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SUSTAINABLE EOQ MODELS FOR EXPIRING GOODS: ADDRESSING CARBON EMISSIONS AND DEMAND FLUCTUATIONS

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ABSTRACT

This study offers a novel approach to inventory management that takes sustainability goals and economic order quantity (EOQ) into account while managing perishable goods. The incorporation of sustainability concerns into traditional EOQ models is aimed at reducing carbon emissions and mitigating environmental damage. The model also incorporates variables like price changes, exponentially distributed demand, shortages, and lifetime demand with allowable delay. Through efficient management of perishable inventory, waste reduction, and environmental sustainability, firms can optimize inventory levels while taking these intricate dynamics into consideration. The suggested approach paves the way for more eco-friendly and effective supply chains by providing decision-makers with a thorough framework for managing perishable inventory while striking a balance between economic and environmental goals. This study uses an exponential price function and lifetime demand forecasts to help organizations adjust their inventory strategy to the changing dynamics of marketplaces and customer preferences. In order to offer useful insights into managing expiring inventory in circumstances of sustainability, carbon emissions reduction, scarcity, and dynamic price and demand, this research uses mathematical modelling and optimization approaches. The results give decision-makers a set of tools to help strike a balance between sustainability objectives and economic efficiency, supporting environmentally friendly inventory management techniques while maintaining financial viability and customer happiness. This research offers a road map for optimizing inventory strategies that not only increase operational efficiency but also support the global imperative of reducing carbon emissions and fostering sustainability management, as businesses endeavor to navigate the complexities of today's business landscape. Variations in various parameters and optimal results are visually represented and further examined in detail. MATHEMATICA 12.0 does sensitivity analysis and determines the optimal solution.

Keywords: Exponentially, Price And Lifetime Demand, Sustainability, Carbon Emissions, Expiring Items, Permissible delay

1. INTRODUCTION

In today's globalized and environmentally sensitive business context, efficient inventory management has become increasingly important for organizations attempting to balance economic aims with sustainability goals. This introduction lays the framework for investigating the integration of Economic Order Quantity (EOQ) models designed primarily for managing expiring products. K.J. Chung **[2004]** discuss about trade credit ordering quantity model. It considers the requirement of sustainability and the reduction of carbon emissions, as well as the constraints provided by scarcity, exponentially changing demand, price changes, and the finite lifetime of items. Inventory management is key to the performance of firms in a variety of industries, and effective management of expiring commodities is especially important in industries such as pharmaceuticals, perishable goods, and consumer products with short shelf life. S.S Sana **[2008]** developed a EOQ model with delay payments and discount offers, after that S. Khanra **[2011]** proceed with permissibly delay in payments model. While useful for managing non-perishable commodities, the typical EOQ approach lacks the ability to account for the inherent complexity involved with products that have an expiration date. To avoid financial losses and environmental effect, these complexities include the need to avoid overstocking, minimize waste, and ensure timely turnover. Furthermore, the study covers the issues raised by shortages, which can have a substantial impact on the supply of critical commodities, affecting consumer happiness and corporate profitability. The proposed EOQ models aim to limit the risks associated with stockouts and assure a continuous supply of critical commodities by accounting for the possibility of supply chain disruptions and fluctuations in demand. In addition, incorporating exponentially shifting demand and price changes recognizes the dynamic character of market settings, in which consumer preferences and economic considerations can fluctuate fast. Management of expiring items within the framework of Economic Order Quantity (EOQ) models has emerged as a critical consideration for organizations across various industries in today's dynamic business environment, characterized by a heightened focus on sustainability and environmental responsibility. The proposed EOQ models seek to achieve a compromise between serving customer demand and optimizing procurement costs while taking price changes into account. Volume flexibility multi-inventory model given by Singhal. S **[2013]**. The research solves the inherent issues connected with managing inventory for things with constrained shelf lives by taking the limited lifetime of products into account. When the time varying dependent demand on freshness of product Chang **[2016]**. The proposed EOQ models emphasize the importance of optimizing ordering amounts and frequencies in order to minimize waste due to product expiration while maintaining adequate stock levels to meet consumer demand. In brief, this project aims to create a new framework that incorporates EOQ models adapted for expiring commodities, with a focus on sustainability, carbon emissions reduction, and scarcity mitigation. Multi item machine breakdown model produced by S Singhal **[2017]**. Further that supply chain model with multiple market demand **[2018]**. The proposed models aim to provide organizations with a comprehensive approach to effectively managing inventory, reducing waste, and contributing to a more sustainable and resilient supply chain by incorporating the complexities of exponentially changing demand, price fluctuations, and limited product lifetimes. This research intends to add to the expanding body of knowledge in the topic of inventory management and sustainability, providing significant insights for firms looking to balance economic goals with environmental stewardship.

