Stochastic Modelling and Computational Sciences

STUDY OF REPAIR RATES IN RELIABILITY MODELS WITH DIFFERENT INSTRUCTIONS

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ABSTRACT

In this paper, we have studied about a two-unit cold standby system which is in repair state before and after instruction. It is assumed that every repairman can repair the failed unit with some perfection and in case if unit is not repaired, then another expert repairman is called to repair the unit. There may be some cases in real life when the repairman may not completely repair the failed unit then there may be some instruction from the expert's side to repair the failed unit. In this paper, we have studied the same by using the mathematical concepts and found MTSF, Availability, Busy period of the repairman, Busy period of the expert repairman (instruction time analysis), Expected number of visits by the expert repairman, Cost benefit analysis.

Keywords: MTSF, Availability analysis, Repaired unit, repairman, expert repairman, instruction.

1. INTRODUCTION

In the study of reliability, a lot of work has been done on different types of one or two-unit standby system as per their use in the modern industry and business system. The standby unit can be explained in two ways: warm standby unit and cold standby unit. Cold standby means that the redundant units cannot fail while they are waiting, and the warm standby means that the inactive component can fail at the standby state. An example of a cold standby unit is a Stephaney and an example of a warm standby is invertor. Many of the researchers have worked on various concepts like repair time, operating time and rest period, availability, two types of repair facilities, regenerative point technique, failure due to error, repair before and after instruction, etc.

Gupta S. K. (2016) examined two distinct cold standby systems based on the requirement for instructions from an assistant repairman in order to resolve a malfunctioning piece of equipment. They examined two models with distinct unit cold standby systems that included training time, replacement, and preventative maintenance, assuming that the backup unit is poor and is replaced upon failure. The twenty-first century is a technological century when everyone gets affected, including businesses like manufacturing that rely entirely on machines to do their tasks. The current major task facing scientists and engineers is to create extremely dependable goods at the lowest possible cost. The fastest-growing sectors must thus use extremely dependable systems. The current article of Goyal and Malhotra (2017) aims to investigate the MTSF, availability, Profits function, and reliability with regard to various system factors, such as failure rate, repair rate, and service rate. Additionally, a tabular and graphical analysis was conducted to emphasize the significant findings. Gupta (2018) considered the duration of the instruction and the potential for a regular repairman to harm the device to the point where it becomes more damaged or possibly irreparable. She used the semi-Markov process and the regenerating point technique to analyse a two-unit cold standby and generate many system efficacy metrics. In repairable systems with dependent failures, a study by Ferreira et al. (2022) focuses on a unique repair assumption known as perfect restoration, where just the failing component is restored to as good as new state. To confirm the model's appropriateness as well as the asymptotic consistency and efficiency features of the maximum likelihood estimators, a comprehensive simulation analysis is carried out under various conditions. The suggested technique can be especially helpful for sectors of the economy that employ repairable systems that need to replace parts as they break and are susceptible to non-quantifiable variables that may affect how quickly these parts fail. Using two real data sets, we demonstrate the proposed model's practical significance. In the first, nine sugarcane harvesters are examined for a predetermined amount of time; throughout this time, the harvesters' cutting blades repeatedly failed. The other concerns a group of five dump trucks, each of which diesel engines experienced intermittent problems during the observation time. Under the power-law process model, which has received a lot of attention in industrial applications, parametric inference is performed.

2. DESCRIPTION OF MODEL AND ASSUMPTIONS:

i. System is made up of two identical units. One unit is operating and the other is kept as cold standby.

ii. If there is a failure in one unit, the standby unit will be in operation automatically and the failed unit will go under repair.

iii. In case of failure of both the units, the system will be in failed state.

