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FUZZY SCALE RELIABILITY ASSESSMENT FOR ANALYSIS OF MATHEMATICS LEARNING STYLES

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ABSTRACT
When using Likert scales, majority of statistical techniques cannot be used directly, & even if they could, doing so would significantly diminish results' interpretability & reliability. It is examined whether fuzzy scale is reliable for assessing survey respondents' opinions. So, validity of a conventional Likert scale-based (SB) questionnaire along with its fuzzy rating counterpart are compared. To do this, a set of students who used both student learning style scale (SLLS) scales provided answers to a few questions (Qs) from each scale. Since we know Likert scale version of this survey to be reliable, we can use corresponding Cronbach's alpha coefficients to gauge how well fuzzy version of survey performs.

Keywords: Fuzzy scale, Reliability, SLSS, Cronbach's $\tilde{\alpha}$ coefficient

1. INTRODUCTION
Likert scales - broadly employed to evaluate characteristics/viewpoints frequently linked to opinions, values, & other concepts. Data is generated from a collection of pre-fixed categories in a questionnaire using a Likert scale. These categories are frequently categorized using integer values from a scale that commonly ranges from 1 to 5, or from 1 to 7. These closed-format questions have become more common in practice since they are straightforward to administer & do not require a general explanation of responses **[1].**

Reliability is degree to which an experiment, test, or other measuring method yields same results throughout a number of runs. The primary focus of this research will be on examining fuzzy scale's suitability in comparison to conventional Likert scale. To do this, a group of students' replies to a few questions from standard SLSS questionnaire will be compared in both Likert & fuzzy formats. Questions were only allowed in a mathematical framework if they matched dependent & independent learning types. To investigate trustworthiness of SLSS questionnaire within context of fuzzy logic, we shall explain an extension of Cronbach's alpha to fuzzy scenario. **[2].**

1.1. PRELIMINARIES
Fuzzy set, also called fuzzy number, is a function that is convex in nature having range [0,1] to $\mathbb{R}$, where each x value of $\mathbb{R}$ is related to membership function $U(x) \in [0,1]$. A fuzzy set (FS) U's 'cuts' or 'levels', stated by "U_α,"; intervals given by a value set substantiating $U(x) \geq \alpha$, per $\alpha \in [0,1]$. In this analysis, we focus on trapezoidal and triangular forms of FSs; latter is a particular case of former **[3].**

A trapezoidal FS that satisfies requirements that $[a,d]$ is 0 -level & $[b,c]$ is 1-level is frequently referred to as $\mathrm{Tr}\,a(a,b,c,d)$. The mathematical equation of fuzzy trapezoidal number having vertices in $\{a,b,c,d\}$ is

$$\mathrm{Tr}\,a(a,b,c,d) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } x \in [a,b) \\ 1 & \text{if } x \in [b,c] \\ \dfrac{d-x}{d-c} & \text{if } x \in (c,d] \\ 0 & \text{otherwise} \end{cases} \quad \dots (1.1)$$

A trapezoidal FS's description & its α-cuts is collected in Figure 1.1

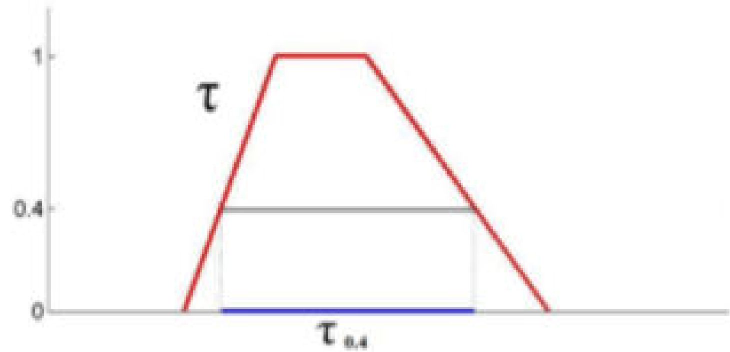

Figure 1.1: A trapezoidal FS's representation **[4].**

2. CRONBACH'S ALPHA PER RANDOM FUZZY SETS

Cronbach's α **[2]** coefficient is used to calculate a test's internal consistency & is frequently used to determine how reliable test results are. By examining correlation between items, it takes values ≤ 1 & reflects how well a group of items assesses a single 1-D latent construct. Higher Cronbach's alpha values are preferred, so closer index is to 1, more reliable scale is**[5]**.

