is a popular method for determining the optimum weight of the assets with the highest Sharpe ratio and the lowest return volatility.

The main measure for determining the optimum asset weights is the portfolio's expected return. The following is the formula to get the same, as per modern portfolio theory:

$$E(r_p) = \sum_{i=1}^{n} W_1 E(r_1) \quad (1)$$

Return and risk are two sides of the same coin. The following formula can be used to determine the variance of a portfolio with more than two assets:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j Cov(r_i, r_j) \quad (2)$$

Using matrix multiplication, choose the best asset weight after obtaining the expected return.

The expected return of portfolio is:

$$E(r_p) = W^T R = [w_1 \quad \cdots \quad w_j]\begin{bmatrix} E(r_1) \\ \vdots \\ E(r_j) \end{bmatrix} \quad (3)$$

R is the vector of expected returns for each individual asset in the portfolio, and W is the vector of weights for those assets.

The portfolio's variance is determined by:

$$\sigma_p^2 = W^t S(W) \quad (4)$$

The following formula is used to compute the portfolio's standard deviation:

$$\sigma_p = \sqrt{W^T S(W)} = \left[[w_1 \quad \cdots \quad w_j]\begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1j} \\ \vdots & \ddots & \vdots \\ \sigma_{j1} & \cdots & \sigma_{jj} \end{bmatrix}\begin{bmatrix} w_1 \\ \vdots \\ w_j \end{bmatrix}\right]^{\frac{1}{2}} \quad (5)$$

Where S is referred to as the variance-covariance matrix of the covariances between each of the asset returns in the portfolio.

The covariance of an asset's returns for the same asset is the variance of the asset's returns. The definition of W remains the same as above.

Sharpe Ratio

A risk-adjusted evaluation measures the level of risk involved in producing a particular return. Using a risk- adjusted formula known as the Sharpe ratio, the return on an investment is compared to its risk. The asset weights in a portfolio that produce the highest Sharpe ratio values are the ones to use. To calculate overall risk, the Sharpe's Index uses the standard deviation. In Sharpe's strategy, each portfolio is ranked in accordance with an evaluation metric. Return and risk premiums are both included in the numerator. Total risk as well as the standard deviation of return are included in the denominator. We will calculate the portfolio's overall risk and return variability with respect to the risk premium.

$$Sp = \frac{E(r_p) - (r_f)}{\sigma_p} \quad (6)$$

$E(r_p)$ stands for Expected return of portfolio whereas r_f is represented as risk free rate, at last these are divided with σ_p that is portfolio standard deviation.

Unit root

It is necessary to test stationarity before conducting a cointegration test. A stationary time series has a consistent mean and variation across time. To test stationarity, we should do a unit root test. The stationarity can be verified using many tests, including the Augmented Dickey-Fuller (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS), Phillips-Perron (PP), etc. The null hypothesis for the test is that there is a unit root. That means the series is non-stationary. If the P-value is greater than 5%, then the null hypothesis can't be rejected, which means the series is non-stationary. The series is stationary if it is less than 5%. Data at level are first evaluated for stationarity. The first difference form will next be examined if the nonstationary null hypothesis cannot be rejected at level. If all the time series are stationary at the first difference, i.e.,I(1), the appropriate test to use is the Johansen cointegration test. If the series are stationary at I(0) or I(2), the Johansen procedure cannot be used, and if the series are stationary at different orders, i.e., I(0) and I(1), then the ARDL bound test procedure is used.

Johansen Cointegration Test

The concept of cointegration is proposed by Granger (Hylleberg, Engle and Granger 1990). It is a statistical technique used to assist in locating equilibrium or long-run parameters for two or more variables. We can apply the Johansen cointegration test if the series are stationary in the same order at first difference I(1). Before conducting a cointegration test, selecting the optimum lag length is crucial because it affects how the ACF (autocorrelation function) on the residual is calculated. The choice of the VAR model's number of lags is predicated on the analysis of a cointegration vector between the series, which would then confirm the presence of a linear combination between them. There are numerous lag duration criteria, including LR, AIC, and SC.

