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STEADY FLOW OF BLOOD WITH PERIODIC BODY ACCELERATION AND MAGNETIC FIELD THROUGH POROUS MEDIUM IN AN INCLINED VESSEL OF SMALL EXPONENTIAL DIVERGENCE

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ABSTRACT

The current study is concerned with a mathematical model for steady flow of blood through a porous medium in an inclined vessel of small exponential divergence under the influence of periodic body acceleration and magnetic field by considering blood as a couple stress fluid. Integral transform techniques have applied for the analytical computation of the physiological parameters that have an impact on the human body, such as axial velocity, flow ratesand shear stresshave been computed as an exact solution in the form of Bessel's Fourier series. It is interesting to see that blood velocity decreases with increasing magnetic field, while blood velocity increases with increasing permeability of the porous media and body acceleration. Graphs are used to illustrate the influence of these parameters. Lastly, this mathematical model's applicationshave been indicated.

Keywords: Couple stress fluid, Porous Medium, Body Acceleration, Magnetic Field, Exponentially Diverging inclined vessel.

1. INTRODUCTION

The human body experiences increase in speed on a regular basis. In general, the human body is exposed to vibrations, much like when traveling in cars, planes, or other transportation, running, operating machinery, drilling, or operating a property hauler, opponents and rivals for their unexpected changes of events. The human body is highly adaptable, but a delayed body's response to these vibrations can cause a number of health problems, including headaches, stomachaches, loss of vision, and elevated heart rate since it can disturb the circulatory system.The electromagnetic theory was introduced in the field of clinical assessment by Kollin [1] in an eccentric manner. According to research presented by Barnothy [2], subjecting biological systems to an external magnetic field lowers their heart rates. In addition to providing data on blood flow, the ECG pattern recorded in the presence of a magnetic field presents a novel noninvasive technique for examining heart function. According to Ramachandra Rao and Deshikachar [3], theoretical research on the impact of an applied magnetic field on blood flow has not gotten much attention up to this point.As a Newtonian fluid, Chaturani and Palanisamy [4] examined the pulsatile flow of blood in arigid circular tube affected by body acceleration. El-Shehawey et al.'s study [5] concentrated on the erratic flow of blood in a circular pipe in the presence of a magnetic field as an electrically conducting, incompressible, and elastico-viscous fluid. The general equation of motion for the flow of a viscous fluid through a porous material was examined by Ahmadi and Manvi [6].In actuality, the fluid-containing permeable substance is a non-homogenous mediumit can be substituted, for analytical purposes, with a homogeneous fluid whose dynamical characteristics are equal to the local averages of the initial non-homogenous medium. According to Dash et al. [7], the distribution of fatty cholesterol and artery-clogging blood clots in the lumen of the coronary artery in some clinical circumstances can be compared to a hypothetical porous medium.Tripathi[8] investigated the blood flow mathematical model in the presence of an inclined magnetic field in an artery that is inclined.

2. Formulation of the Problem:

Let us consider a one-dimensional steady flow of blood through a porous medium in a straight rigid and non-conducting exponentially diverging inclined tube with impermeable walls by considering blood as couple stress, non-Newtonian, incompressible and electrically conducting fluid in the presence of magnetic field. The flow is considered as axially symmetric, steady and fully developed through a uniform; rigid circular tube with impermeable walls that diverge exponentially is given by the relation:

$$R(z)=\frac{R_0}{2}\left(1+e^{\varepsilon z/z_0}\right) \qquad (1)$$

where ε is the divergence parameter over a characteristic axial distance z_0 and is considered to be very less than one and R_0 is the tube radius.The pressure gradient and body acceleration G are given by:

$$-\frac{\partial p}{\partial z}=A_0+A_1 \qquad (2)$$

$$G=a_0\cos(\phi) \qquad (3)$$

where A_0 is the steady – state part of the pressure gradient, A_1 is the amplitude of the oscillatory part, a_0 is the amplitude of body acceleration, ϕ is its phase difference, z is the axial distance. Based on Stokes [9] model, a one-dimensional steady flow through a uniform, straight, rigid and non-conducting tube in the presence of transverse magnetic field in an inclined exponentially diverging vessel has been formulated as

$$\begin{aligned}&\eta\nabla^2(\nabla^2 u)-\mu\nabla^2 u+\sigma B_0^2 u=-\frac{\partial p}{\partial z}+\rho G-\frac{\mu}{K}u+\rho g\cos\theta\\&where\quad \nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\end{aligned} \qquad (4)$$

where u(r) is velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field, K is the permeability of the isotropic porous medium K and r is the radial coordinate.