The Cronbach's alpha for related measuring scale is defined as follows in classical framework; k items expressed via k real random variables $X_1, \ldots, X_k$ with sample - replies per $i \in \{1, \ldots, k\}$:

$$\alpha = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{i=1}^{k} \hat{\sigma}_{X_i}^2}{\hat{\sigma}_T^2}\right) \ldots. (1.6)$$

Where, $n = n_1 + \cdots + n_k$ and $\hat{\sigma}_T^2$ is variance of all observed values, where n is the sample of students. Since variance previously mentioned for random FSs is a marker of distribution of fuzzy values in relation with sample mean (SM), Cronbach's alpha may be applied to fuzzy framework in same way it was in classical situation.

In light of k RFSs $\mathcal{X}_1, \ldots, \mathcal{X}_k$ & n_i answers per $i \in \{1, \ldots, k\}$,

Cronbach's $\tilde{\alpha}$ per RFSs:

$$\tilde{\alpha} = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{i=1}^{k} \hat{\sigma}_{\mathcal{X}_i}^2}{\hat{\sigma}_{\mathcal{T}}^2}\right) \ldots. (1.7)$$

Where, $\hat{\sigma}_{\mathcal{T}}^2$ is variance of all observed fuzzy values.

3. METHODOLOGY

Usually, idea of dependability provides a study of a construct's internal structure. The Cronbach's $\tilde{\alpha}$ index for RFSs is used to examine SLSS questionnaire's reliability in fuzzy context.

To evaluate a test's dependability, there are two key conditions **[2].** The test must, first & foremost, consist of a set of things that may be added together to provide a final score. The desired attribute must also be measured consistently across all goods. 20 questions from SLSS survey have been taken & used in this study. A dependent LS in mathematics was subject of 10 questions, while an independent LS was subject of 10 questions. These inquiries are compiled in appendix.

The degree program in elementary teaching at the asked a group of 110 students to respond to these 20 questions using both fuzzy and Likert scales.

In 1st scenario, students select an answer ranging - 1 to 5, where 1 denotes utter disagreement, 2 moderate disagreement, 3 undecided, 4 moderate agreement, & 5 utter agreement.

In second instance, respondents used trapezoidal FSs on a scale from 0 to 10 (0 shows total disagreement & 10 shows absolute agreement). Each response's 0-level represents set of values that student believes, to some extent, to be compatible with his or her view (i.e., student believes their opinion cannot exist outside this set).

Alternatively, the student views set of values in trapezoidal FS's first level to be wholly consistent with his or her viewpoint. Finally, a trapezoidal can be created by linearly interpolating appropriate bounds of 0-level & 1-level.

The viewpoints of 2 students A & B provided by trapezoidal FSs on 3 questionnaire Qs are shown in Figures 2 & 3 below. It should be noted that when both students were asked to respond to identical questions utilising a value on a Likert scale - 1 to 5, their responses - 4, 1 & 5, which demonstrates adaptability & greater diversity of fuzzy-type responses compared to Likert-type responses **[6].**

