Stochastic Modelling and Computational Sciences

EDGE CONGRUENCE SQUARE SUM LABELING

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ABSTRACT

A new concept of labeling called the one or two modulo 4 square sum labeling is introduced and investigated for *the comb graph $P_n \odot K_1$, the star graph $K_{1,n}$, the subdivision of the star graph $K_{1,n}$, the one point union of cycle C_3 with star graph $K_{1,n}$, the crown graph C_n^+, the wheel graph W_n, the fan graph F_n, and the friendship graph T_n.*

Keywords: Square sum labeling, one or two modulo 4 square sum labeling.

1 INTRODUCTION

All graphs considered in this paper are finite, simple and undirected graphs. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G .One of the most important achievement made in graph theory is graph labeling. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions in the mid 1960's the labeling was introduced and more than 200 different types of labels have been derived. For a wide range of graph labeling applications include addressing communication networks, wi-fi security and secrete sharing schemes.

The concept of graph labeling was introduced by Rosa[7] in 1967. Gallian [2] has published a book named a dynamic survey of graph labeling and it contains the latest update related to the labeling of graphs. The square difference labeling was introduced by J.Shiama [8].

T Geetha and D.Kalamani [4] were derived the square difference labeling for some graphs and also proved that the square difference prime labeling for some graphs [5]. In 2020, Vanu Esakki and Syed Ali Nisaya [10] was established the two modulo three sum graph. Germina K A, Arumugam S and Ajitha V[3] were established on square sum graphs. In this paper, a new labeling called one or two modulo four square sum labeling are introduced. Further notations and terminologies are followed from Harary [6] and Bondy and Murty [1].

2 PRELIMINARIES

In this section, some basic definitions namely square sum labeling, path graph P_n, bipartite graph, complete bipartite graph $K_{m,n}$, comb graph $P_n \odot K_1$, one point union of cycle C_3 with $K_{1,n}$, star graph $K_{1,n}$, the subdivision of the star graph $K_{1,n}$, crown graph C_n^+, the wheel graph W_n, the fan graph F_n, and the friendship graph T_n are given.

Definition 2.1. A **path P_n** is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \le i \le n-1$.

Definition 2.2. A closed walk $v_0 v_1 v_2 \ldots v_n = v_0$ in which $n \ge 3$ and $v_1, v_2, \ldots, v_n$ are distinct is a **cycle** . It is denoted by C_n.

Definition 2.3. A **bipartite graph** is one whose vertex set can be partitioned into subsets X and Y, so that each edge has one end in X and one end in Y; such a partition (X,Y) is called a bipartition of the graph.

Definition 2.4. A **complete bipartite graph** $K_{1,n}$ is called a **star** and it has n+1 vertices and n edges. It is also denoted by S_n.

Definition 2.5. The **comb** is the graph obtained from a path P_n by attaching a pendent edges to each vertex of the path. It is denoted by $P_n \odot K_1$.

Definition 2.6. The graph obtained by joining n pendent edges at one vertex of the cycle C_3 is called **one point union of C_3 with $K_{1,n}$.**

Definition 2.7. A subdivision graph S(G) is obtained from G by subdividing each edge of G

with a vertex.

Definition 2.8. The **crown graph C_n^+** is obtained from C_n by attaching a pendent vertex from each vertex of the graph C_n. n

Definition 2.9. A **wheel graph** is a graph formed by connecting a single universal vertex to all vertices of a cycle.Wheel graph of (n+1) vertices denoted by W_n.

Definition 2.10. A **fan graph F_n** can be constructed from a wheel graph by deleting one edge on the n- cycle. Fan graph has (n+1) vertices.

Definition 2.11. The **friendship graph T_n** is a set of n triangles having a common central vertex.

Definition 2.12. A graph G = (V, E) with p vertices and q edges is said to be **square sum labeling** if there exists a bijection mapping $f : V(G) \rightarrow \{0,1, 2, ..., p-1\}$ such that the induced function $f^* : E(G) \rightarrow N$ defined by $f^*(uv) = |[f(u)]^2 + [f(v)]^2|$ for every $uv \in E(G)$ are all distinct.

3 MAIN RESULTS

In this section , we proved one or two modulo four square sum labeling for the comb graph $P_n \odot K_1$, the star graph $K_{1,n}$, the subdivision of the star graph $K_{1,n}$, the one point union of cycle C_3 with star graph $K_{1,n}$, the crown graph C_n^+, the wheel graph W_n, the fan graph F_n, and the friendship graph T_n.