The successful handling of expiring commodities provides particular challenges in the dynamic context of modern supply chain management, necessitating novel solutions. This overview establishes the groundwork for further investigation into Economic Order Quantity (EOQ) models tailored specifically for expiring items. The analysis incorporates critical factors such as sustainability, carbon emissions reduction, scarcity, exponentially growing demand, price variations, and lifetime demand, all while accounting for allowable product delivery delays. Mushed. A **[2020]** developed a model on different types of demands with deteriorating inventory and Shan N **[2021]** upper lower bound for-profit inventory underprice stock lifetime e demand. Traditional EOQ models encounter difficulties when applied to items with limited shelf lives as industries develop towards more sustainable and ecologically responsible methods. Furthermore, taking into account lifetime demand in conjunction with allowable delay provides a subtle element to the management of expiring commodities. The proposed EOQ models seek to find a balance between minimizing loss due to product expiration and mitigating product delivery delays, ensuring that products reach customers within acceptable time constraints.

This study aims to close the gap by establishing specialized EOQ models that not only optimize inventory management for expiring items but also align with broader sustainability goals, such as lowering carbon emissions and reducing waste related with the disposal of expired products. The inclusion of shortages as a key point recognizes the complex nature of supply chain dynamics, where disruptions can have a significant impact on product availability. The study aims to equip organizations with ways to handle uncertainty and maintain consistent product availability by extending the existing EOQ framework to account allowable delays in product delivery. Ghosh P **[2021]** introduced permissible items under multiple advanced and delay payments policy. The incorporation of exponentially increasing demand and price fluctuations acknowledges market unpredictability. Industries that are experiencing frequent fluctuations in consumer tastes and economic situations can benefit from EOQ models that react to these changes by optimizing procurement decisions and inventory levels. In summary, this study aims to promote inventory management methods by presenting EOQ models designed specifically for expiring products. The study intends to provide organizations with a holistic framework by integrating sustainable principles, addressing carbon emissions reduction, shortages, exponentially changing demand, price variations, and lifetime demand with allowable delays. Chaudhary A and Shah N **[2022]** introduced non instantaneous deteriorating model with inflation and green effect deteriorating inventory model respectively. This paradigm promotes not only economic efficiency but also environmental responsibility and resilience in the face of supply chain uncertainty. The findings of this study have the potential to provide practical recommendations for firms looking to optimize their inventory management procedures while satisfying both economic and environmental goals.

Furthermore, taking into account lifetime demand in conjunction with allowable delay provides a subtle element to the management of expiring commodities. The proposed EOQ models seek to find a balance between minimizing loss due to product expiration and mitigating product delivery delays, ensuring that products reach customers within acceptable time constraints. In summary, this study aims to promote inventory management methods by presenting EOQ models designed specifically for expiring products. The study intends to provide organizations with a holistic framework by integrating sustainable principles, addressing carbon emissions reduction, shortages, exponentially changing demand, price variations, and lifetime demand with allowable delays. This paradigm promotes not only economic efficiency but also environmental responsibility and resilience in the face of supply chain uncertainty. The findings of this study have the potential to provide practical recommendations for firms looking to optimize their inventory management procedures while satisfying both economic and environmental goals.

2. ASSUMPTIONS AND NOTATIONS

Notations:

M	Expiration date
m	Trade credit period
Q	Ordering quantity
θ	Deterioration rate
n	Price elasticity
R	Shortage level
r	Inflation rate
p	Selling price
C_O	Ordering cost
C_S	Shortage cost
C_P	Purchase cost
C_s	Sales revenue
C_d	Deterioration cost
C_h	Holding cost
C_L	Lost sale cost
I_P	Interest charged by supplier
I_e	Interest earned
T	Total replenishment time cycle
t_1	Time at which inventory level reaches zero
$\prod_1$	Total profit when $t_1 \geq m$
$\prod_2$	Total profit when $t_1 < m$

Assumptions:

Numerous factors have an influence on fresh produce, including weather (temperature), condition, preservation, and time in stock. Getting an explicit newness of the item seems to be difficult. It is noteworthy, nevertheless, that fresh product has an expiration date. The most extreme lifespan, $\frac{M-t}{M}$, 0 < t < M, where M is the product's expiry period, is what we might anticipate in order to simplify and manage the problem.

- Demand for the item is exponentially, price and lifetime dependent demand.

$\{\lambda e^{-\lambda t}\}p^{-n}\frac{M-t}{M}$, $0 < t < M$

- Shortages are allowed and partially backlogged.
- Deterioration rate is constant.
- Inflation is also considered in this model.
- Also considered carbon emission reduction and sustainability.
- Time horizon is infinite.