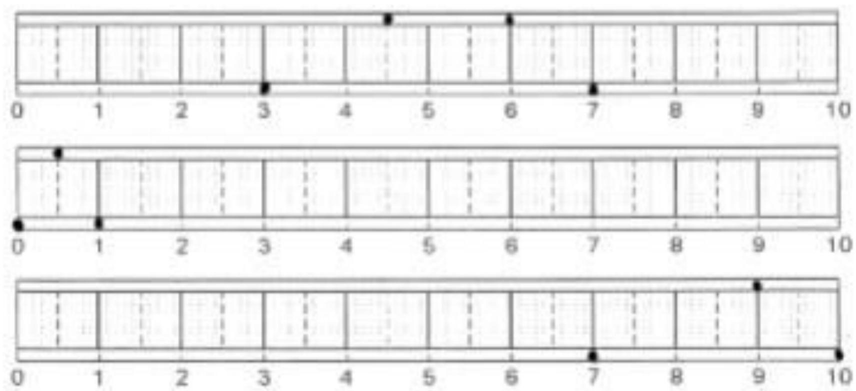

Figure 1.2: Student *A's* responses

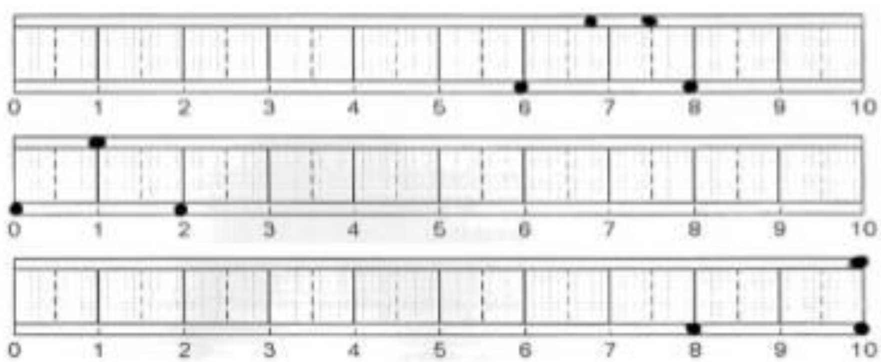

Figure 1.3: Student *B's* responses

4. Experimental results

To examine SLSS questionnaire's reliability when Likert & fuzzy replies are used. A descriptive analysis of responses is provided as a first step. Trapezoidal FSs' SMs that encode responses to SLSS questionnaire's questions corresponding to dependent & independent LSs are gathered in Figures 1.4 & 1.5. Additionally, Table 1.1 below shows SMs of Likert-type responses.

Table 2: Likert answers' sample means

Question	Dependent style	Independent style
1	4.194	3.12

2	4.037	2.806
3	4.454	2.407
4	3.676	2.954
5	2.852	3.398
6	3.657	3.620
7	3.602	3.611
8	3.843	2.861
9	4.593	3.324
10	2.583	3.639

Considerations about fuzzy-type & Likert-type sample means are noteworthy, particularly:

1. In regards to questions that correspond to dependent LS, if we pay close attention to supremum of 0-levels of fuzzy SMs, we observe those supremum's optimal values are attained per Qs D_3 & D_9; they are also highest means attained for Likert answers. Additionally, questions D_5 & D_{10} obtain lowest values for maximum of 0-levels in fuzzy case as well as least values of category means.
2. However, in independent LS questions, questions I_6, I_7 & I_{10} have highest values of supremum of 0 -levels of fuzzy means & categorical means, whereas questions I_2 & I_3 have lowest values in both circumstances.

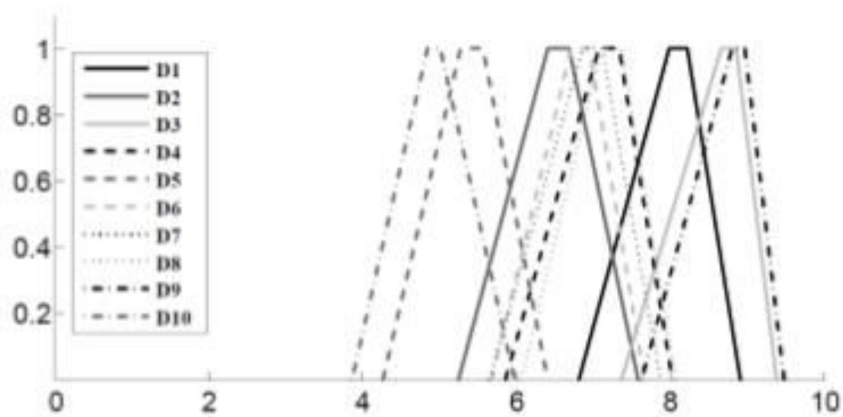

Figure 1.4: Examples of ways for responses to questions about dependent LSs

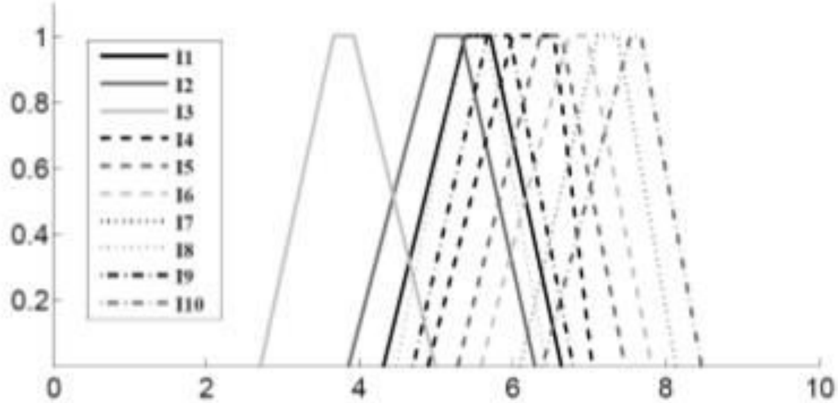

Figure 1.5: Examples of ways for responses to questions about independent LSs

Table 1.3: Sample variances in both cases

Question	Dependent style	Independent style
1	4.194	3.12
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4	3.676	2.954
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9	4.593	3.324
10	2.583	3.639

These findings confirm that fuzzy-based responses to SLSS questionnaire are coherent with respect to traditional categorical approach.

The overall sample mean of fuzzy answers for dependent & independent LSs is shown in Figure 1.6.