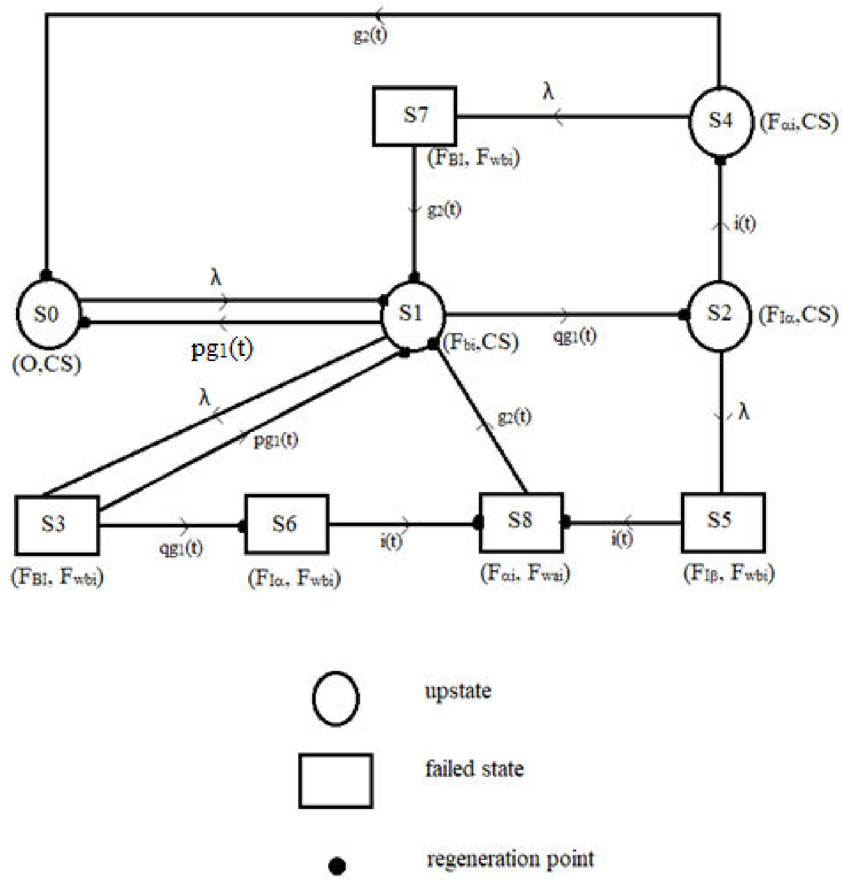
iv. After the repair, the failed unit will behave like new one.

v. The time to failure for each unit is in exponential distribution and the repair time and instruction time are in arbitrary distribution.

vi. All the random variables are mutually independent.

3. NOMENCLATURE

p	Probability that the repairman repairs system without instructions
q	Probability that the repairman fails to repair system without instructions
λ	Constant failure rate of the operative unit
O	Operative unit
CS	Cold standby
$g_1(t)$	p.d.f. of time to repair by repairman before instructions are given
$g_2(t)$	p.d.f. of time to repair by repairman after instructions are given
$G_1(t)$	c.d.f. of time to repair by repairman before instructions are given
$G_2(t)$	c.d.f. of time to repair by repairman after instructions are given
$i(t)$	p.d.f. when expert gives instructions to the repairman
$I(t)$	c.d.f. when expert gives instructions to the repairman
F_{bi}	Failed unit under the repair of repairman before getting instructions
F_{ai}	Failed unit under the repair of repairman after getting instructions
F_{BI}	Repair by the repairman is continuing from the previous state before getting instructions
F_{AI}	Repair by the repairman is continuing from the previous state after getting instructions
$F_{I\alpha}$	Expert is giving instructions to the repairman
$F_{I\beta}$	Instructions by the expert are continuing from the previous state
F_{wbi}	Failed unit waiting for repair before getting instructions
F_{wai}	Failed unit waiting for repair after getting instructions



4. Transition probabilities

The transition probabilities are:

$$dQ_{01}(t) = \lambda e^{-\lambda t} dt$$
$$dQ_{10}(t) = p g_1(t) e^{-\lambda t} dt$$
$$dQ_{12}(t) = q g_1(t) e^{-\lambda t} dt$$
$$dQ_{13}(t) = \lambda e^{-\lambda t} \overline{G_1(t)}\, dt$$
$$dQ_{16}^{(3)}(t) = \left(\lambda e^{-\lambda t} © q\right) g_1(t)\, dt = q\left(1 - e^{-\lambda t}\right) g_1(t)\, dt$$
$$dQ_{11}^{(3)}(t) = \left(\lambda e^{-\lambda t} © p\right) g_1(t)\, dt = p\left(1 - e^{-\lambda t}\right) g_1(t)\, dt$$
$$dQ_{24}(t) = i(t) e^{-\lambda t} dt$$
$$dQ_{28}^{(5)}(t) = \left(\lambda e^{-\lambda t} © 1\right) i(t)\, dt = \left(1 - e^{-\lambda t}\right) i(t)\, dt$$
$$dQ_{25}(t) = \lambda e^{-\lambda t} \overline{I(t)}\, \mathrm{dt}$$
$$dQ_{40}(t) = g_2(t) e^{-\lambda t} dt$$
$$dQ_{41}^{(7)}(t) = \left(\lambda e^{-\lambda t} © 1\right) g_2(t)\, dt = \left(1 - e^{-\lambda t}\right) g_2(t)\, dt$$
$$dQ_{47}(t) = \lambda e^{-\lambda t} \overline{G_2(t)}\, dt$$
$$dQ_{68}(t) = i(t)\, dt$$
$$dQ_{81}(t) = g_2(t)\, dt \qquad \text{..........(1-14)}$$

The non-zero elements p_{ij} are as follows:

$p_{01} = 1, p_{10} = p g_1^*(\lambda)$, $p_{12} = q g_1^*(\lambda)$, $p_{13} = 1 - g_1^*(\lambda)$, $p_{16}^{(3)} = q(1 - g_1^*(\lambda))$, $p_{11}^{(3)} = p(1 - g_1^*(\lambda))$, $p_{24} = i^*(\lambda)$, $p_{28}^{(5)} = 1 - i^*(\lambda)$, $p_{25} = 1 - i^*(\lambda)$, $p_{40} = g_2^*(\lambda)$, $p_{41}^{(7)} = 1 - g_2^*(\lambda), p_{47} = 1 - g_2^*(\lambda)$, $p_{68} = p_{81} = 1$

From the transition probabilities, it can be verified that

$$p_{01} = 1$$
$$p_{10} + p_{12} + p_{13} = p_{10} + p_{12} + p_{16}^{(3)} + p_{11}^{(3)} = 1$$
$$p_{24} + p_{25} = p_{24} + p_{28}^{(5)} = 1$$
$$p_{40} + p_{47} = p_{40} + p_{41}^{(7)} = 1$$
$$p_{68} = p_{81} = 1 \qquad \text{......(15-19)}$$

5. Mean Sojourn Time

If T denotes mean sojourn time in state 0, then

$$\mu_0 = \int P(T > t)dt = \frac{1}{\lambda}, \quad \mu_1 = \frac{1-g_1^*(\lambda)}{\lambda}, \quad \mu_2 = \frac{1-i^*(\lambda)}{\lambda}, \quad \mu_4 = \frac{1-g_2^*(\lambda)}{\lambda},$$
$$\mu_6 = \int_0^\infty \overline{I(t)}dt = \int_0^\infty t dI(t) = \text{Mean instruction time}, \ \mu_8 = \int_0^\infty \overline{G_2(t)}dt \qquad \text{.........(20)}$$

The unconditional mean time taken by the system to transit to any regenerative state i when it is counted from the epoch of entrance into that state is, mathematically, stated as

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -\frac{d}{ds} q_{ij}^*|_{s=0} \qquad \text{.........(21)}$$

Thus, $m_{01} = \mu_0$, $m_{10} + m_{12} + m_{13} = \mu_1$, $m_{24} + m_{25} = m_{24} + m_{28}^{(5)} = \mu_2$, $m_{40} + m_{47} = m_{40} + m_{41}^{(7)} = \mu_4, m_{10} + m_{12} + m_{11}^{(3)} + m_{16}^{(3)} = k_1$ ....(22)

6. Mean time to system failure

To determine the MTSF of the system, we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t)$$
$$\phi_1(t) = Q_{13}(t) + Q_{12}(t) Ⓢ \phi_2(t) + Q_{10}(t) Ⓢ \phi_0(t)$$
$$\phi_2(t) = Q_{25}(t) + Q_{24}(t) Ⓢ \phi_4(t)$$
$$\phi_4(t) = Q_{47}(t) + Q_{40}(t) Ⓢ \phi_0(t) \qquad \text{......(23-26)}$$