Let us introduce the following dimensionless quantities:

$$u^*=\frac{u}{\omega R},\ r^*=\frac{r}{R},\ A_0^*=\frac{R}{\mu\omega}A_0,\ A_1^*=\frac{R}{\mu\omega}A_1,\ a_0^*=\frac{\rho R}{\mu\omega}a_0,\ z^*=\frac{z}{R},\ g^*=\frac{\rho R}{\mu\omega}g,\ K^*=\frac{K}{R^2} \qquad (5)$$

In terms of these variables, equation (4) [if dropping the stars] becomes

$$\nabla^2\left(\nabla^2 u\right)-\bar{\alpha}^2\,\nabla^2 u-\bar{\alpha}^2(A_0+A_1+a_0\cos\phi+g\cos\theta)+\bar{\alpha}^2(H^2+h^2)u=0 \qquad (6)$$

where $\bar{\alpha}^2=\dfrac{R^2\mu}{\eta}$ - Couple stress prameter , $H=B_0 R\left(\dfrac{\sigma}{\mu}\right)^{\frac{1}{2}}$ is the Hartmann number, $h=\sqrt{1/K}$ and R is the radius of the pipe.

The boundary conditions for this problem are:

$$\begin{aligned}&u\ \ and\ \ \nabla^2 u\ \ \ are\ \ all\ \ finite\ at\ \ r=0\\&u=0,\quad \nabla^2 u=0 \qquad at\quad r=r_1\end{aligned} \qquad (7)$$

where $\quad r_1=\dfrac{R(z)}{R_0}=\dfrac{1}{2}\left(1+e^{\varepsilon z/z_0}\right)$, non - dimensional divergence parameter

3. Required Integral Transform: The finite Hankel transform as per Tranter [10] is defined to be

$$u^*(\lambda_n) = \int_0^{r_1} r\, u J_0(r\lambda_n)\, dr \ , \qquad (8)$$

wherethe λ_n are the roots of the equation $J_0(r) = 0.$ Then at each point of the interval at which $f(r)$ is continous:

$$u(r) = \frac{2}{r_1^2}\sum_{n=1}^{\infty} u^*(\lambda_n)\frac{J_0(r\lambda_n)}{J_1^2(r_1\lambda_n)} \qquad (9)$$

where the sum is taken over all positive roots of $J_0(r) = 0,\ J_0$ *and* J_1 are Bessel function of the first kind.

Now applying finite Hankel transform (8) to eqn. (6) in the light of (7) we obtain:

$$u^* = \frac{J_1(\lambda_n)\bar{\alpha}^2}{\lambda_n}\ \frac{\left(A_0 + A_1 + a_0\cos\varphi + g\cos\theta\right)}{\left(\lambda_n^4 + \bar{\alpha}^2\left(\lambda_n^2 + H^2 + h^2\right)\right)} \qquad (10)$$

Now the finite Hankel inversion of (10) gives the final solution as:

$$u(r) = 2\sum_{n=1}^{\infty}\frac{J_0(r\lambda_n)\bar{\alpha}^2}{r_1\lambda_n J_1(r_1\lambda_n)}.\frac{[A_0 + A_1 + a_0\cos\varphi + g\cos\theta]}{[\lambda_n^4 + \bar{\alpha}^2\ (\lambda_n^2 + H^2 + h^2)]} \qquad (11)$$

The expression for the flow rate can be written as:

$$Q = 2\pi\int_0^{r_1} r\, u\, dr \qquad (12)$$

then

$$Q(r) = 4\pi\sum\frac{\bar{\alpha}^2}{\lambda_n^2}.\frac{[A_0 + A_1 + a_0\cos\varphi + g\cos\theta]}{[\lambda_n^4 + \bar{\alpha}^2\,(\lambda_n^2 + H^2 + h^2)]} \qquad (13)$$