Johansen (1988) developed a cointegration, based on the rank (r) of the matrix Π and the number of vectors determined by the knowledge of matrix rank (r), according to Eq. (7):

$$\Delta X_1 = \delta + \Gamma_1 \Delta X_{t-1} + \cdots \Gamma_{p-1} \Delta X_{t-p+1} + \Pi Y_{t-1} + \varepsilon_{x,t} \tag{7}$$

There are two likelihood ratios to test for Johansen cointegration, known as the Trace and Max statistics, respectively:

The trace test focuses on testing the null hypothesis, in which the number of distinct cointegration vectors is less than or equal to r, or the alternative hypothesis. In the alternative hypothesis, the number of vectors will be greater than r, according to Eq. (8).

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{g} In\left(1 - \hat{\lambda}_i\right) \tag{8}$$

where $\hat{\lambda}_i$ is the estimated value for the ith ordered eigenvalue and r is the number of cointegrating vectors under the null hypothesis. Intuitively, the larger is $\hat{\lambda}_i$, the larger and more negative will be $In\left(1 - \hat{\lambda}_i\right)$ and hence the larger will be the test statistic. Each eigenvalue will have associated with it a different cointegrating vector, which will be eigenvectors. A significantly non-zero eigenvalue indicates a significant cointegrating vector.

λ_{trace} is a joint test where the null is that the number of cointegrating vectors is less than or equal to r against an unspecified or general alternative that there are more than r. It starts with p eigenvalues, and then successively the largest is removed $\lambda_{trace} = 0$ when all the $\lambda_i = 0$, for i = 1,..., g.

$$\lambda_{max}(r, r+1) = -T\ In(1 - \hat{\lambda}_{r+1}) \tag{9}$$

In the maximum eigenvalue test the null hypothesis indicates that the number of vectors is r and consequently, the alternative hypothesis is the existence of $r+1$ cointegration vectors, represented by Eq. (9). λ_{max} conducts separate tests on each eigenvalue, and has as its null hypothesis that the number of cointegrating vectors is r against an alternative of $r+1$.

Data

The study makes use of nine commodity futures, one cryptocurrency, and a stock index for testing cointegration and the construction of portfolios. The daily data was retrieved from investing.com and collected for the four years and seven months between November 1, 2018, and June 15, 2023. Only those closing prices that coincide with the same date in each time series of data are used for the cointegration test as well as portfolio construction; all other prices are disregarded.

The S&P 500 was used in this analysis as a proxy for global stock indices. Since bitcoin has the biggest market cap as of 2023, according to coinmarketcap.com, it has been chosen as a representative of the cryptocurrency market. Agricultural, metal, and energy futures are the commodities taken for the analysis; for each commodity, there were three futures included in the analysis. The investment avenues used in this investigation are listed in Table 1.

5	
Stock Index	**S&P 500**
Metals futures	Gold
	Silver
	Copper
Energy futures	Crude Oil
	Natural gas
	Gasoline
Agricultural futures	US Coffee
	US Wheat
	US Corn
Cryptocurrency	Bitcoin

Table 2: Descriptive statistics

	S&P 500	Gold	Silver	Copper	Crude Oil	Natural gas
Mean	3630.281	1701.996	20.96974	3.464686	65.18983	3.60357
Medium	3759.79	1775.9	21.805	3.49675	62.52	2.7575
Maximum	4796.56	2069.4	29.418	4.9375	123.7	9.647
Minimum	2237.4	1201.4	11.772	2.1005	-37.63	1.482
Std. Dev.	636.5674	220.0152	4.190702	0.746746	20.96065	1.878085
skewness	-0.096171	-0.728232	-0.110668	0.105489	0.195582	1.352346
Kurtosis	1.722471	2.412244	1.689163	1.569219	3.410392	3.951446
Jarque-Bera	79.28112	117.1705	83.9459	99.35312	15.26801	390.4791
Probability	0.00000	0.00000	0.00000	0.00000	0.000484	0.00000
Obs.	1140	1140	1140	1140	1140	1140
	Gasoline	US Coffee	US Wheat	US Corn	Bitcoin	
Mean	1.800149	151.2319	658.6038	513.4768	22947.85	
Medium	1.61795	130.675	628.75	530.375	19129.25	
Maximum	3.0618	258.45	1425.25	818.25	67527.9	
Minimum	0.8394	88	418.5	302.75	3247.8	