3. MODEL DEVELOPMENT

This section generates a model for perishable goods with a constant rate of degradation in an inflationary environment, where the demand rate is determined by the product of exponential cost, the lifespan, and selling cost. At t = 0, the inventory system has an order quantity of (Q+R) units. At that time, the impact of item decay and customer interest started to lower the inventory level. The inventory level drops to zero at time t = T. Assumed to be constant is the rate of degradation.

The instantaneous inventory level I(t) at any time t during the cycle time $[0, t_1]$ and $[t_1, T]$ is governed by the following differential equation

$$I_1'(t) = -\theta I_1'(t) + (-p^{-n})\lambda e^{-\lambda t}\left(\frac{M-t}{M}\right) \qquad 0 \le t \le t_1 \qquad (1)$$

$$I_2'(t) = (-p^{-n})\lambda e^{-\lambda t}\left(\frac{M-t}{M}\right) \qquad t_1 \le t \le T \qquad (2)$$

With boundary condition $I(t_1) = 0$, Solutions of these differential equations are given below:

$$I_1(t) = \left[-p^{-n}\lambda\left(e^{-\lambda t}\left(\frac{1}{(\theta-\lambda)} - \frac{t}{M} + \frac{1}{M(\theta-\lambda)}\right) - e^{(\theta-\lambda)t_1-\theta t}\left(\frac{1}{(\theta-\lambda)} - \frac{t_1}{M} + \frac{1}{M(\theta-\lambda)}\right)\right)\right], \qquad (3)$$

$$I_2(t) = \left[-p^{-n}\left(e^{-\lambda t}\left(1 - \frac{\lambda t}{M} + \frac{1}{M}\right) - e^{-\lambda t_1}\left(1 - \frac{\lambda t_1}{M} + \frac{1}{M}\right)\right)\right], \qquad (4)$$

With boundary condition $I_1(t) = Q$, $t = 0$, Ordering quantity (Q) is given by:

$$I_1(t) = Q = \left[-p^{-n}\lambda\left(\left(\frac{1}{(\theta-\lambda)} + \frac{1}{M(\theta-\lambda)}\right) - e^{(\theta-\lambda)t_1}\left(\frac{1}{(\theta-\lambda)} - \frac{t_1}{M} + \frac{1}{M(\theta-\lambda)}\right)\right)\right], \qquad (5)$$

With boundary condition $I_2(t) = -R$, $t = T$, Shortage Level (R) is given by:

$$I_2(t) = R = \left[p^{-n}\left(e^{-\lambda T}\left(1 - \frac{\lambda T}{M} + \frac{1}{M}\right) - e^{-\lambda t_1}\left(1 - \frac{\lambda t_1}{M} + \frac{1}{M}\right)\right)\right], \qquad (6)$$

Different type of costs is used in this model are as follows:

- **Ordering Cost:**

$$C_O = O \qquad (7)$$

- **Purchase Cost:**

$$C_p = (C_p + C_p')(Q + R) \qquad (8)$$

- **Holding Cost:**

$$C_h = (C_h + C_h^{'})\left[\int_0^{t_1} e^{-rt} I_1(t)dt\right]$$

$$C_h = (C_h + C_h^{'})\left[-p^{-n}\lambda\left(\begin{array}{c} t_1\left(\frac{2}{(\theta-\lambda)} + \frac{2}{M(\theta-\lambda)}\right) - t_1^2\left(\frac{(r+\lambda)+(r+\theta)}{2(\theta-\lambda)} + \frac{1}{2M} + \frac{(r+\lambda)+(r+\theta)}{2M(\theta-\lambda)}\right) \\ +t_1^3\left(\frac{(r+\lambda)}{3M} - \frac{(\theta-\lambda)}{M} + \frac{(r+\theta)}{2M}\right)\end{array}\right)\right] \quad (9)$$

- **Sales Revenue:**

$$C_s = (C_s + C_s^{'})\left[\int_0^T e^{-rt}\left[\left(p^{-n}\lambda e^{-\lambda t}\right)\left(\frac{M-t}{M}\right)\right]dt\right]$$

$$C_s = (C_s + C_s^{'})\left[-p^{-n}\lambda T - \frac{p^{-n}T^2\lambda}{2}\left(r + \lambda + \frac{1}{M}\right) + \frac{p^{-n}T^3\lambda}{3M}(r+\lambda)\right] \quad (10)$$

- **Deterioration Cost:**

$$C_h = (C_d + C_d^{'})\left[\int_0^{t_1} \theta e^{-rt} I_1(t)dt\right]$$

$$C_h = (C_d + C_d^{'})\left[-p^{-n}\lambda\theta\left(\begin{array}{c} t_1\left(\frac{2}{(\theta-\lambda)} + \frac{2}{M(\theta-\lambda)}\right) - t_1^2\left(\frac{(r+\lambda)+(r+\theta)}{2(\theta-\lambda)} + \frac{1}{2M} + \frac{(r+\lambda)+(r+\theta)}{2M(\theta-\lambda)}\right) \\ +t_1^3\left(\frac{(r+\lambda)}{3M} - \frac{(\theta-\lambda)}{M} + \frac{(r+\theta)}{2M}\right)\end{array}\right)\right] \quad (11)$$