Similarly the expression for the shear stress can be obtained from

$$\tau_w = \mu\frac{\partial u}{\partial r} \qquad (14)$$

Then

$$\tau_w = -2\mu\sum_{n=1}^{\infty}\frac{J_0(r\lambda_n)\bar{\alpha}^2}{\lambda_n J_1^2(r_1\lambda_n)}.\frac{[A_0 + A_1 + a_0\cos\varphi + g\cos\theta]}{[\lambda_n^4 + \bar{\alpha}^2\ (\lambda_n^2 + H^2 + h^2)]} \qquad (15)$$

4. RESULTS, DISCUSSIONS AND CONCLUSIONS

The velocityprofile for a steady blood flow computed by using (11) for different values of the divergence parameter ε and have been shown through Fig. 1, a slight decrease in the velocity profile is observed. When the Hartmann number (H) increases the velocity of the blood decreases shown in Fig 2. From Figure 3 it is observed that with the increase of the body acceleration (a_0),the velocity of the blood also increases. The velocity profile increases initially and then decreaseswith an increase in couple stress parameter $\bar{\alpha}$ which is shown in Fig. 4. As the permeability of the porous media K increases the velocity slightly decreases as shown in Fig. 5, the velocity profile decreases with an increase in the inclination angle θ as can be seen in Fig. 6. The wall shear stress decreases with an increase of Hartmann number H and a reverse flow is observed which is shown in Fig. 7. The wall shear stress increases with an increase of body acceleration (a_0) and permeability of the porous media K, reverse flow is observed which are shown through Figures 8 to 9.

The current model gives a most broad type of velocity expression from which the other numerical models can without much of a stretch be acquired by legitimate replacement.It is essential to observe that thevelocity expression (11) got for the current model provides different velocity expressions for different mathematical models given underneath:

•The velocity expression for steady blood flow with periodic body acceleration in the presence of magnetic field in an exponentially diverging vessel can be obtained by substituting$\theta = 90^0$ or g=0 in (11), reduces to the result of Shakera [11].

- The velocity expression for steady blood flow with periodic body acceleration in the absence of magnetic field can be obtained by substituting H=0 and $\theta = 90^0$ or g=0 in (11), reduces to the result of Shakera [12].
- The velocity expression for steady blood flow through a porous medium with periodic body acceleration and magnetic field can be obtained by substituting g=0 or $\theta = 90^0$ in (11), gives the result of Shakera [13].
- The velocity expression for steady blood flow with periodic body acceleration can be obtained by substituting h=0, H=0 and $\theta = 90^0$ or g=0 which is the result of Rathod et al [14]
- The velocity expression for steady blood flow with periodic body acceleration and magnetic field can be obtained by substituting h=0, and $\theta = 90^0$ or g=0 which is the result of Rathod et al [15]
- The velocity expression for steady blood flow with periodic body acceleration and magnetic field through porous media in an inclined tube can be obtained by substituting ε=0 which is the result of Shakera [16]

5. CONCLUSIONS

Thus, the present mathematical model gives an exact form ofvelocity expression for the blood flow and explanation for the circulation system so it will help not only people working in the field of physiological fluid mechanics nevertheless the clinical experts having simple data on Mathematics. Believing that this assessment could help for the further examinations in the field of clinical investigation, the use of magnetic field for the treatment of explicit cardiovascular sicknesses.

REFERENCES

[1]. Kollin, A.: Electromagnetic flowmeter: Principle of method and its Application to blood flow measurement. Proc.Soc.Exp.Biol.Med.**35**,(1936)33.

[2]. M. F. Barnothy (ed), Biological effects of magnetic fields, **1**&**2** Plenum Press, New York (1964,1969).

[3]. Ramachandra Rao A, Deshikachar. K. S.:Physiological type flow in a circular pipe in the presence of a transverse magnetic field. J. Ind.Ins. Sci.**68**, (1988)247-260.

[4]. Chaturani, P, Palanisamy V.: Pulsatile flow of blood with periodic body acceleration. Int. J. Engg. Sci. **29**(1), (1991)113-121.

[5]. El-Shehawey, E.F., Elsayed, M.E. Elbarbary, Afifi, N. A. S., Mostafa Elshahed.: MHD flow of an elastico-viscous fluid under periodic body acceleration.Int.J.Math. & Math. Sci. **23**(11), (2000)795-799.

[6]. G.Ahmadi, Manvi, Equation of motion for viscous flowthrough a rigid porous medium.IndianJ. Tech. 9(1971)441-444.