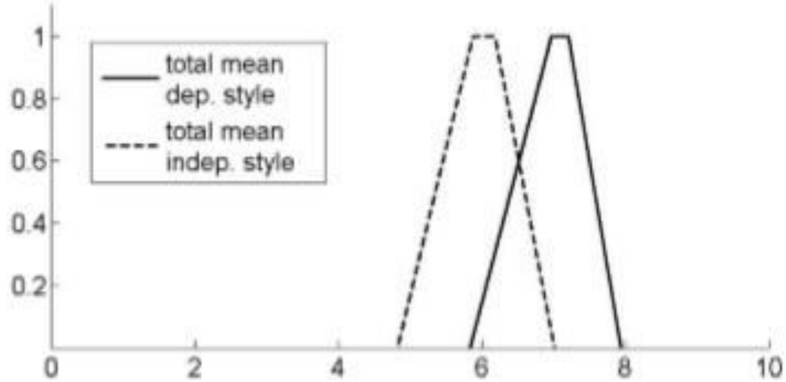

Figure 1.6: SMs of answers corresponding to independent LS questions

Additionally, overall sample averages for Likert responses are 3.174 for independent case & 3.749 for dependent case. In light of findings from both frameworks, we can therefore say that respondents generally had more reliant than independent LSs. Regarding replies' variability, Table 1.3 compiles matching variances of Likert & fuzzy responses for dependent & independent learning methods.

Table 1.3: Sample variances in both cases

	Fuzzy	Likert		Fuzzy	Likert
D_1	3.261	0.879	I_1	5.126	0.884
D_2	4.392	1.017	I_2	2.872	0.434
D_3	1.713	0.507	I_3	5.438	0.982
D_4	2.9	0.497	I_4	4.671	0.748
D_5	5.867	1.293	I_5	3.969	0.74
D_6	4.967	1.003	I_6	6.139	1.272
D_7	3.56	0.795	I_7	3.68	0.96
D_8	4.469	1.003	I_8	7.728	1.305
D_9	1.539	0.501	I_9	5.789	0.96
D_{10}	7.597	1.447	I_{10}	4.017	0.823

Changes in Likert-type responses are less than those of fuzzy-type replies given that answer variation scale spans from 1 to 5 in 2nd instance, yet from 0 to 10 in 1st. Nevertheless, it is simple to verify that for both response types, questions D_{10} and I_6 have highest variances while questions D_9 and I_2 have lowest variances. This fact supports previously observed consistency between fuzzy & Likert responses.

Both traditional Cronbach's alpha & RFS version of Cronbach's alpha account for both dependent & independent LSs when calculating reliability..

Cronbach's α and $\tilde{\alpha}$ are calculated with variances of numerators shown in Table 1.3. In addition, values acquired by adding each person's responses to specialized LS questions are used to calculate variances of total observed values utilised in denominators, $\hat{\sigma}_T^2$ and $\hat{\sigma}_{\tilde{T}}^2$.

Table 1.4 displays calculated findings.

	Independent style	Dependent style
α	0.7356	0.6276
$\tilde{\alpha}$	0.7258	0.6049

Table 4: Mentioned Cronbach's $\tilde{\alpha}$ per mathematical dependent & independent LSs, as well as for categorical & fuzzy situations

The results shown in Table 1.4 indicate that Cronbach's α and $\tilde{\alpha}$ on both answer styles are extremely close. However, a widely accepted criteria for expressing a test's internal consistency using Cronbach's alpha states that when index is between range $[0.6, 0.7)$ it is acceptable, however it is desirable when it exceeds value 0.7. As a result, α and $\tilde{\alpha}$ are acceptable for independent LS questions & acceptable for dependent LS questions using both types of answers.

The most noteworthy element is that, even if values found in Table 1.4 are not particularly high, reliability of SLSS questionnaire's questions about dependent & independent LSs is maintained in presence of ambiguous responses. This suggests that using a fuzzy scale to respond to opinion surveys is highly advised because it seems to maintain test's internal consistency & offers some benefits (such as those discussed throughout this work) that help capture inherent human fallibility in forming accurate judgments better.

5. CONCLUSIONS

When categorical values & FSs are given as answers, internal consistency (or reliability) of some questions from SLSS questionnaire regarding a dependent or independent LS in mathematics was examined in this work. For this reason, a Cronbach's α has been created for variables with values in space of FSs. As a result, tests with fuzzy responses were shown to be just as reliable as tests with values selected from a Likert scale. The first scale is a particularly suitable instrument to capture & express imprecision inherent in human beliefs since fuzzy scale, in contrast to categorical one, has some advantages. Additionally, instructors might gain from usage of fuzzy scale since data from fuzzy questionnaire may help them develop their teaching techniques & broaden their perspectives on subjects they teach.

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