Stochastic Modelling and Computational Sciences

OBSERVATIONAL CONSTRAINTS OF BIANCHI TYPE-I COSMOLOGICAL MODEL WITH COSMOLOGICAL CONSTANT Λ

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ABSTRACT

In this work, we examine Einstein's field equations where a variable cosmological constant is taken into account for a Bianchi type-I universe with a perfect fluid present. We make the assumption that the cosmological term has a proportionality to the square of the Hubble parameter. On the basis of quantum field estimation in a curved growing backdrop, numerous researchers have recently developed the variation law for vacuum density. Both the model and the cosmological term tend asymptotically towards a de-Sitter universe and a true cosmological constant, respectively. The current universe is speeding with a significant fraction of cosmological density in the form of a cosmological term, according to some recent findings that were acquired here using a somewhat different methodology from that of other researchers. The cosmological model's geometrical and physical interpretations were examined.

1. INTRODUCTION

The cosmological constant problem is a highly significant observational discovery in recent times that captivates academics worldwide. Einstein initially included the cosmic constant Λ in his field equations. Given the dynamic nature of the cosmos, it is logical to see this constant as a variable that changes throughout time. The Λ term is a concept that emerges naturally in the context of general relativistic quantum field theory. It is understood as the energy density of the vacuum [1]-[4].

Many researchers [5]-[13] have argued for the dependence of Cosmological constant Λ. Several scholars [14]-[27] have examined cosmological models featuring variable values of G and Λ, using a homogeneous and isotropic FRW line element. Furthermore, investigations have been conducted on Bianchi type-I models by employing varying values for the gravitational constant G and the cosmological constant Λ, as documented in references[28]-[34]. Schutzhold [35, 36] has recently suggested that the vacuum energy density is directly proportional to the Hubble parameter. This implies that the vacuum energy density decreases as a power of $\Lambda \approx$ m3H, where $m \approx 150$MeV represents the energy scale of the chiral phase transmission of QCD. Borges and Carnerio [37] have examined a flat space that is both isotropic and homogeneous. This space is filled with matter and has a cosmological term that is directly proportional to the Hubble parameter H. The equation of state for this space follows that of a vacuum. Tiwari and Divya Singh[38] have examined the anisotropic Bianchi type-I model with a variable Λ term. Tiwari and Sonia [39] examined the absence of shear in Bianchi type-III string cosmological models by considering the effects of bulk viscosity and a time-dependent Λ. Tiwari and Sonia [40] examined the cosmological model of the Bianchi type-I string, taking into account the presence of bulk viscosity and a time-dependent Λ term. To examine the potential impact of anisotropy in the early cosmos on current findings, several scholars [28]-[48] have analyzed Bianchi type-I models from various perspectives.

The paper is organized as follows: Section 2 examines the homogeneous anisotropic Bianchi type-I space time with variable cosmological constant Λ containing matter in the form of a perfect fluid. In Section 3, We find the solution of the Einstein field equations assuming that cosmological term is proportional to Hubble parameter H for stiff matter and also we calculate various physical parameters such as scale factors, Hubble parameter, expansion scalar, deceleration parameter, and more. Finally, we provide concluding remarks in the last section.

2. THE MODEL AND THE FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type-I space-time is given by

$$ds^2 = -dt^2 + X_1^2(t)dx^2 + X_2^2(t)dy^2 + X_3^2(t)dz^2, \qquad (2.1)$$

Where, $X_1(t)$, $X_2(t)$ and $X_3(t)$ are the metric functions of cosmic time t only.

The energy-momentum tensor of a perfect fluid is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \qquad (2.2)$$

where $g_{\mu\nu}^{uv} = -1$; u_μ is the four-velocity vector; $R_{\mu\nu}$ is Ricci tensor; R is Ricci scalar; ρ and p are energy density and thermodynamic pressure respectively. Let us assume that the matter content yield to an equation of state,

$$P = \omega\rho, 0 \leq \omega \leq 1 \qquad (2.3)$$

The Einstein's field equation with varying cosmological constant Λ are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -T_{\mu\nu} + \Lambda g_{\mu\nu}, \qquad (2.4)$$

Spatial volume V as an average scale factor R of the line element can be defined as

$$V = R^3 = (X_1X_2X_3). \qquad (2.5)$$

The hubble parameter H can be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3}\right). \qquad (2.6)$$

Where, dot means the ordinary time derivative of the concerned quantity are

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \qquad (2.7)$$

where, $H_1 = \frac{X_1}{X_2}, H_2 = \frac{X_2}{X_2}, H_3 = \frac{X_3}{X_3}$, are directional hubble factors in the x, y and z directions respectively.