Taking the L. S. T. of the equation $(23 - 26)$ and solving them for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{12}^{**}(s)Q_{25}^{**}(s) + Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{47}^{**}(s)]}{1 - Q_{01}^{**}(s)[Q_{10}^{**}(s) + Q_{12}^{**}(s)Q_{24}^{**}(s)Q_{40}^{**}(s)]} \qquad \text{.........(27)}$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = lim_{s \to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D} \qquad \text{.........(28)}$$

where $N = \mu_{0+}\mu_1 + P_{12}\mu_2 - P_{12}p_{24} + P_{12}p_{24}\mu_4$ ....(29)

and $D = p_{01} - p_{10} - p_{12}p_{24}p_{40}$ ....(30)

7. Availability Analysis

$M_i(t)$ denotes the probability that the system starting in up regenerative state is up at time t without passing through any regenerative state.

Thus, we have $M_0(t) = e^{-\lambda t}$, $M_1(t) = e^{-\lambda t}\overline{G_1(t)}$, $M_2(t) = e^{-\lambda t}\overline{I(t)}$, $M_4(t) = e^{-\lambda t}\overline{G_2(t)}$ ....(31)

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t) + q_{11}^{(3)}(t)©A_1(t) + q_{16}^{(3)}(t)©A_6(t)$$
$$A_2(t) = M_2(t) + q_{24}(t)©A_4(t) + q_{28}^{(5)}(t)©A_8(t)$$
$$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{41}^{(7)}(t)©A_1(t)$$
$$A_6(t) = q_{68}(t)©A_8(t)$$
$$A_8(t) = q_{81}(t)©A_1(t) \quad ......(32\text{-}37)$$

Taking Laplace transform of equation (31) and solving for $s \to 0$, we get

$$M_0^*(0) = \mu_0\,,\ M_1^*(0) = \mu_1\,, M_2^*(0) = \mu_2\,,\ M_4^*(0) = \mu_4 \quad ....(38)$$

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \quad ....(39)$$

where $N_1 = \mu_0[1 - p_{12}p_{28}^{(5)} - p_{16}^{(3)} - p_{11}^{(3)} - p_{12}p_{24}p_{41}^{(7)}] + \mu_1 + \mu_2 p_{12} + \mu_4 p_{12}p_{24}$ ......(40)

and $D_1 = \mu_0[p_{10} + p_{12}p_{24}p_{40}] + k_1 + \mu_2 p_{12} + \mu_4 p_{12}p_{24} + \mu_8[p_{12}p_{28}^{(5)} + 2p_{16}^{(3)}]$ ......(41)

8. Busy period analysis

Busy period of the repairman:

$$B_0^r(t) = q_{01}(t)©B_1^r(t)$$
$$B_1^r(t) = W_1(t) + q_{10}(t)©B_0^r(t) + q_{12}(t)©B_2^r(t) + q_{11}^{(3)}(t)©B_1^r(t) + q_{16}^{(3)}(t)©B_6^r(t)$$
$$B_2^r(t) = q_{24}(t)©B_4^r(t) + q_{28}^{(5)}(t)©B_8^r(t)$$
$$B_4^r(t) = W_4(t) + q_{40}(t)©B_0^r(t) + q_{41}^{(7)}(t)©B_1^r(t)$$
$$B_6^r(t) = q_{68}(t)©B_8^r(t)$$
$$B_8^r(t) = W_8(t) + q_{81}(t)©B_1^r(t) \quad ......(42\text{-}47)$$

where $W_1(t) = e^{-\lambda t}\overline{G_1(t)} + [\lambda e^{-\lambda t}©1]\,\overline{G_1(t)}$, $W_4(t) = e^{-\lambda t}\overline{G_2(t)} + [\lambda e^{-\lambda t}©1]\,\overline{G_2(t)}$, $W_8(t) = \overline{G_2(t)}$ ....(48)

Taking Laplace transform of equation (48) and solving for $s \to 0$, we get

$$W_1^*(0) = k_1\,,\ W_4^*(0) = \mu_4 = W_8^*(0) \quad ....(49)$$

In the steady-state, the total fraction of time for which repairman is busy is

$$B_0^r = \lim_{s\to 0} s\,B_0^{r^*} = \frac{N_2}{D_1} \quad ....(50)$$

Where $N_2 = \mu_4[p_{12}p_{28}^{(5)} + p_{16}^{(3)} + p_{12}p_{24}] + k_1$ ....(51)

And D_1 is same as in equation (41).