Std. Dev.	0.535293	48.52012	170.7886	146.0311	16769.36	
Skewness	0.42428	0.491539	1.184328	0.249542	0.759598	
Kurtosis	1.935607	1.76995	4.38264	1.665984	2.408824	
Jarque-Bera	88.0169	117.7746	357.3057	96.36241	126.2286	
Probability	0.00000	0.00000	0.00000	0.00000	0.00000	
Obs.	1140	1140	1140	1140	1140	

Table 2 shows summary statistics of the return of 11 investment avenues for four years and seven months. There are 1140 observations there. The table shows that the mean returns for all markets are positive. Cryptocurrency has the highest mean, and it is a highly volatile market. The skewness and kurtosis of time series as well as the Jarque-Bera statistic test reject the null hypothesis of a normal distribution for all return series.

RESULTS AND EMPIRICAL FINDINGS

Tables 3 and 4 show the Unit root test with both the ADF and PP tests. Table 3 contains the estimated τ-statistics values of the indices at level, and Table 4 contains the estimated τ-statistics at first difference. At the level, it has been observed that all the assets are not mean-reverting and hence non-stationary. So, they are tested again by taking the first differences (i.e., returns), and the results prove the stationarity of the series, i.e., the series are integrated into Order 1, i.e., I (1). Therefore, this study can use the Johansen cointegration test.

Table 3: Unit root test on level

	Intercept		Intercept and Trend	
Variables	ADF	PP	ADF	PP
S&P 500	-1.0173	-1.0875	-2.1812	-2.1078
Bitcoin	-1.5553	-1.1092	-2.1205	-1.723
Gold	-0.5754	-0.6328	-1.781	-1.9341
Silver	-1.4446	-1.4958	-2.2655	-2.2385
Copper	-1.0786	-1.0509	-1.9208	-1.8983
Crude Oil	-1.5466	-1.6821	-2.0355	-2.1984
Natural Gas	-1.7336	-1.7297	-2.2912	-2.2931
Gasoline	-1.6353	-1.6591	-1.3525	-1.3755
US Coffee	-1.2859	-1.2028	-2.0954	-1.9981
US Wheat	-1.4726	-1.7371	-3.0791	-3.0519
US Corn	-1.1085	-1.1478	-2.5703	-2.4251

Table 4: Unit root test on first difference

	Intercept		Intercept and Trend	
Variables	ADF	PP	ADF	PP
S&P 500	-10.5173*	-37.5897*	-10.5161*	-37.575*
Bitcoin	-5.6554*	-33.9919*	-5.6618*	-33.9871*
Gold	-11.6512*	-35.0583*	-11.6656*	-35.0686*
Silver	-8.7843*	-37.4771*	-8.7962*	-37.4681*
Copper	-24.8214*	-33.7065*	-24.8144*	-33.6947*
Crude Oil	-23.7792*	-45.9995*	-23.7687*	-45.9762*
Natural Gas	-35.9327*	-35.9734*	-35.9227*	-35.9635*
Gasoline	-16.6247*	-35.7803*	-16.658*	-35.8793*
US Coffee	-34.2935*	-34.3963*	-34.2784*	-34.3802*
US Wheat	-8.3382*	-31.1311*	-8.3601*	-31.124*
US Corn	-9.5219*	-32.6614*	-9.5227*	-32.6487*

Source: Author's calculations.

The critical values for unit root test are: −3.43 and −2.86 (without trend) and −3.96, −3.41 (with trend) for 1 and 5 percent levels, respectively. *, **Imply stationarity at 1 and 5 percent levels, respectively.