Definition 3.1. A graph G = (V, E) is said to be **one or two modulo four square sum labeling** if there is a bijection $f : V(G) \rightarrow \{1, 2, ..., |v|\}$ and the induced function $f^* : E(G) \rightarrow N$ defined by $f^*(uv) = |[f(u)]^2 + [f(v)]^2| \equiv$ 1 or 2(mod4) if $uv \in E(G)$ are all distinct.

Theorem 3.1. The star graph $K_{1,n}$ admits one or two modulo 4 square sum labeling.

Proof. Let G be a star graph $K_{1,n}$.

Let $V(G) = \{v, v_i : 1 \le i \le n\}$ and $E(G) = \{vv_i : 1 \le i \le n\}$.

Hence G has $n+1$ vertices and n edges.

Define $f : V(G) \rightarrow \{1,2,...,n+1\}$ as follows:

$f(v) = 1$

$f(v_i) = i+1, 1 \le i \le n$

Clearly f is bijective and f induces a bijective function. $f^* : E(G) \rightarrow \{1,2\}$ as follows:

$f^*(vv_i) = 1, 1 \le i \le n$ (i is even)

$f^*(vv_i) = 2, 1 \le i \le n$ (i is odd)

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4).

Therefore $G = K_{1,n}$ is a one or two modulo four square sum labeling.

Example 3.1. The star graph $K_{1,7}$ is a one or two modulo four square sum labeling which is shown in the Figure 1

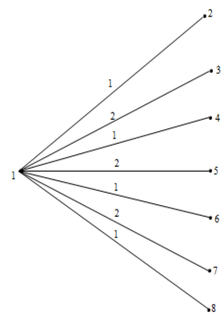

Figure 1: The star graph $K_{1,7}$.

Theorem 3.2. The comb graph $P_n \odot K_1$ admits one or two modulo 4 square sum labeling.

Proof. Let G be the comb graph $P_n \odot K_1$.

Let $V(G) = \{u_i\ v_i : 1 \le i \le n\}$ and $E(G) = \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_iv_i : 1 \le i \le n\}$.

Hence G has 2n vertices and $2n-1$ edges.

Define $f : V(G) \to \{1,2,3,...,2n\}$ as follows:

$f(u_i) = 2i-1,\ 1 \le i \le n\ f(v_i) = 2i, \quad 1 \le i \le n$

Clearly f is injective and f induces a bijective function $f^* : E(G) \to \{1,2\}$ as follows:

$f^*(u_iu_{i+1}) = 2,\ 1 \le i \le n-1$ (i is odd)

$f^*(u_iv_i) = 1,\ 1 \le i \le n$ (i is even)

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = $P_n \odot K_1$ is a one or two modulo 4 square sum labeling.

Example 3.2. The comb graph $P_6 \odot K_1$ is a one or two modulo 4 square sum labeling which is shown in the Figure 2.

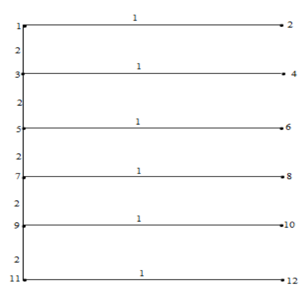

Figure 2: The comb graph $P_6 \odot K_1$.

Theorem 3.3: The subdivision of the edges of the star $k_{1,n}$ admits one or two modulo 4 square sum labeling.

Proof. Let G be a graph obtained by the subdivision of the edges of the star $k_{1,n}$.

Let V(G) = $\{v, u_i, w_i : 1 \le i \le n\}$ and E(G) = $\{ vu_i, u_iw_i : 1 \le i \le n \}$.

Hence G has 2n+1 vertices and 2n edges.

Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 2n + 1\}$ as follows

$f(v) = 1$

$f(u_i) = 2i + 1, \quad 1 \leq i \leq n$

$f(w_i) = 2i, \quad 1 \leq i \leq n$

Clearly f is bijective and f induces a bijective function. $f^*: E(G) \rightarrow \{1,2\}$ as follows:

$f^*(vu_i) = 2, \quad 1 \leq i \leq n$

$f^*(u_i w_i) = 1, \quad 1 \leq i \leq n$

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = subdivision of the edges of the star K_{1n} is a one or two modulo 4 square sum labeling.

Example 3.3: The subdivision of the edges of the star $K_{1,7}$ is a one or two modulo 4 square sum labeling which is shown in the figure 3.