So, in the moving coordinate system, we have the metric Eq. (1) and energy moment tensor Eq. (2). The field equation Eq. (4) gives us the following result:

$$\frac{\ddot{X}_2}{X_2}+\frac{\ddot{X}_3}{X_3}+\frac{\dot{X}_2\dot{X}_3}{X_2X_3}=-P+\Lambda, \qquad (2.8)$$

$$\frac{\ddot{X}_1}{X_1}+\frac{\ddot{X}_3}{X_3}+\frac{\dot{X}_1\dot{X}_3}{X_1X_3}=-P+\Lambda, \qquad (2.9)$$

$$\frac{\ddot{X}_1}{X_1}+\frac{\ddot{X}_2}{X_2}+\frac{\dot{X}_1\dot{X}_2}{X_1X_2}=-P+\Lambda, \qquad (2.10)$$

$$\frac{\dot{X}_1\dot{X}_2}{X_1X_2}+\frac{\dot{X}_2\dot{X}_3}{X_2X_3}+\frac{\dot{X}_1\dot{X}_3}{X_1X_3}=\rho+\Lambda, \qquad (2.11)$$

The vanishing divergence of the Einstein tensor, we obtain

$$\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right)\right]+\dot{\Lambda}=0. \qquad (2.12)$$

The non-vanishing component of shear tensor $\sigma_{\mu\nu}$ can be defined as $\sigma_{\mu\nu}=$ $u_{\mu;\nu}+u_{\nu;\mu}-\frac{2}{3}g_{\mu\nu}u^{\gamma}_{\gamma}$ are obtain by

$$\sigma^1_1=\frac{4}{3}\frac{\dot{X}_1}{X_1}-\frac{2}{3}\left(\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right), \qquad (2.13)$$

$$\sigma^2_2=\frac{4}{3}\frac{\dot{X}_2}{X_2}-\frac{2}{3}\left(\frac{\dot{X}_3}{X_3}+\frac{\dot{X}_1}{X_1}\right), \qquad (2.14)$$

$$\sigma^3_3=\frac{4}{3}\frac{\dot{X}_3}{X_3}-\frac{2}{3}\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}\right), \qquad (2.15)$$

The shear scalar σ is defined as

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{X}_1^2}{X_1^2}+\frac{\dot{X}_2^2}{X_2^2}+\frac{\dot{X}_3^2}{X_3^2}-\frac{\dot{X}_1\dot{X}_2}{X_1X_2}-\frac{\dot{X}_2\dot{X}_3}{X_2X_3}-\frac{\dot{X}_1\dot{X}_3}{X_1X_3}\right), \qquad (2.16)$$

$$\frac{\dot{\sigma}}{\sigma}=-\left(\frac{\dot{X}_1}{X_1}+\frac{\dot{X}_2}{X_2}+\frac{\dot{X}_3}{X_3}\right)=-3H. \qquad (2.17)$$

The Einstein's field equations from (8)-(11) in terms of physical quantities Hubble parameter H, shear scalar σ and declaration parameter q written as

$$H^2(2q-1)-\sigma^2 = p-\Lambda, \tag{2.18}$$

$$3H^2-\sigma^2 = \rho-\Lambda, \tag{2.19}$$

Where, the declaration parameter q is

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}, \tag{2.20}$$

After integrating Eqns

$$\frac{X_1}{X_1} - \frac{X_2}{X_2} = \frac{c_1}{(X_1X_2X_3)} = \frac{c_1}{R^3} \tag{2.21}$$

$$\frac{\dot{X}_1}{X_1} - \frac{\dot{X}_3}{X_3} = \frac{c_2}{(X_1X_2X_3)} = \frac{c_2}{R^3} \tag{2.22}$$

$$\frac{\dot{X}_2}{X_2} - \frac{\dot{X}_3}{X_3} = \frac{c_3}{(X_1X_2X_3)} = \frac{c_3}{R^3} \tag{2.23}$$

Where, c_1, c_2 & c_3 are integrating constants.

We assume energy conservation equation $T_{\mu\nu} = 0$ obeys as

$$\dot{\rho} + \rho(1+\omega)\left(\frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3}\right) = 0. \tag{2.24}$$

By using Eqns. (5) and (24), we have

$$\rho = \frac{c_4}{R^{3(1+\omega)}}, \tag{2.25}$$

Where, c4 is the integration constant. Again, integrating from Eqns. (21), (22) & (23), we have

$$\frac{X_1}{X_2} = d_1.exp\left(c_1 \int \frac{1}{R^3} dt\right), \tag{2.26}$$

$$\frac{X_1}{X_3} = d_2.exp\left(c_2 \int \frac{1}{R^3} dt\right), \tag{2.27}$$

$$\frac{X_2}{X_3} = d_3.exp\left(c_3 \int \frac{1}{R^3} dt\right), \tag{2.28}$$

Where, d1, d_2 and d_3 are constant of integration

From the Eq. (20), we have

$$\frac{3\sigma^2}{\Theta^2} = 1 - \frac{3\rho}{\Theta^2} - \frac{3\Lambda}{\Theta^2}, \tag{2.29}$$

which implying that

$$\Lambda \geq 0, 0 < \frac{\sigma^2}{\Theta^2} \leq \frac{1}{3}, 0 < \frac{\rho}{\Theta^2} < \frac{1}{3}.$$