The results of the Johansen cointegration test with both the trace and Maximum eigenvalue tests are shown in Tables 5 and 6, respectively. This study adopts a bivariate approach to examine the common trends between each pair of assets in the sample. With 11 assets in the sample, there are 10 pairings. However, the interest is in the trends that exist between the S&P 500 and the other assets by using the trace and maximum eigenvalue tests. Before conducting the Johansen cointegration test, the selection of the optimum lag length is necessary. The appropriate lag values are taken based on the Akaike information criterion (AIC). The optimal lag length of each pair is 8 except for S&P 500-Silver, which is shown at 3. Tables 5 and 6 shows that without any contradiction both trace and maximum eigenvalue test fails to reject the null hypothesis of no cointegration for the majority of the pairings. Out of 10 pairings for the entire sample, 8 pairs are non-cointegrated. From November 2018 to June 2023, the analysis found that only S&P 500-US Coffee and S&P 500-US Corn would be cointegrated. Energy futures, precious metals, and bitcoin are not cointegrated with the S&P 500. The most interesting finding from Table 5 is that US corn has no long-run relationship with the S&P 500, even though the other two agricultural futures show cointegration.

Table 5: Unrestricted Cointegration Rank Test (Trace)

Asset Combinations	Hypothesized No. of CE(s)	Eigenvalue	Trace statistics	0.05 Critical value	Prob.**
S&P 500 - Bitcoin	None (r = 0)	0.008224	10.08224	15.49471	0.2744
	At most 1 (r<1)	0.000640	0.725678	3.841466	0.3943
S&P 500 - Gold	None (r = 0)	0.003973	5.011704	15.49471	0.8076
	At most 1 (r<1)	0.000442	0.500794	3.841466	0.4792
S&P 500- Silver	None (r = 0)	0.007705	10.07004	15.49471	0.2754
	At most 1 (r<1)	0.001114	1.268107	3.841466	0.2601
S&P 500 - Copper	None (r = 0)	0.011650	13.73296	15.49471	0.0906
	At most 1 (r<1)	0.000423	0.478933	3.841466	0.4889
S&P 500 - Crude Oil	None (r = 0)	0.005059	7.667465	15.49471	0.5016
	At most 1 (r<1)	0.001694	1.920587	3.841466	0.1658
S&P500-Natural Gas	None (r = 0)	0.007284	9.695516	15.49471	0.3049
	At most 1 (r<1)	0.001246	1.4121	3.841466	0.2347
S&P 500-Gasoline	None (r = 0)	0.003445	7.550400	15.49471	0.5146
	At most 1 (r<1)	0.003220	3.647771	3.841466	0.0561
S&P 500-US Coffee	None* (r = 0)	0.017429	21.61497	15.49471	0.0053
	At most 1 (r<1)	0.001494	1.693557	3.841466	0.1931
S&P 500-US Wheat	None* (r = 0)	0.014918	17.44169	15.49471	0.0251
	At most 1 (r<1)	0.000391	0.442268	3.841466	0.5060
S&P 500-US Corn	None (r = 0)	0.010602	12.66450	15.49471	0.1277
	At most 1 (r<1)	0.000519	0.587939	3.841466	0.4432

Table 6: Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Asset Combinations	Hypothesized No. of CE(s)	Eigenvalue	Trace statistics	0.05 Critical value	Prob.**
S&P 500 - Bitcoin	None (r = 0)	0.008224	9.356561	14.2646	0.2577
	At most 1 (r<1)	0.000640	0.725678	3.841466	0.3943
S&P 500 - Gold	None (r = 0)	0.003973	4.510910	14.2646	0.8019
	At most 1 (r<1)	0.000442	0.500794	3.841466	0.4792

S&P 500- Silver	None (r = 0) At most 1 (r<1)	0.007705 0.001114	8.801936 1.268107	14.26460 3.841466	0.3030 0.2601
S&P 500 - Copper	None (r = 0) At most 1 (r<1)	0.011650 0.000423	13.25403 0.478933	14.26460 3.841466	0.0717 0.4889
S&P 500 - Crude Oil	None (r = 0) At most 1 (r<1)	0.005059 0.001694	5.746878 1.920587	14.2646 3.841466	0.6459 0.1658
S&P500-Natural Gas	None (r = 0) At most 1 (r<1)	0.007284 0.001246	8.283417 1.412100	14.26460 3.841466	0.3506 0.2347
S&P 500-Gasoline	None (r = 0) At most 1 (r<1)	0.003445 0.003220	3.902628 3.647771	14.2646 3.841466	0.8695 0.0561
S&P 500-US Coffee	None* (r = 0) At most 1 (r<1)	0.017429 0.001494	19.92141 1.693557	14.26460 3.841466	0.0057 0.1931
S&P 500-US Wheat	None* (r = 0) At most 1 (r<1)	0.014918 0.000391	16.99943 0.442268	14.26460 3.841466	0.018 0.5060
S&P 500-US Corn	None (r = 0) At most 1 (r<1)	0.010602 0.000519	12.07656 0.587939	14.2646 3.841466	0.1078 0.4432