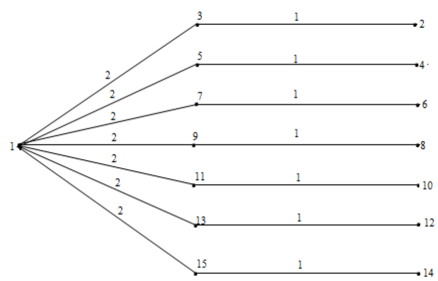

Figure 3: Subdivision of edges of $K_{1,7}$

Theorem 3.4. The one point union of cycle C_3 with star graph $K_{1,n}$ admits one or two modulo 4 square sum labeling.

Proof. Let G be the one point union of cycle C_3 with star graph $K_{1,n}$.

Let $V(G) = \{u, v, w, v_i : 1 \leq i \leq n\}$ and $E(G) = \{uv, vw, wu\} \cup \{uv_i : 1 \leq i \leq n\}$. Hence G has $n+3$ vertices and $n+3$ edges.

Define $f : V(G) \rightarrow \{1,2,3,\ldots, n+3\}$ as follows:

$f(u) = 1$

$f(v) = n+2$

$f(w) = n+3$

$f(v_i) = i+1, \ 1 \leq i \leq n$

Clearly f is injective and f induces a bijective function $f^*: E(G) \rightarrow \{1,2\}$ as follows:

$f^*(uv) = 1$

$f^*(uw) = 2$

$f^*(vw) = 1$

$f^*(uv_i) = 1, 1 \le i \le n$ (i is even)

$f^*(uv_i) = 2, 1 \le i \le n$ (i is odd)

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4).Therefore G = one point union of cycles C_3 with star graph $K_{1,n}$ admits one or two modulo 4 square sum labeling.

Example 3.4. The one point union of cycles C_3 with star graph $K_{1,9}$ admits one or two modulo 4 square sum labeling which is shown in the Figure 4.

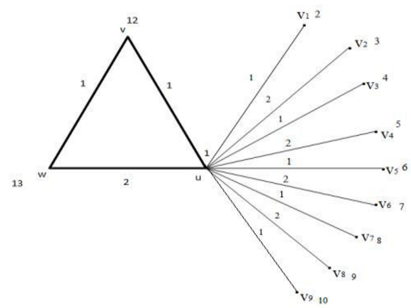

Figure 4: One point union of cycle C_3 with star graph $K_{1,9}$

Theorem 3.5. The crown graph C_n^+ admits one or two modulo 4 square sum labeling.

Proof. Let G be the crown graph C_n^+.

Let $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and

$E(G) = \{u_n u_1\} \cup \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$. Hence G has 2n vertices and 2n edges.

Define $f : V(G) \to \{1, 2, ..., 2n\}$ as follows:

$f(u_i) = 2i - 1, 1 \le i \le n$ $f(v_i) = 2i, \quad 1 \le i \le n$

Clearly f is injective and f induces a bijective function $f^* : E(G) \to \{1,2\}$ as follows:

$f^*(u_i u_{i+1}) = 2, \ 1 \le i \le n-1$ (i is odd)

$f^*(u_n u_1) = 2$

$f^*(u_i v_i) = 1, 1 \le i \le n$ (i is even)

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = crown graph C_n^+ admits one or two modulo 4 square sum labeling.

Example 3.5. The crown graph C_8^+ admits one or two modulo 4 square sum labeling which is shown in the Figure 5.

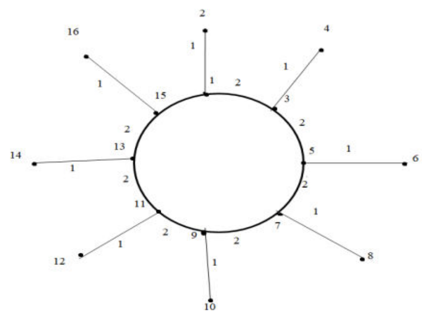

Figure 5: The Crown graph C_8^+.

Theorem 3.6. The wheel graph W_n (n is even) admits one or two modulo 4 square sum labeling.

Proof. Let G be the wheel graph W_n.

Let $V(G) = \{v_i : 1 \leq i \leq n+1\}$ and

$E(G) = \{v_2v_{n+1}\} \cup \{v_1v_{i+1} : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 2 \leq i \leq n\}$. Hence G has n+1 vertices and 2n edges.