Busy period analysis for expert repairman/ Instruction time analysis

$$B_0^i(t) = q_{01}(t)©B_1^i(t)$$
$$B_1^i(t) = q_{10}(t)©B_0^i(t) + q_{12}(t)©B_2^i(t) + q_{11}^{(3)}(t)©B_1^i(t) + q_{16}^{(3)}(t)©B_6^i(t)$$
$$B_2^i(t) = W_2(t) + q_{24}(t)©B_4^i(t) + q_{28}^{(5)}(t)©B_8^i(t)$$
$$B_4^i(t) = q_{40}(t)©B_0^i(t) + q_{41}^{(7)}(t)©B_1^i(t)$$
$$B_6^i(t) = W_6(t) + q_{68}(t)©B_8^i(t)$$
$$B_8^i(t) = q_{81}(t)©B_1^i(t) \quad ......(52\text{-}57)$$

Where $W_2(t) = e^{-\lambda t}\overline{I(t)} + [\lambda e^{-\lambda t}©1]\,\overline{I(t)}$, $W_6(t) = \overline{I(t)}$ ....(58)

Taking Laplace transform of equation (58) and solving for $s \to 0$, we get

$W_2^*(0) = \mu_2 = W_6^*(0)$ ....(59)

In steady state, the total fraction of time for which the expert repairman is busy in giving instructions is

$B_0^i = lim_{s\to 0}\, s\, B_0^{i^*} = \frac{N_3}{D_1}$ ....(60)

where $N_3 = \mu_2[p_{12} + p_{16}^{(3)}]$ ....(61)

and D_1 is same as in equation (41).

9. Expected Number of Visits by Expert Repairman

$$V_0(t) = q_{01}(t) Ⓢ V_1(t)$$
$$V_1(t) = q_{10}(t) Ⓢ V_0(t) + q_{12}(t) Ⓢ [1 + V_2(t)] + q_{16}^{(3)}(t) Ⓢ [1 + V_6(t)] + q_{11}^{(3)}(t) Ⓢ V_1(t)$$
$$V_2(t) = q_{24}(t) Ⓢ V_4(t) + q_{28}^{(5)}(t) Ⓢ V_8(t)$$
$$V_4(t) = q_{40}(t) Ⓢ V_0(t) + q_{41}^{(7)}(t) Ⓢ V_1(t)$$
$$V_6(t) = q_{68}(t) Ⓢ V_8(t)$$
$$V_8(t) = q_{81}(t) Ⓢ V_1(t)$$ ......(62-67)

In steady-state, the number of visits per unit time is given by taking $s \to 0$ and $t \to \infty$

$V_0 = \lim_{t\to\infty} \frac{V_0(t)}{t} = \lim_{s\to\infty} [sV_0^{**}(s)] = \frac{N_4}{D_1}$ ....(68)

Where $N_4 = p_{12} + p_{16}^{(3)}$ ....(69)

And D_1 is same as in equation (41).

10. Cost-Benefit analysis

The expected cost-profit of the system in steady state is given by

$P_1 = C_0A_0 - C_1B_0^r - C_2B_0^i - C_3V_0$ ....(70)

where C_0 is revenue per unit up-time of the system.

C_1 is cost per unit time for which the assistant repairman is busy.

C_2 is cost per unit for which expert repairman is busy.

C_3 is cost per visit of expert repairman.

11. Comparison Analysis

MTSF vs. Failure Rate of the Main Unit with Initial State S_0

The MTSF has been determined by taking different values of the failure rate (λ) of the operative unit as shown in Table 1.1 and the graphs corresponding to these cases have been shown in Figure 1. This has been done by taking specific values of the probability of the system that needs repair (p_{12}) & the probability that it needs replacement (p_{24}) for the different cases (i) $p_{12} > p_{24}$ $(p_{12} = 0.75;\ p_{24} = 0.25)$ (ii) $p_{12} = p_{24}$ $(p_{12} = 0.50;\ p_{24} = 0.50)$

(iii) $p_{12} < p_{24}$ $(p_{12} = 0.25; p_{24} = 0.75)$.