Portfolio Optimization

Portfolios with different investment avenues are displayed in Tables 7 and 8. Each table contains three portfolios with six alternative weighted combinations of the assets. The portfolios in Table 7 take the results of the cointegration test into account, i.e., the cointegration results show that US Coffee and US Wheat are cointegrated with the S&P 500. Consequently, the asset selection is based on the cointegration test, and portfolios are built without including the above-mentioned cointegrated assets. In Table 7, Portfolio I consist of the stock index, metals futures, energy futures, agricultural futures, and cryptocurrency, specifically the S&P 500 index, Gold, silver, copper, crude oil, natural gas, gasoline, US corn, and bitcoin. Portfolio II includes stock index, metals futures, agricultural futures, and cryptocurrency, specifically the S&P 500 index, Gold, silver, copper, US corn, and bitcoin. Portfolio III is constructed with stock index, energy futures, agricultural futures, and cryptocurrency, i.e., the S&P 500 index, crude oil, natural gas, gasoline, US corn, and bitcoin. The study employs agricultural futures in these three portfolios, namely US Corn, but these three portfolios don't include US coffee and US wheat because they are cointegrated with the S&P 500.

In order to determine whether the cointegration test enhances the efficiency of modern portfolio theory, this study compares the portfolio return and risk in Table 7 with Table 8, which includes portfolios without considering the cointegration test, i.e., the combination of assets in Table 8 is the same as Table 7 but includes the cointegrated assets in that combination. In Table 8, Portfolio I include stock index, metals futures, energy futures, agricultural futures, and cryptocurrency, i.e., the S&P 500 index, Gold, silver, copper, crude oil, natural gas, gasoline, US corn, US coffee, US wheat, and bitcoin, Portfolio II includes stock index, metals futures, agricultural futures, and bitcoin, namely the S&P 500 index, Gold, silver, copper, corn futures, US coffee, US wheat, and bitcoin. Portfolio III includes stock index, energy futures, agricultural futures, and cryptocurrency, i.e., the S&P 500 index, crude oil, natural gas, gasoline, US corn, US coffee, US wheat, and bitcoin.

Table 7: Portfolio optimization with cointegration test

Portfolio I: Stock index, Metal futures, Energy futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.000497	0.000847	0.001197	0.001547	0.001897	0.002245
Portfolio risk	0.007836	0.011835	0.018845	0.026811	0.035562	0.044821
S&P 500	0.23067	0.09861	0	0	0	0
Gold	0.497589	0.393703	0.175284	0	0	0

Silver	0	0	0.025497	0.037477	0	0
Copper	0.111853	0	0	0	0	0
Crude oil	0	0	0	0	0	0
Natural gas	0.016551	0.040211	0.059375	0.068735	0.069602	5.29E-05
Gasoline	0	0.074454	0.112982	0.090293	0.018125	0
UScorn futures	0.143336	0.210915	0.258057	0.237199	0.130382	0
Bitcoin	0	0.182107	0.368806	0.566296	0.781891	0.999947
Portfolio II: Stock index, Metal futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.000494	0.000844	0.001194	0.001544	0.001894	0.002245
Portfolio risk	0.007868	0.012024	0.019162	0.027033	0.035638	0.044821
S&P 500	0.238165	0.14894	0.024799	0	0	0
Gold	0.500818	0.411152	0.245976	0	0	0
Silver	0	0	0.035391	0.102559	0	0
Copper	0.114333	0.011389	0	0	0	0
US corn	0.146684	0.240237	0.315807	0.330345	0.216429	5.02E-05
Bitcoin	0	0.188282	0.378028	0.567096	0.783571	0.99995
Portfolio III: Stock index, Energy futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.00057	0.000906	0.001242	0.001578	0.001914	0.002245
Portfolio risk	0.011093	0.013967	0.020056	0.027568	0.036001	0.044821
S&P 500	0.542116	0.289862	0.040511	0	0	0
Crude oil	0	0	0	0	0	0
Natural gas	0.030611	0.04853	0.068713	0.070489	0.069555	5.29E-05
Gasoline	0.078618	0.122741	0.161566	0.098169	0.013859	0
US corn	0.348655	0.348061	0.347755	0.245724	0.124235	0
Bitcoin	0	0.190806	0.381455	0.585617	0.792351	0.999947