Thus, the presence of positive cosmological constant Λ lowers the upper limit of anisotropy and a negative value of the cosmological constant Λ gives more room for anisotropy. Eq.(29) can be defined as

$$\frac{3\sigma^2}{3H^2} = 1 - \frac{\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c}. \tag{2.30}$$

Where, ρ_c= $3H^2$ and ρ_v= = Λ denotes the critical density and vacuum density respectively.

$$\frac{d\Theta}{dt} = -\frac{3}{2}(\rho + p) - 3\sigma^2 \tag{2.31}$$

Demonstrating that the rate of volumetric expansion diminishes over time, and the existence of a positive cosmological constant Λ decelerates this drop, while a negative cosmological constant Λ would accelerate it. From Eqns. (19) and (20), we have

$$\Lambda = (2 - q)H^2 - \frac{(1 - \omega)\rho}{2}, \tag{2.32}$$

which implies that $\Lambda \leq 0$ for $q \geq 2$.

3. SOLUTIONS OF THE FIELD EQUATIONS

The system of equations (3) and (8), (9), (10) and (11) have five independent equations in six unknowns X_1, X_2, X_3, ρ, p and Λ. Therefore, we require one extra condition to solve the system completely. Here, we will take the cosmological term proportional to the Hubble parameter, as many authors considered that it as cosmological constant Λ decay. Schutzhold [26] consider variation law for vacuum density, Borges and Carnerio [27], R. K. Tiwari and Divya Singh [28], Tiwari and Sonia [29,30] have considered a cosmological term proportional to H. Thus we take the decaying vacuum energy density is given by

$$\Lambda = \alpha H^2, \tag{3.1}$$

Where, α is the positive constant.

Let us assume that $\Omega = \Lambda/\rho$ is the ratio of the vacuum and matter density. From the Eqns. (19) & (23), we have

$$\alpha = \frac{3\Omega}{1+\Omega}\left(1 - \frac{\sigma^2}{27\Theta^2}\right), \tag{3.2}$$

Therefore, the value of α is reduced in an anisotropic background as compared to its value in an isotropic background.

When $\omega = 1$ i.e the stiff fluid, Eqns. (18), (19) and (33) produce a differential equation

$$\dot{H} + (3-\alpha)H^2 = 0. \tag{3.3}$$

After integrating Eqn. (35), we have

$$R = \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{1}{(3-\alpha)}}. \tag{3.4}$$

Where, k_1 and k_2 are the constant of integration

$$H = \frac{\dot{R}}{R} = k_1.\Big[(3-\alpha)(k_1 t + k_2)\Big]^{-1}. \tag{3.5}$$

$$X_1 = \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{1}{(3-\alpha)}}.exp\left[\frac{2c_1 + c_2}{6[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right]$$

$$X_2 = \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{1}{(3-\alpha)}}.exp\left[\frac{c_2 - c_1}{3[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right]$$

$$X_3 = \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{1}{(3-\alpha)}}.exp\left[\frac{2c_2 - c_1}{2[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right]$$

Therefore, the line element Eqn. (1) reduces to

$$ds^2 = -dt^2 + \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{2}{(3-\alpha)}} . \Bigg\{ exp\left[\frac{2c_1 + c_2}{3[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right] dx^2$$

$$+ exp\left[\frac{2(c_2 - c_1)}{3[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right] dy^2 + exp\left[\frac{2c_2 - c_1}{[(3-\alpha)(k_1 t + k_2)]^{\frac{3}{(3-\alpha)}}}\right] dz^2 \Bigg\}$$

The matter density ρ, pressure p, cosmological term Λ, shear scalar σ and expansion scalar Θ are given for this model as,

$$\rho = p = c_4 \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{-6}{(3-\alpha)}},$$

$$\Lambda = \alpha k_1^2 \Big[(3-\alpha)(k_1 t + k_2)\Big]^{-2},$$

$$\Theta = \frac{H}{3} = \frac{k_1}{3}\Big[(3-\alpha)(k_1 t + k_2)\Big]^{-1},$$

$$\sigma = (k_1 t + k_2)^{\frac{-3k_1^2}{(3-\alpha)}} k_3,$$

The ratio of the vacuum and matter density given as

$$\Omega = \frac{\Lambda}{\rho} = \frac{\alpha k_1^2}{c_4}\Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{2\alpha}{(3-\alpha)}},$$