- **Shortage Cost:**

$$C_S = (C_S + C_S^{'})\left[\int_{t_1}^T -e^{-rt} I_2(t)dt\right]$$

$$C_S = (C_S + C_S^{'})\left[-p^{-n}\left(\begin{array}{c} 2T\left(1+\frac{1}{M}\right) - T^2\left(\frac{2r(M+1)+\lambda(M+2)}{2M}\right) + T^3\left(\frac{\lambda(r+\lambda)}{3M}\right) - Tt_1\left(\frac{2\lambda(M+2)+\lambda(rT+2\lambda t_1)}{2M}\right) \\ -2t_1\left(\frac{M+1}{M}\right) + t_1^2\left(\frac{2r(M+1)+3\lambda(M+2)}{2M}\right) - t_1^3\left(\frac{\lambda(r+\lambda)}{3M} + \frac{\lambda(r+2\lambda)}{2M}\right)\end{array}\right)\right] \quad (12)$$

- **Lost Sale Cost:**

$$C_L = (C_L + C_L^{'})\left[\int_{t_1}^T e^{-rt}\left(1 - e^{-\delta t}\right)\left[p^{-n}\lambda e^{-\delta t}\left(\frac{M-t}{M}\right)\right]dt\right]$$

$$C_L = (C_L + C_L^{'})\left[\frac{-p^{-n}\lambda\delta}{2}(T^2 - t_1^2) - \frac{-p^{-n}\lambda\delta}{3M}(T^3 - t_1^3)\right] \quad (13)$$

We have the two possible cases of permissible delay:

❖ **Case 1: $t_1 \geq m$** (i.e. when the inventory period is greater than permissible delay period)

By the end of the allowable delay, the buyer must sell all of the units and pay the provider. Starting at time m, the provider charges the products in stock at interest rate I_p. After then, because of consistent sales and revenue, the buyer gradually decreases the amount of the funded loan from the supplier. The annual interest that is due is:

$$IP_1 = pI_p\left[\int_m^{t_1} I_1(t)\,dt\right]$$

$IP_1 = pI_p$

$$\left[p^{-n}\left(\begin{array}{l} t_1^2\left[\left(\lambda^2\left(\frac{2\lambda-2\theta-M-1}{2M(\theta-\lambda)}\right) + \lambda\left(\frac{\lambda+M\theta-2\lambda\theta m+2m\theta^2}{2M(\theta-\lambda)}\right)\right)\right] - t_1^3\left[\lambda^2\left(\frac{-2}{3M}\right) + \lambda\left(\frac{\theta}{2M}\right)\right] + \\ t_1\left[\lambda\left(\frac{\theta\lambda m^2-\theta^2m^2-2m\lambda-2Mm\theta}{2M(\theta-\lambda)}\right) + \lambda^2\left(\frac{Mm+m}{M(\theta-\lambda)}\right)\right] - m^2\left[\lambda\left(\frac{\theta-2\lambda-M\theta}{2M(\theta-\lambda)}\right) + \lambda^2\left(\frac{M+1}{2M(\theta-\lambda)}\right)\right] + \frac{m^3\lambda^2}{3M}\end{array}\right)\right] \quad (14)$$

$$IE_1 = pI_e\int_0^m t\left(p^{-n}\lambda e^{-\lambda t}\right)\left(\frac{M-t}{M}\right)dt$$

$$IE_1 = pI_e\left[p^{-n}\left[\lambda\left(\frac{m^2}{2}-\frac{m^3}{3M}\right)-\lambda^2\left(\frac{m^3}{3}-\frac{m^4}{4M}\right)\right]\right] \tag{15}$$

Total Profit ($\prod_1$) for case 1

$$\prod_1 = \frac{1}{T}[C_s-(C_O+C_P+C_H+C_d+C_S+C_L)-IP_1+IE_1] \tag{16}$$

The following equations may be concurrently solved to find the best values of t_1 and p, which will maximize the predicted total yearly profit.

$$\frac{\partial\Pi_1(t_1,p)}{\partial t_1}=0 \tag{17}$$

$$\frac{\partial\Pi_1(t_1,p)}{\partial p}=0 \tag{18}$$

And they satisfied the following condition,

$$\left.\begin{array}{c}\frac{\partial^2\Pi_1(t_1,p)}{\partial t_1^2}>0,\frac{\partial^2\Pi_1(t_1,p)}{\partial p}>0\\ \left(\frac{\partial^2\Pi_1(t_1,p)}{\partial t_1^2}\right)\left(\frac{\partial^2\Pi_1(t_1,p)}{\partial p}\right)-\left(\frac{\partial^2\Pi_1(t_1,p)}{\partial t_1\partial p}\right)^2>0\end{array}\right\} \tag{19}$$