Source: Author's calculation

Table 8: Portfolio optimization without cointegration test

Portfolio I: Stock index, Metal futures, Energy futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.000501	0.000850	0.001198	0.001546	0.001897	0.002245
Portfolio risk	0.00768	0.011653	0.018638	0.026662	0.035458	0.044821
S&P 500	0.22176	0.082637	0	0	0	0
Gold	0.468433	0.34907	0.101562	0	0	0
Silver	0	0	0.034135	0.006361	0	0
Copper	0.090567	0	0	0	0	0
Crude oil	0	0	0	0	0	0
Natural gas	0.013536	0.035582	0.052782	0.061178	0.064743	5.28E-05
Gasoline	0	0.060399	0.089824	0.062198	0	0
US coffee	0.062779	0.106011	0.131708	0.111505	0.056204	0
US wheat	0.034532	0	0	0	0	0
US corn	0.108392	0.187593	0.227722	0.192552	0.09882	0
Bitcoin	0	0.178707	0.362267	0.566207	0.780233	0.999947

Portfolio II: Stock index, Metal futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.000499	0.000849	0.001199	0.001549	0.001899	0.002245
Portfolio risk	0.007702	0.011812	0.018929	0.026923	0.035702	0.044821
S&P 500	0.227536	0.124879	0	0	0	0
Gold	0.470406	0.361756	0.155496	0	0	0
Silver	0	0	0.042698	0.040462	0	0
Copper	0.091851	0	0	0	0	0
US coffee	0.06481	0.120062	0.160024	0.145115	0.080831	5.14E-05
US wheat	0.035022	0	0	0	0	0
US corn	0.110375	0.209749	0.271257	0.243726	0.132468	0
Bitcoin	0	0.183555	0.370525	0.570697	0.786701	0.999949
Portfolio III: Stock index, Energy futures, Agricultural futures and Cryptocurrencies						
	1	2	3	4	5	6
Portfolio return	0.000562	0.000898	0.001234	0.00157	0.001906	0.002245
Portfolio risk	0.010296	0.013325	0.019521	0.027249	0.03577	0.044821
S&P 500	0.468698	0.230167	0	0	0	0
Crude oil	0	0	0	0	0	0
Natural gas	0.021505	0.040782	0.05813	0.061613	0.064636	5.43E-05
Gasoline	0.038063	0.088442	0.123908	0.059693	0	0
US coffee	0.151038	0.162038	0.163381	0.109185	0.053523	0
US wheat	0.098295	0.018359	0	0	0	0
US corn	0.222402	0.276088	0.280149	0.188501	0.094208	0
Bitcoin	0	0.184124	0.374431	0.581009	0.787634	0.999946

Source: Author's calculation

When evaluating portfolio I in tables 7 and 8, portfolio II in tables 7 and 8, and portfolio III in tables 7 and 8, it shows that the results are nearly the same regardless of the use of cointegration. However, an in-depth analysis of the risk-return characteristics of the portfolios reveals certain information: the return is a little bit decreased and the risk is increased in the portfolio I in table 8 (portfolio without cointegration) compared to the portfolio I in table 7 (portfolio with cointegration). As well, Portfolio II also shows that there is a slight decrease in return and a slight increase in the risk portfolio of Table 8 compared to Table 7. In all of these instances, it is clear that using modern portfolio theory alone can lead to better results than incorporating cointegration test results. Conversely, when compared to portfolio III in table 7, the returns for the one to five combinations of that portfolio are only slightly higher than portfolio III in table 8, at the same time that risks are increased. In terms of the sharpe ratio, all three portfolios exhibit the same result. The sixth combination of assets in each table yields the highest sharpe ratio of 0.547%. The 1year Treasury bill rate from the date of 2018 to 2023 is used as the risk-free rate of return in the Sharpe ratio calculation, which is 0.20%.