The deceleration parameter q for this model is given by

$$q = 2 - \alpha$$

The vacuum energy density ρv and the critical density ρc can be defined as

$$\rho_v = \alpha k_1^2 \Big[(3-\alpha)(k_1 t + k_2)\Big]^{-1},$$

$$\rho_c = 3k_1^2 \Big[(3-\alpha)(k_1 t + k_2)\Big]^{-2},$$

Spatial volume is given as

$$V = R^3 = \Big[(3-\alpha)(k_1 t + k_2)\Big]^{\frac{3}{3-\alpha}}.$$

4. GRAPHICAL REPRESENTATION OF PHYSICAL QUANTITIES

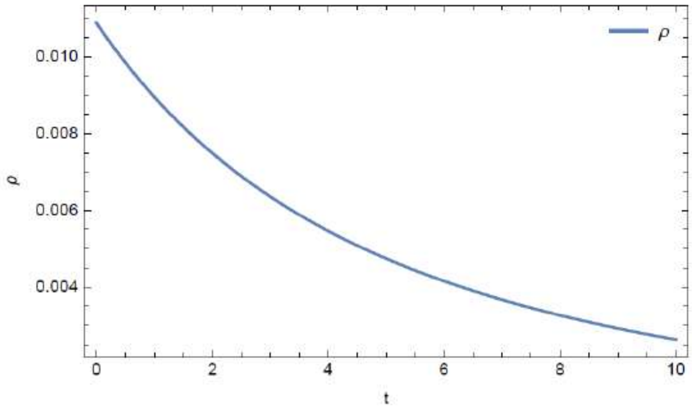

Figure 1. The variation matter density ρ is shown against t. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1.$

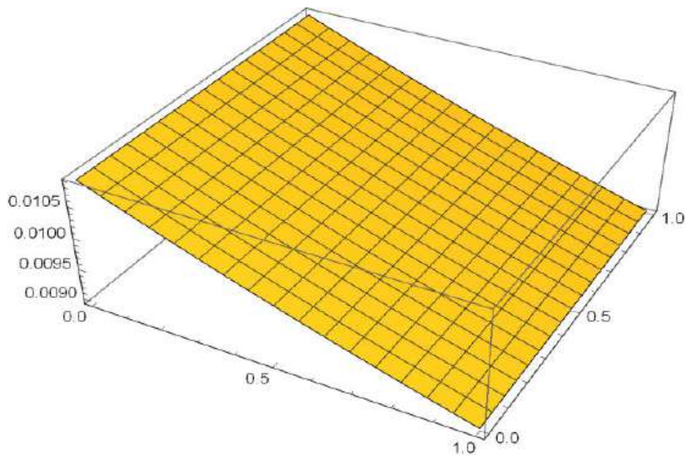

Figure 2. The variation matter density ρ is shown against t in 3- Dimensional plot. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

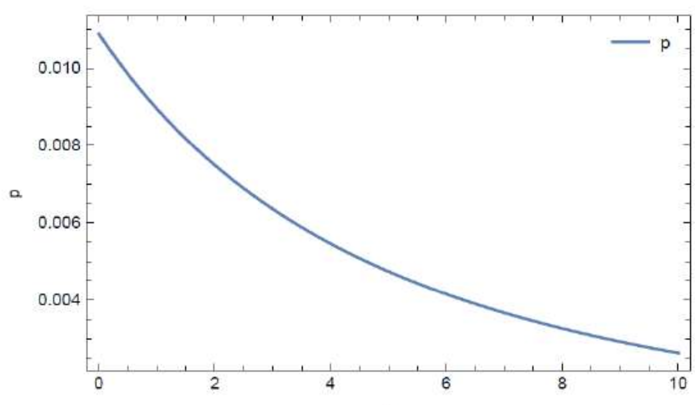

Figure 3. The variation pressure p is shown against t. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1.$

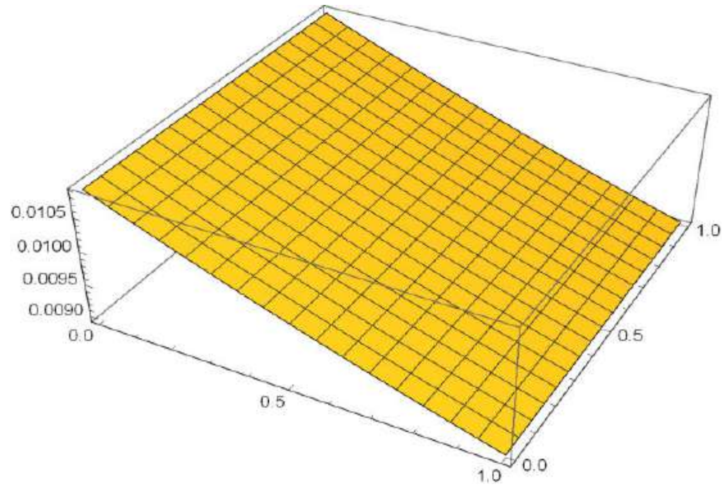

Figure 4. The variation matter density ρ is shown against t in 3- Dimensional plot. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