❖ **Case 2**: $\mathbf{t_1 < m}$ (when the inventory period is less than permissible delay period)

In this instance, the customer must pay the supplier in full before the end of the credit term m if they sell the required number of units by the end of the replenishment cycle time, t_1. As a result, no interest is required. But the annual interest earned is

$$IE_2 = pI_e\left[\int_0^{t_1} tp^{-n}\lambda e^{-\lambda t}\left(\frac{M-t}{M}\right)+t_1p^{-n}\lambda e^{-\lambda t}\left(\frac{M-t}{M}\right)(m-t_1)\right]$$

$$IE_2 = pI_e\left[p^{-n}\left[\begin{array}{c}t_1^2\lambda\left(\frac{1}{2}-m\right)-t_1^3\left(\lambda^2\left(\frac{1}{3}-\frac{m}{2}\right)+\lambda\left(\frac{1}{3M}+\frac{m}{2M}+1\right)\right)+\\ t_1^4\left(\frac{\lambda}{2M}+\lambda^2\left(\frac{1}{4M}+\frac{m}{3M}+\frac{1}{2}\right)\right)\end{array}\right]\right] \tag{20}$$

$$IP_2 = 0 \tag{21}$$

Total Profit ($\prod_2$) for case 2

$$\prod_2 = \frac{1}{T}[C_s-(C_O+C_P+C_H+C_d+C_S+C_L)+IE_2] \tag{22}$$

The following equations may be concurrently solved to find the best values of t_1 and p, which will maximize the predicted total yearly profit.

$$\frac{\partial\Pi_2(t_1,p)}{\partial t_1}=0 \tag{23}$$

$$\frac{\partial\Pi_2(t_1,p)}{\partial p}=0 \tag{24}$$

And they satisfied the following condition,

$$\left.\begin{array}{c}\frac{\partial^2\Pi_2(t_1,p)}{\partial t_1^2}>0,\frac{\partial^2\Pi_2(t_1,p)}{\partial p}>0\\ \left(\frac{\partial^2\Pi_2(t_1,p)}{\partial t_1^2}\right)\left(\frac{\partial^2\Pi_2(t_1,p)}{\partial p}\right)-\left(\frac{\partial^2\Pi_2(t_1,p)}{\partial t_1\partial p}\right)^2>0\end{array}\right\} \tag{25}$$

4. Numerical Examples And Sensitivity Analysis

Example 1: Let us consider the values for case 1 as

C_s = 2 units, C_s' = 8 units, n = 10, λ = 2, T = 15 months, r = 6%, M = 10, O = 140, C_p = 5 units,

C_p' =10 units, C_h = 12 units, C_h' = 13 units, θ = 0.05, C_d =15 units, C_d' = 5 units, C_S = 2 units,

C_S' = 8 units, C_L = 6 units, C_L' = 18 units, δ = 0.05, I_p = 9, I_e = 11, m = 12.9 months

Using MATHEMATICA 12.0 we are getting an optimal solution t_1 = 10.1181 months, p = \$ 1.28421 and with maximum total profit $\prod_1$ = \$ 285.666

The effect of changes in different parameters like elasticity, Demand rate, holding cost, lost sale cost etc. on the value of optimal time and total profit. Sensitivity analysis is performed by changing each parameter at -20%, -10%, 10%, 20%.

Table 1: Sensitivity Analysis of case 1 for Different Parameters

Parameters	% change	t_1	p	TP ($\prod_1$)
C_s	-20%	10.1293	1.27647	286.392
	-10%	10.1236	1.28040	286.023
	10%	10.1129	1.28789	285.318
	20%	10.1077	1.29147	284.980
n	-20%	9.93276	1.40480	266.975
	-10%	10.0292	1.33856	276.677
	10%	10.1970	1.24133	294.191
	20%	10.2642	1.20875	302.329
C_S	-20%	10.1193	1.28344	285.738
	-10%	10.1187	1.28382	285.702
	10%	10.1176	1.28459	285.712
	20%	10.1171	1.28497	285.760
r	-20%	9.78055	1.29148	206.568
	-10%	9.94684	1.28850	245.400
	10%	10.2945	1.27896	327.424
	20%	10.4759	1.27303	370.737
C_d	-20%	10.1165	1.28521	285.572
	-10%	10.1173	1.28471	285.619
	10%	10.1190	1.28370	285.712
	20%	10.1198	1.28319	285.760
O	-20%	10.1181	1.28421	294.999
	-10%	10.1181	1.28421	290.332
	10%	10.1181	1.28421	280.999
	20%	10.1181	1.28421	276.332
C_p	-20%	10.1001	1.29544	284.623
	-10%	10.1089	1.28996	285.133
	10%	10.1280	1.27815	186.224
	20%	10.1386	1.27174	286.811
C_h	-20%	10.0941	1.29936	284.257
	-10%	10.1056	1.29203	284.940
	10%	10.1318	1.27581	286.439
	20%	10.1468	1.26673	287.269
θ	-20%	10.1373	1.27001	276.858
	-10%	10.1294	1.27611	281.322
	10%	10.1096	1.29102	290.195
	20%	10.0987	1.29930	294.666