For any fixed value of p_{12}/p_{24}, Table 1.1 and Figure 1 shows that the system's MTSF (T_0) rapidly reduces when the operating unit's failure rate (λ) increases. The percentage decrease in MTSF has been reported to range from 75% to 18% approximately when λ changes between 0.0015 and 0.0095. Notably, this % decrease in MTSF is nearly identical in all three examples. In contrast, at a given value of the operating unit's failure rate (λ), MTSF falls with the likelihood of system repair decreasing or the likelihood of replacement increasing. The value of MTSF reduces by 26.34% $for\ \lambda = 0.0095$ and by 28.22% for $\lambda = 0.001$ when p_{12} drops from 0.75 to 0.25.

TABLE - 1.1

MTSF T_0 vs. Failure Rate λ				
Sr. No.	λ	T_0 $p_{12} = 0.75$; $p_{24} = 0.25$	T_0 $p_{12} = 0.50$; $p_{24} = 0.50$	T_0 $p_{12} = 0.25$; $p_{24} = 0.75$
1	0.0125	210728.5110	176109.0422	151304.4172
2	0.0135	53306.5852	44629.5031	38409.5578
3	0.0145	23997.4309	20129.9914	17359.0735
4	0.0155	13686.8713	11504.1047	9941.3078
5	0.0165	8880.6877	7488.2028	6485.3903
6	0.0175	6271.4460	5294.1316	4595.9084
7	0.0185	4685.9777	3963.5988	3449.2866
8	0.0195	3651.2690	3095.1237	2700.3349
9	0.0205	2968.4382	2496.3410	2183.6086

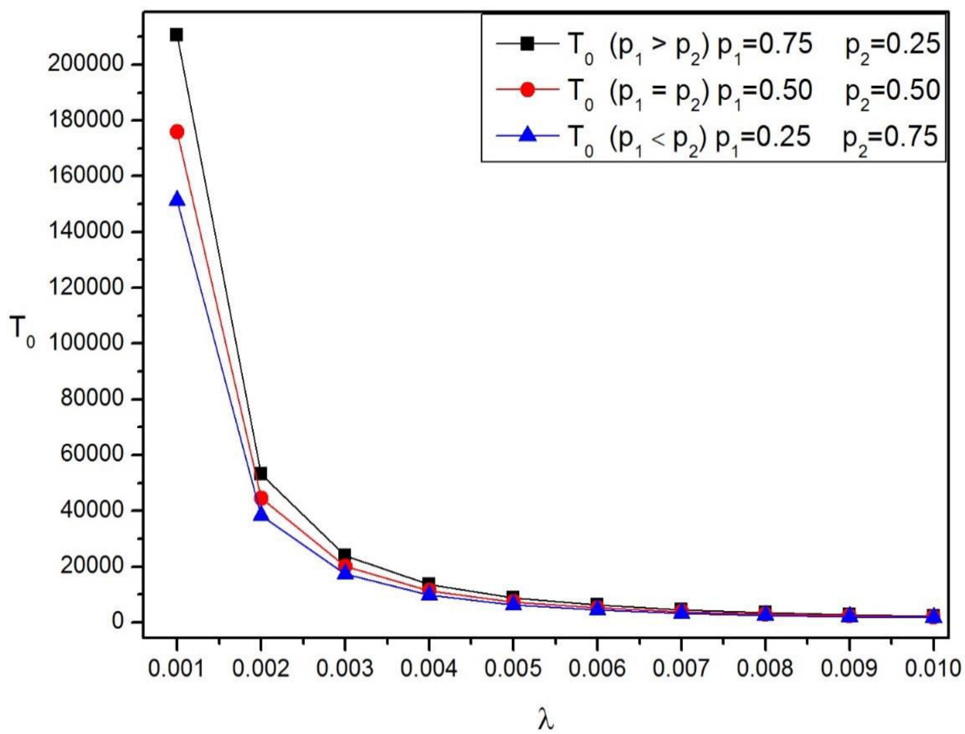


Figure 1 (MTSF T_0 vs. Failure Rate of the system)

MTSF vs. Failure Rate of the Main Unit with Initial State S_1

Examining the various scenarios of probability of repair (p_{12}) and replacement (p_{24}) that the system requires, the MTSF has been assessed for the various operative unit failure rate values (λ) as indicated in Table 1.2. The corresponding graphs for these scenarios are displayed in Figure 2.