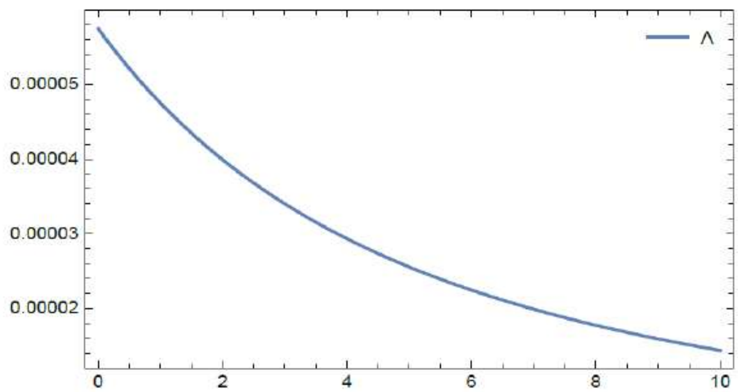

Figure 5. The variation cosmological constant Λ is shown against t. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

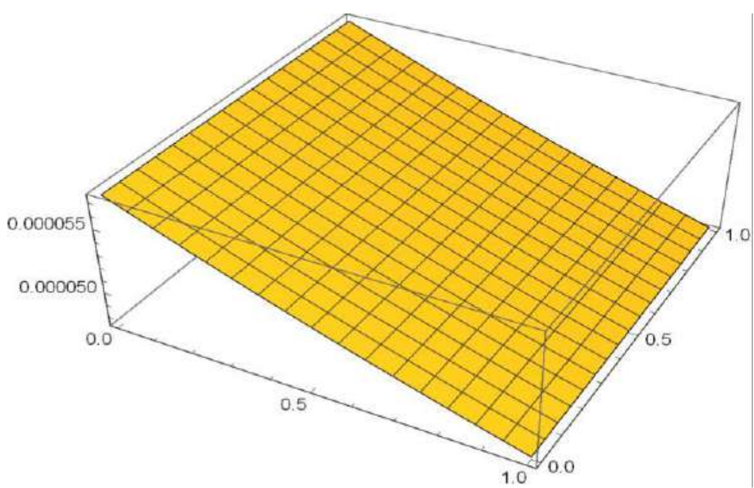

Figure 6. The variation cosmological constant Λ is shown against t in 3-Dimensional plot. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

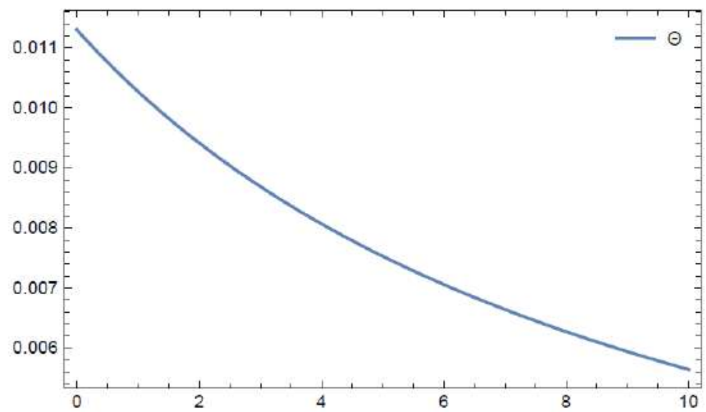

Figure 7. The variation expansion scalar Θ is shown against t. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

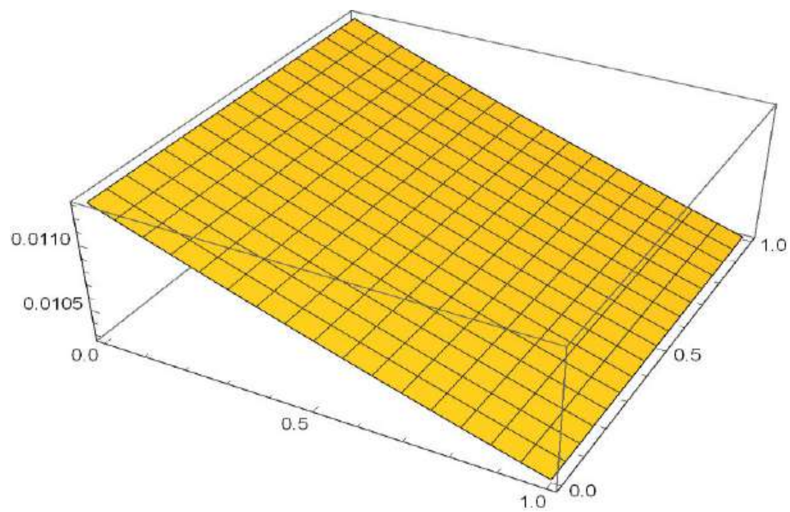

Figure 8. The variation expansion scalar Θ is shown against t in 3-Dimensional plot. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