δ	-20%	10.1156	1.28601	285.495
	-10%	10.1169	1.28511	285.580
	10%	10.1194	1.28330	285.752
	20%	10.1208	1.28238	285.838
C_L	-20%	10.1175	1.28466	285.623
	-10%	10.1178	1.28443	285.644
	10%	10.1185	1.28398	285.687
	20%	10.1188	1.28375	285.708

Example 2: Let us consider the values for case 2 as

$C_s = 200$ units, $C_s' = 80$ units, $n = 1.1$, $\lambda = 2$, $T = 15$ months, $r = 0.08\%$, $M = 10$, $O = 140$, $C_p = 20$ units, $C_p' = 10$ units, $C_h = 5$ units, $C_h' = 3$ units, $\theta = 0.1$, $C_d = 15$ units, $C_d' = 5$ units, $C_S = 20$ units, $C_S' = 15$ units, $C_L = 10$ units, $C_L' = 18$ units, $\delta = 3.5$, $I_e = 11$, $m = 10$ months

Using MATHEMATICA 12.0 we are getting an optimal solution $t_1 = 4.25608$ months, $p = \$\ 93.8985$ and with maximum total profit $\prod_2 = \$\ 1927.5$

The effect of changes in different parameters like elasticity, Demand rate, holding cost, lost sale cost etc. on the value of optimal time and total profit. Sensitivity analysis is performed by changing each parameter at -20%, -10%, 10%, 20%.

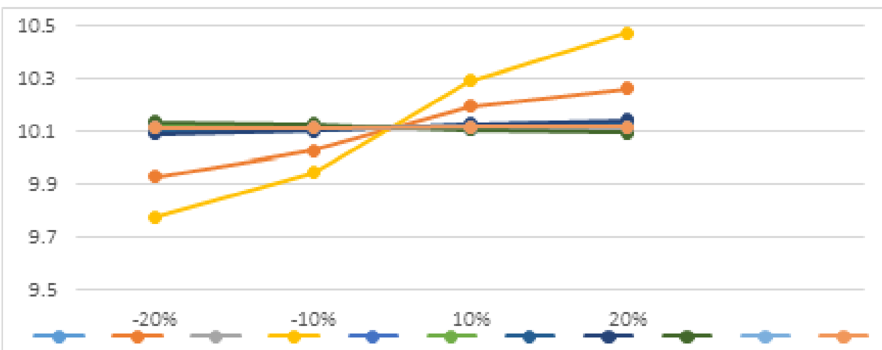

Figure 1: Sensitivity analysis for t_1 (Case 1)

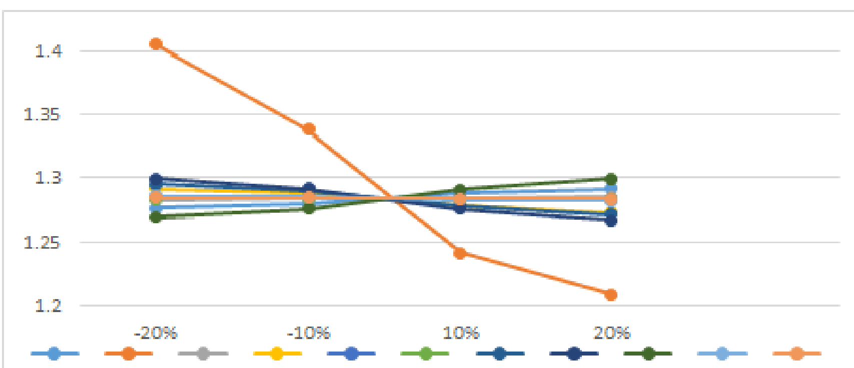

Figure 2: Sensitivity analysis for p (Case 1)