In summary, while analysing the 36 combinations of portfolios, 31 of them yield the same result: the portfolios with the cointegration test do not overlap the portfolios without the cointegration test; instead, the risk and return in the portfolios with the cointegration test are slightly decreased and increased, respectively. However, these changes are not statistically significant. Therefore, Sharpe ratio reveals the same results at the same time. It can be concluded that modern portfolio theory has the ability to construct long term investment portfolios.

The optimization process is performed using the MS Excel solver, which is available as a plug-in for MS Excel. The minimum variance is set as the objective function, and the optimal weights (asset allocation) are found using the Excel solver. The constraints for the portfolios are set so that the sum of the weights equals one.

The graphical representation of the risk and return characteristics of the three portfolios can be seen below in Figs. 2, 3, and 4. It clearly demonstrated the slight variations in risk among the portfolios.

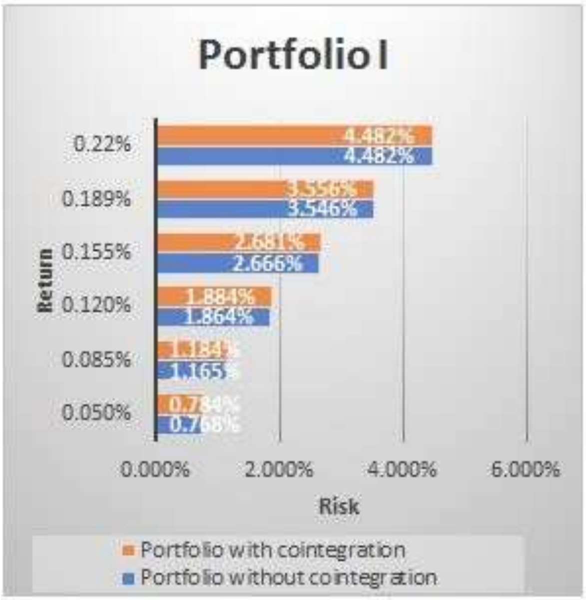

Figure 2: Risk-return profile of portfolio I

Figure 3: Risk-return profile of portfolio II

Fig 4: Risk-return profile of portfolio III

CONCLUSION

The current study bridges the gap in the literature by combining the cointegration result into a portfolio based on modern portfolio theory and conducting a comprehensive risk-return analysis with a risk-adjusted evaluation. Due to the fact that numerous studies have criticised modern portfolio theory for employing correlation and instead recommend using the cointegration test, this study empirically examines how effective cointegration is compared to correlation in the investment decision-making process. To solve this objective, this study constructed portfolios based on modern portfolio theory with and without combining the results of the cointegration test.

This study constructed 36 portfolios with different combinations of optimal weights for the assets. The analysis of these 36 portfolios led to the conclusion that the results from portfolios with and without the cointegration test are essentially the same. Additionally, the Sharpe ratio remains consistent across all portfolios. But when we go through a comprehensive analysis of portfolios with and without the cointegration test, there is only a slight increase and decrease in the risk return of the portfolios, which is not statistically significant. However, the application of correlation in modern portfolio theory, which refers to short-term knowledge about assets, could have the ability to provide information that will aid in making long-term investment decisions. On the other hand, cointegration shows the integration of the assets, but its inclusion in portfolio construction does not significantly impact the outcomes. It may be due to the fact that the cointegration test is conducted on the first difference, so the long-term information is lost by first differencing the data.

This article provides important implications for investors, portfolio managers, and financial advisors who are interested in long-term diversification. They should not spend time conducting cointegration tests in order to construct portfolios in accordance with modern portfolio theory. This study is also helpful to researchers interested in understanding the practical implications of cointegration and correlation. This study suggests that there are other ways to use the cointegration test in the investment decision-making process than this. If more studies are conducted on the potential analysis of cointegration in the investment decision-making process, it will be beneficial.

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