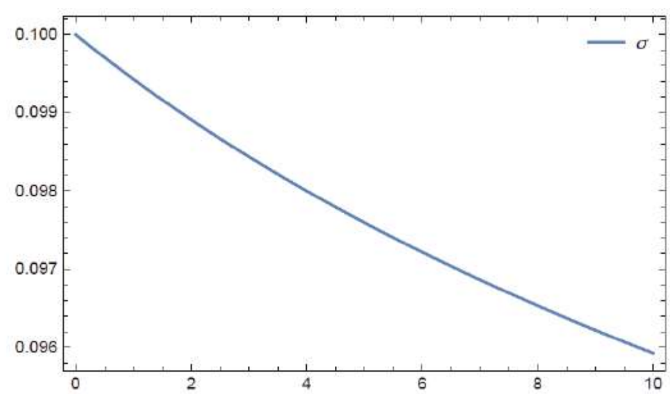

Figure 9. The variation of shear scalar σ is shown against t. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

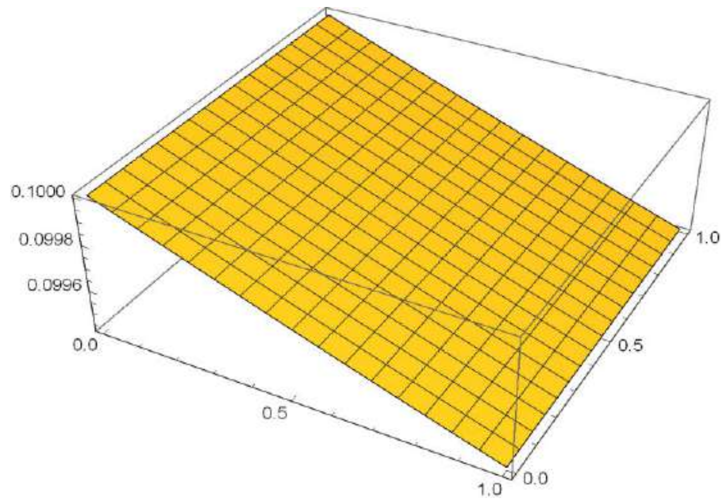

Figure 10. The variation of shear scalar σ is shown against t in 3-Dimensional plot. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

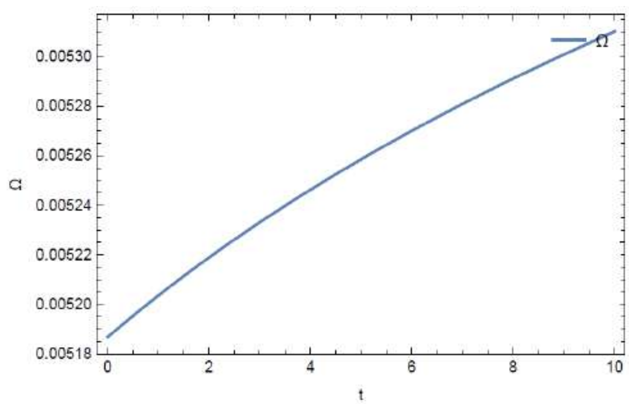

Figure 11. The ratio of vacuum and mater density Ω is shown against t. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1. Figure

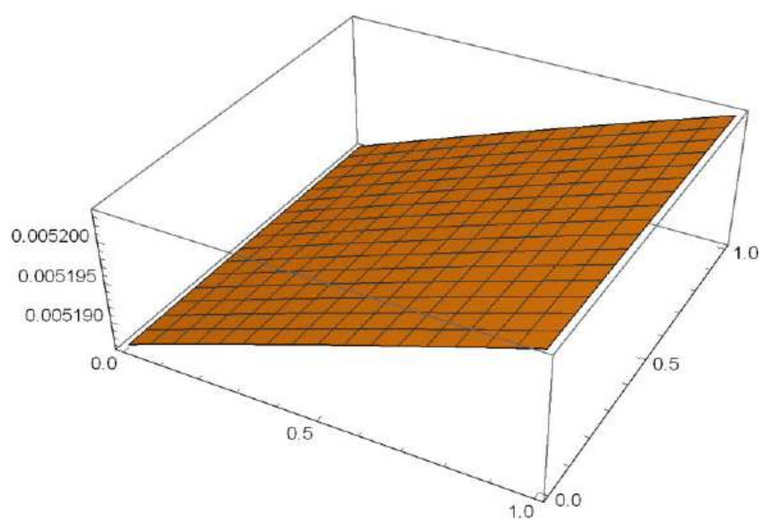

Figure 12. The ratio of vacuum and mater density Ω is shown against t in 3-Dimensional plot. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