[7]. R. K. Dash, G. Jayaraman, K. N. Mehta, Casson fluid flow in a pipe filled with a homogenous porous medium, Int. J. Eng. Sci.34, (1996) 1145-1156.

[8]. Tripathi, D.: A mathematical model for blood flow through an inclined arteryunder the influence of an inclined magnetic field, Journal of Mechanics in Medicine and Biology, 12, (2012) 1–18.

[9]. Stokes, V.K.: Couple stresses in fluids. The Physics of Fluids **9,** (1966)1709-1715.

[10]. Tranter, C J. (1965) Integral Transforms in Mathematical Physics, Infinite Transforms. 3rd ed. London pp.88-89.

[11]. Shakera Tanveer, Steady blood flow through a porous medium with periodic body acceleration. Int. J. of Sci and Innovative Math. Research 3(5), (2015)135-142.

[12]. Shakera Tanveer, Effect of Porous Medium on the Steady Flow of Blood with Periodic Body acceleration and Magnetic Field in a Vessel of Small Exponential Divergence. Int. J. of All Res. Edu.and Sci. Methods (IJARESM), 8(8), (2020)429-433

[13]. Shakera Tanveer, Steady blood flow through a porous medium with periodic body accelerationand magnetic field. Comp. and Appl. Math. J. 2(1) (2016)1-5.

[14]. V. P. Rathod, ItagiSheebaRani, Shakera Tanveer, G. G. Rajput.: Steady blood flow with periodic body acceleration. ActaCienciaIndica.XXX M, No. 2(2004)405-410.

[15] V. P. Rathod, Shakera Tanveer, ItagiSheebaRani, G. G. Rajput.: Steady blood flow with periodic body acceleration and magnetic field. ActaCienciaIndica. XXXI M, No.1 (2005)41-46

[16]. Shakera Tanveer, Steady Blood Flow through Porous Medium with Periodic Body Acceleration and Magnetic Field in An Inclined Tube. Int. J. of All Res. Edu. and Sci. Methods (IJARESM), 9(7), (2021)4126-4132

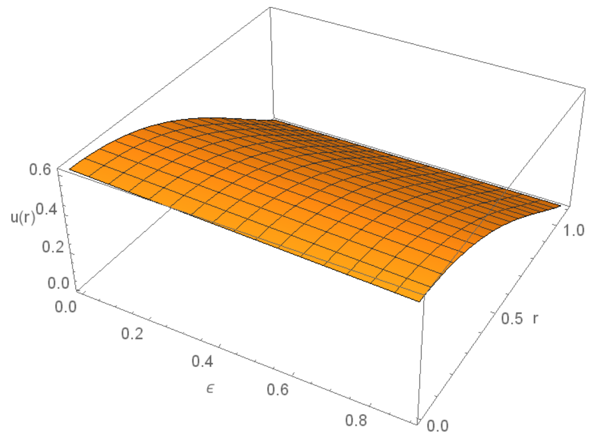

Fig. 1 Variation of velocity profile for different values of ε :

$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, a_0 = 3, K = 2.5, \theta = 30^0,$

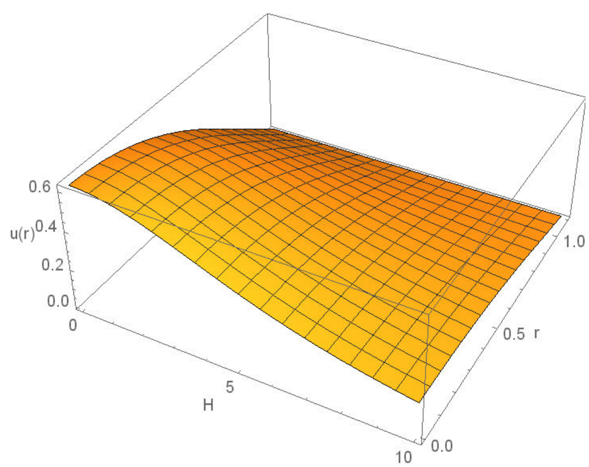

Fig. 2 Variation of velocity profile for different values of H:

$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, a_0 = 3, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, K = 2.5, \theta = 30^0,$