Table 1.2 and Figure 2 demonstrate that, for any fixed value of p_{12}/p_{24}, the system's MTSF (T_1) rapidly drops when the operating unit's failure rate (λ) increases. It has been noticed that the percentage decrease in MTSF ranges from around 74.85% to 18.51% when λ fluctuates between 0.0015 and 0.0095, and in all three situations, this percentage decrease in T_1 is nearly identical. Conversely, for a given value of λ, T_1 reduces as p_{12} falls or p_{24} increases. The value of T_1 reduces by 27.54% for $\lambda = 0.0095$ and by 28.35% for $\lambda = 0.0015$ when p_{12} drops from 0.75 to 0.25. This indicates that the fluctuations in MTSF (T_1) are less for a given variation in p_{12} at greater failure rates (λ).

TABLE- 1.2

MTSF T_1 vs. Failure Rate λ				
Sr. No.	λ	T_1 $p_{12} = 0.75$; $p_{24} = 0.25$	T_1 $p_{12} = 0.50$; $p_{24} = 0.50$	T_1 $p_{12} = 0.25$; $p_{24} = 0.75$
1	0.0125	210019.1672	175377.5735	150548.1828
2	0.0135	52997.3391	44303.6750	38083.8391
3	0.0145	23821.5442	19953.0500	17074.1887
4	0.0155	13577.6652	11396.6037	9727.8420
5	0.0165	8721.4904	7322.3679	6314.7746
6	0.0175	6029.9206	5156.0730	4453.8601
7	0.0185	4562.5041	3845.3811	3427.6435
8	0.0195	3542.0839	2991.7866	2593.9958
9	0.0205	2840.3669	2404.5774	2089.1727

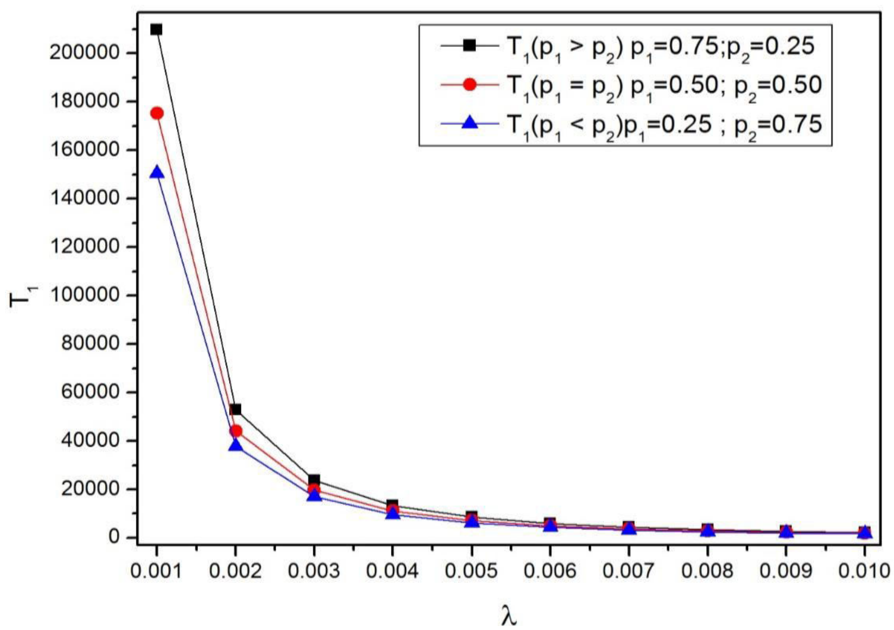


Figure 2 (MTSF T_1 vs. Failure Rate λ of the Main Unit)

12. CONCLUSION

The MTSF and availability of the two-unit cold standby system, have been obtained easily and quickly by using the path analysis. It has been verified that MTSF being a positional measure, it depends upon the initial state. Whereas the steady state availability of the system being the global measure, has been found to be the same, although determined separately, by assuming the state S_0 and the base state S_1 as the initial states.

It has further been observed from the analysis of the system that in general all the MTSF of the system w.r.t. S_0, S_1 (as the initial states), decrease rapidly with the increase in failure rate (λ) of operating unit, for any fixed value of the probability of minor or major failures and the inspection, replacement and repair rates.

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