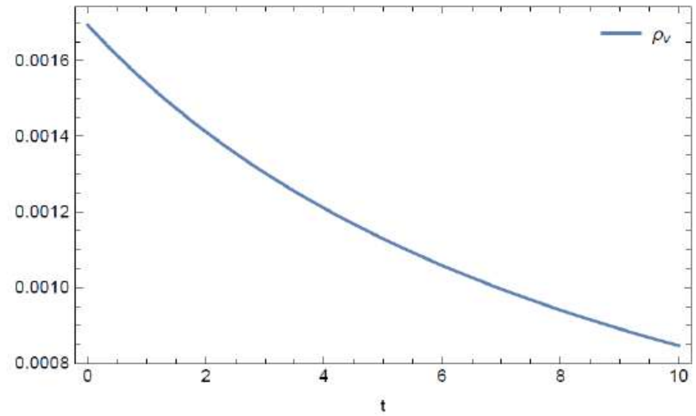

Figure 13. The variation of vacuum energy density ρ_v is shown against t. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

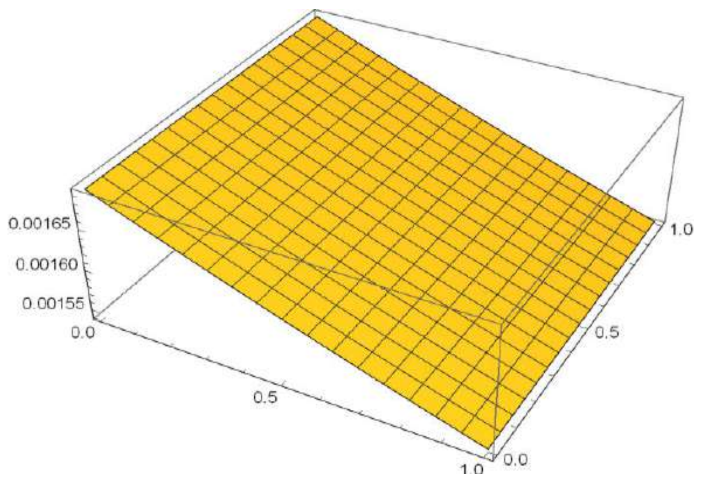

Figure 14. The variation of vacuum energy density ρv is shown against t in 3-Dimensional plot. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

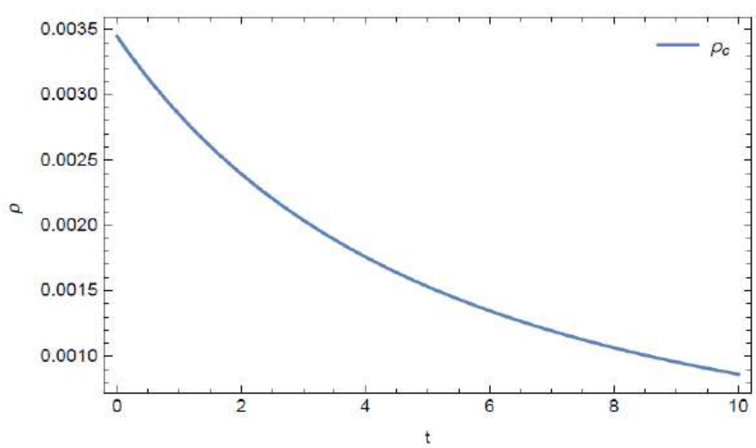

Figure 15. The variation of critical energy density ρ_c is shown against t. We use constants $C_4 = 0.1$ and $\alpha = 0.05, k_1 = 0.1, k_2 = 1$.

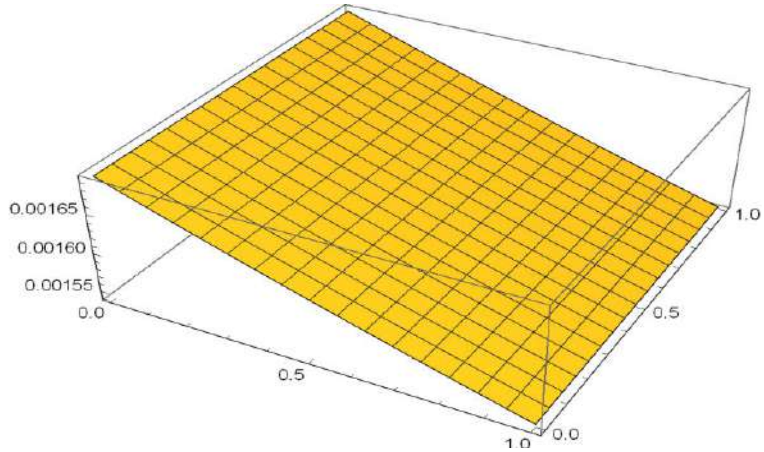

Figure 16. The variation of critical energy density ρc is shown against t in 3-Dimensional plot. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