Define $f : V(G) \rightarrow \{1,2,...,n+1\}$ as follows:

$f(v_i) = i,\ 1 \leq i \leq n+1$

Clearly f is injective and f induces a bijective function $f^* : E(G) \rightarrow \{1,2\}$ as follows:

$f^*(v_1v_i) = 1,\ 1 \leq i \leq n+1$ (i is even)

$f^*(v_1v_i) = 2,\ 1 \leq i \leq n+1$ (i is odd)

$f^*(v_i v_{i+1}) = 1,\ 2 \leq i \leq n$

$f^*(v_2v_{n+1}) = 1,$

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = wheel graph W_n (n is even) admits one or two modulo 4 square sum labeling.

Example 3.6 The wheel graph W_8 (n is even) admits one or two modulo 4 square sum labeling which is shown in the Figure 6.

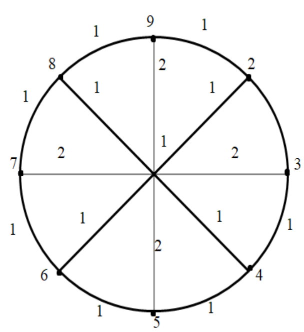

Figure 6: The wheel graph W_8.

Theorem 3.7. The fan graph F_n admits one or two modulo 4 square sum labeling.

Proof. Let G be the an graph F_n

Let $V(G) = \{v_i : 1 \le i \le n+1\}$ and

$E(G) = \{v_1 v_{i+1} : 1 \le i \le n\} \cup \{v_i v_{i+1} : 2 \le i \le n\}$.

Hence G has n+1 vertices and 2n-1 edges.

Define $f : V(G) \rightarrow \{1,2,...,n+1\}$ as follows:

$f(v_i) = i,\ 1 \le i \le n$

Clearly f is injective and f induces a bijective function $f^* : E(G) \rightarrow \{1,2\}$ as follows:

$f^*(v_1 v_i) = 1,\ 1 \le i \le n+1$ (i is even)

$f^*(v_1 v_i) = 2,\ 1 \le i \le n+1$ (i is odd)

$f^*(v_i v_{i+1}) = 1, 2 \le i \le n$

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = fan graph F_n admits one or two modulo 4 square sum labeling.

Example 3.7. The fan graph F_9 admits one or two modulo 4 square sum labeling which is shown in the Figure 7.

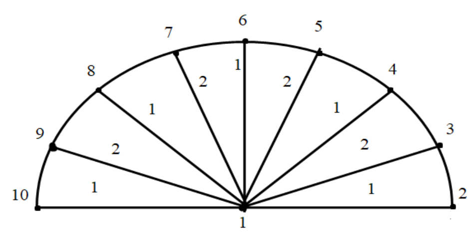

Figure 7: Fan graph F_9

Theorem 3.8. The friendship graph Tn admits one or two modulo 4 square sum labeling.

Proof. Let G be the friendship graph T_n.

Let $V(G) = \{v_i : 1 \le i \le 2n+1\}$ and

$E(G) = \{v_1 vi : 2 \le i \le 2n+1\} \cup \{v_{2i} v_{2i+1} : 1 \le i \le n\}$.

Hence G has 2n vertices and 2n edges.

Define $f : V(G) \rightarrow \{1,2,...,2n+1\}$ as follows:

$f(v_i) = i,\ 1 \le i \le 2n+1$

Clearly f is injective and f induces a bijective function $f^* : E(G) \rightarrow \{1,2\}$ as follows:

$f^*(v_1 v_i) = 1,\ 1 \le i \le 2n+1$ (i is even)

$f^*(v_1 v_i) = 2,\ 1 \le i \le 2n+1$ (i is odd)

$f^*(v_i v_{i+1}) = 1, 2 \le i \le 2n$

Hence the edge labels are all distinct and congruence to 1 or 2(mod 4). Therefore G = friendship graph Tn admits one or two modulo 4 square sum labeling.

Example 3.8. The friendship graph T_4 admits one or two modulo 4 square sum labeling which is shown in the Figure 8.

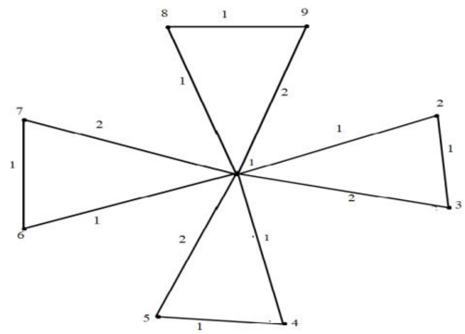

Figure 8: Friendship graph T_4

CONCLUSION

In this paper, one or two modulo four square sum labeling are introduced and proved for some standard graphs like comb graph, star graph, wheel graph and so on. This work contributes several new result to the theory of graph labeling.

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