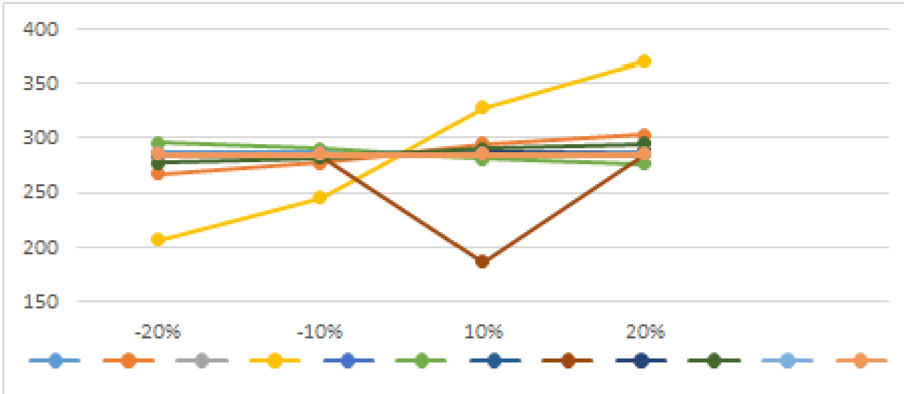

Figure 3: Sensitivity Analysis for $\prod_1$ (Case 1)

Table 2: Sensitivity Analysis of case 2 for Different Parameters

Parameters	% change	t_1	p	TP ($\prod_2$)
C_s	-20%	4.26701	94.1143	1535.43
	-10%	4.26095	93.9944	1731.43
	10%	4.25208	93.8199	2123.54
	20%	4.24874	93.7544	2319.58
n	-20%	4.45145	250.652	2599.85
	-10%	4.35086	217.236	2506.65
	10%	4.19911	43.8235	1252.56
	20%	4.2346	28.9347	869.682
C_S	-20%	3.3562	94.6553	1927.21
	-10%	3.8632	93.5662	1927.56
	10%	4.3985	92.9563	1927.64
	20%	4.9521	91.6523	1927.89
r	-20%	4.2512	95.7709	1923.16
	-10%	4.25362	94.8351	1925.31
	10%	4.2586	92.9612	1929.71
	20%	4.26116	92.0231	1931.95
O	-20%	4.24727	93.7256	1934.08
	-10%	4.25168	93.8121	1930.79
	10%	4.26047	93.9849	1924.21
	20%	4.26484	94.0712	1920.93
C_p	-20%	4.22566	85.0828	1946.94
	-10%	4.24163	89.5047	1936.94
	10%	4.26923	98.2685	1918.54
	20%	4.28125	102.618	1910.03
C_h	-20%	4.24292	93.879	1927.04
	-10%	4.24949	93.8919	1927.26
	10%	4.26269	93.8989	1927.74
	20%	4.26931	93.893	1927.99
θ	-20%	4.48471	94.1483	2000.01
	-10%	4.36936	93.9814	1963.33
	10%	4.14723	93.8895	1852.52
	20%	4.04252	93.9462	1858.38

δ	-20% -10% 10% 20%	4.31479 4.28378 4.23155 4.20992	91.3964 92.3494 95.9014 98.2594	1603.36 1766.32 2087.32 2244.93
C_L	-20% -10% 10% 20%	3.63073 3.95337 4.54022 4.80699	92.9726 93.2857 94.7241 95.7024	1650.48 1793.03 2054.49 2174.51

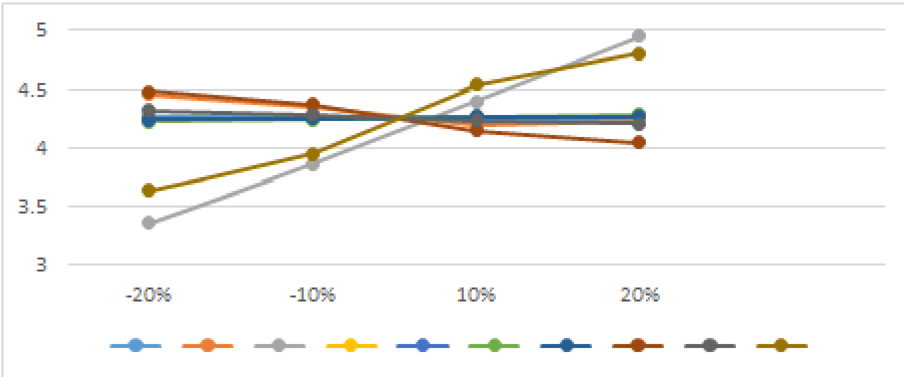

Figure 4: Sensitivity Analysis for t_1 (Case 2)