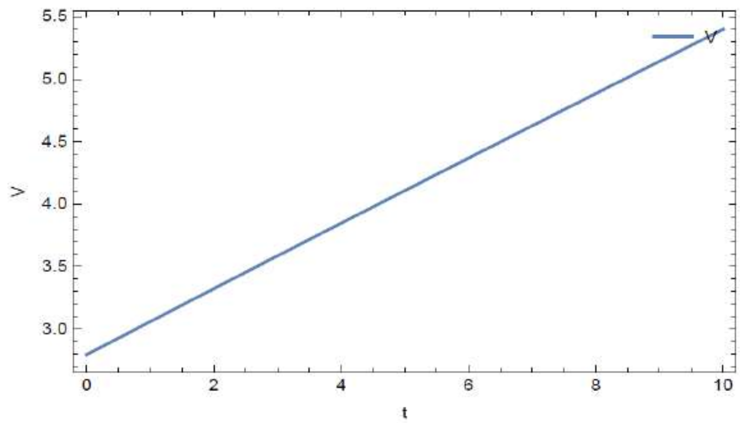

Figure 17. The variation of Spatial volume V is shown against t. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

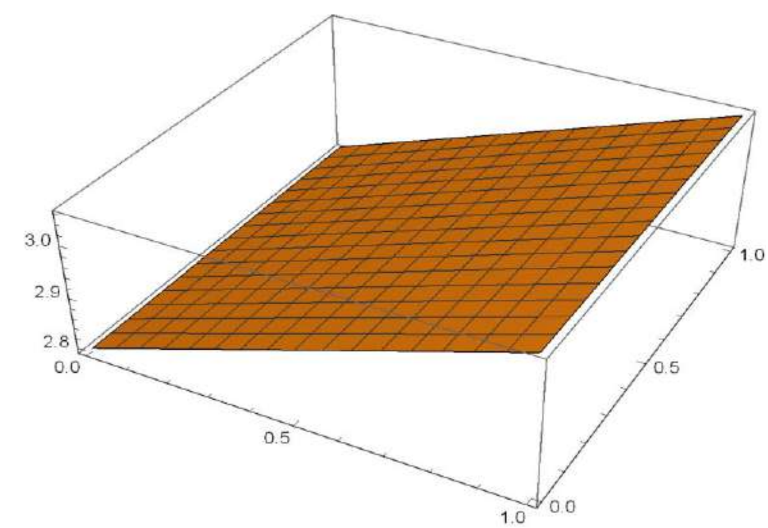

Figure 18. The variation of Spatial volume V is shown against t in 3-Dimensional plot. We use constants C4 = 0.1 and α = 0.05, k1 = 0.1, k2 = 1.

5. RESULTS AND CONCLUSIONS

The spatial volume V becomes 0 when the time t is equal to the negative ratio of k_2 and k_1, and the expansion scalar becomes infinite at the same time. It demonstrates that the Universe begins its development with a volume of zero and experiences an expansion with an infinite rate. The scale factor R is 0 at $t = -k_2/k_1$, indicating that the space-time experiences a singularity of point type during the initial age. When t approaches $-k_2/k_1$, both ρ and σ tend towards infinity. As time progresses, the scale factor R and spatial volume V increase, while the expansion scalar drops, indicating a deceleration in the pace of expansion. As time approaches infinity, the values of R, V, and Λ all tend to infinity, while the values of ρ, ρ_c and ρ_v all tend to zero. Thus, the model predicts that the cosmos will become empty as time approaches infinity. This outcome aligns with observations gathered by other astronomers [12, 28, 30, 36]. The behaviour of physical quarantines is shown in the figures (1)-(18) in 2D and 3D- dimensional space. To summarize, we have examined the Bianchi type-I cosmology model that includes a stiff fluid and a cosmological factor represented by $\Lambda = \alpha H^2$. The deceleration parameter q for the model is determined to be 2 at α, it is 0 at α = 2, and it declines with increasing α. The cosmological term Λ, initially of significant magnitude, subsequently transitions to a bona fide cosmological constant, aligning with recent discoveries [34-38]. The model approaches the de-Sitter universe as a limit.

ACKNOWLEDGMENT

PKD would like to thank the COST Action CA18108 "Quantum gravity phenomenology in the multi-messenger approach" for providing financial support to visit the University of Rijeka, Croatia and also PKD acknowledges IUCAA, Pune, India for giving him a visiting associateship position. KS would like to express her gratitude to Institute of Physics, UK for awarding B F Funds support.

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