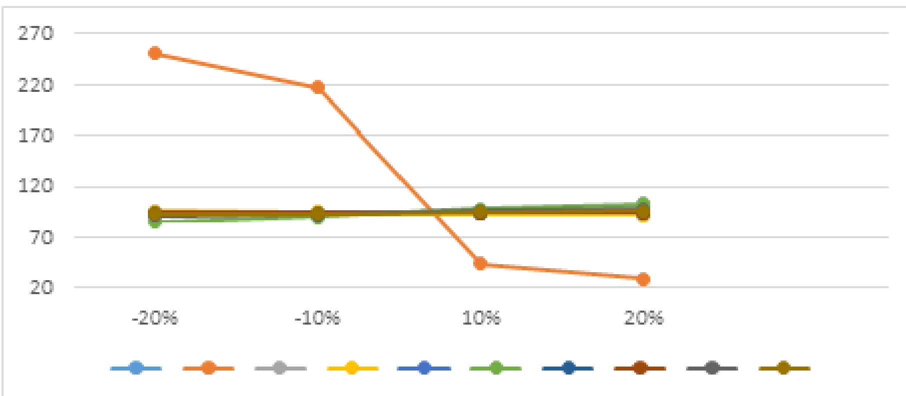

Figure 5: Sensitivity Analysis for p (Case 2)

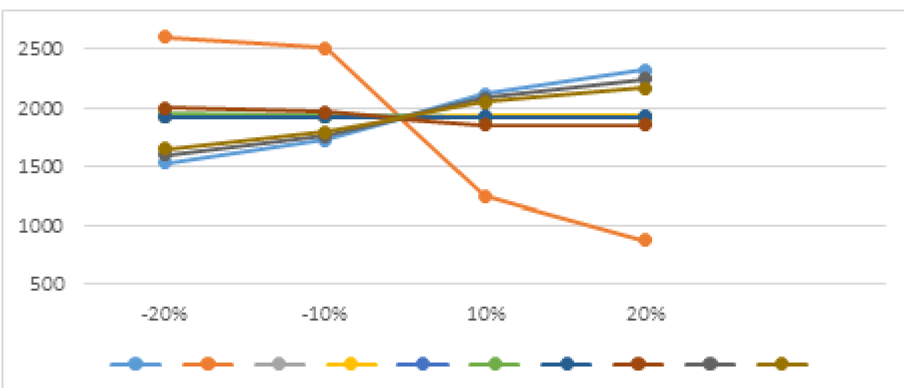

Figure 6: Sensitivity Analysis for ∏2 (Case 2)

- Price elasticity, ordering cost, and purchasing cost all have reversible effects on total profit.
- Figure 3 and 4 demonstrate the reversible effects of markup and scale demand on cycle time.
- Price elasticity fluctuates in both the cases.
- Deterioration cost fluctuates more in figure 4 and less in figure 6.
- Table 1 indicates that an increase in the allowable delay duration will correspondingly result in an increase in the inventory period. However, the profit is rising while the selling price is falling.
- Table 2 indicates that an increase in the allowable delay duration will correspondingly result in a decrease in the inventory period. Additionally, the profit is rising while the selling price is falling.

5. CONCLUSION

This research looked into the integration of EOQ models designed for expiring items while addressing the multifarious issues provided by sustainability goals, carbon emissions reduction, shortages, exponentially changing demand, price changes, and limited product lifetimes. The limits of typical EOQ models in effectively managing inventory for commodities with short shelf lives were emphasized in our analysis, emphasizing the importance of building specialized frameworks to address the complexity involved with expiring goods. By incorporating sustainability concepts into inventory management, organizations can not only improve operational efficiency but also reduce the environmental effect of discarded products, so contributing to a more sustainable and responsible supply chain ecosystem. Furthermore, the consideration of shortages, which are caused by disruptions in the supply chain and fluctuations in demand, emphasizes the significance of maintaining optimal inventory levels to ensure the continued availability of important commodities. Organizations can limit the risks associated with stockouts and improve customer satisfaction by using the recommended EOQ models, encouraging long-term customer loyalty and corporate resilience. The incorporation of exponentially changing demand and price changes into EOQ models acknowledges the volatile nature of market dynamics, allowing organizations to adapt their inventory management methods in response to evolving consumer preferences and economic conditions. Organizations can establish a balance between cost-efficiency and addressing consumer needs by optimizing procurement decisions in light of variable demand and prices, strengthening their competitive advantage in the marketplace.

The outcomes of this study emphasize the need of accounting for a product's limited lifetime in the management of expiring items. Organizations can efficiently minimize waste caused by product expiration by optimizing ordering amounts and frequency, thereby decreasing financial losses and contributing to a more sustainable and ecologically conscientious approach to inventory management. Finally, the incorporation of specialized EOQ models for managing expiring items, while taking into account sustainability, carbon emissions reduction, scarcity, exponentially changing demand, price fluctuations, and limited product lifetimes, provides a comprehensive and forward-thinking approach to inventory management. Organizations can not only improve their operational efficiency and financial performance by applying these techniques, but they can also contribute to the global drive to create a more sustainable and resilient business ecosystem. This research lays the groundwork for the practical use of these models, providing valuable insights for firms looking to align their inventory management practices with both economic and environmental goals.

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