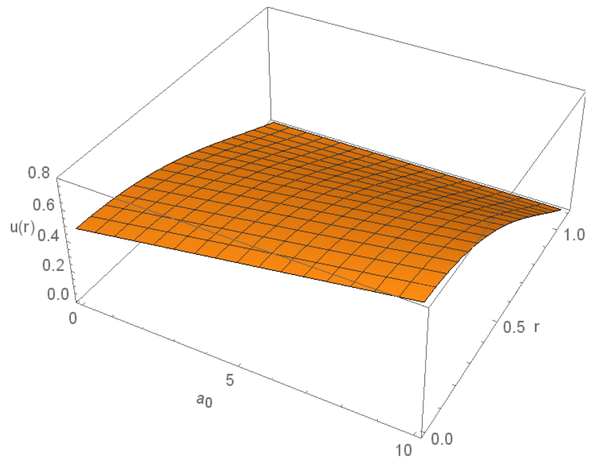

Fig. 3 Variation of velocity profile for different values of a_0:

$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, \mathrm{H} = 4, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, K = 2.5, \theta = 30^0,$

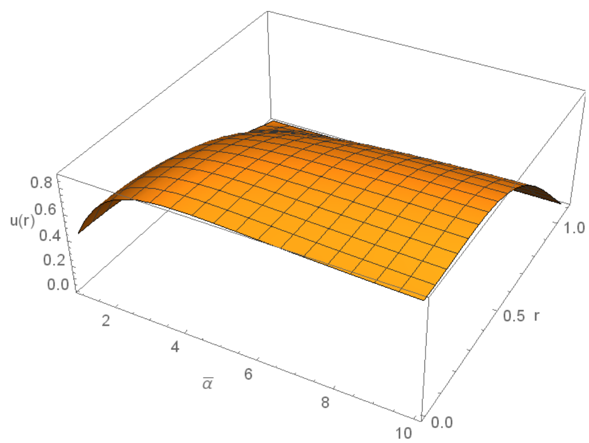

Fig. 4 Variation of velocity profile for different values of $\bar{\alpha}$:

$a_0 = 3, \phi = 15^0, A_0 = 2, A_1 = 4, \mathrm{H} = 4, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, K = 2.5, \theta = 30^0,$

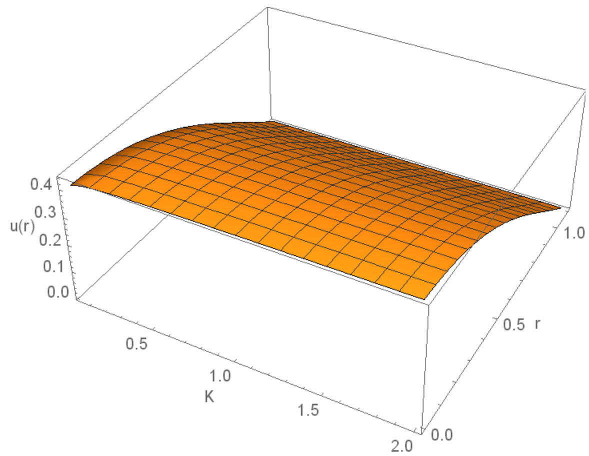

Fig. 5 Variation of velocity profile for different values of K:

$\bar{\alpha} = 1, a_0 = 3, \phi = 15^0, A_0 = 2, A_1 = 4, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, \mathrm{H} = 4, \theta = 30^0,$

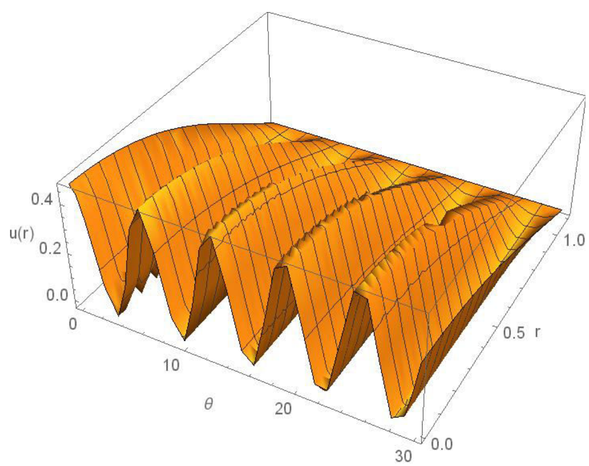

Fig. 6 Variation of velocity profile for different values of θ:

$\bar{\alpha} = 1, a_0 = 3, \phi = 15^0, A_0 = 2, A_1 = 4, \mathrm{H} = 4, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, K = 2.5,$

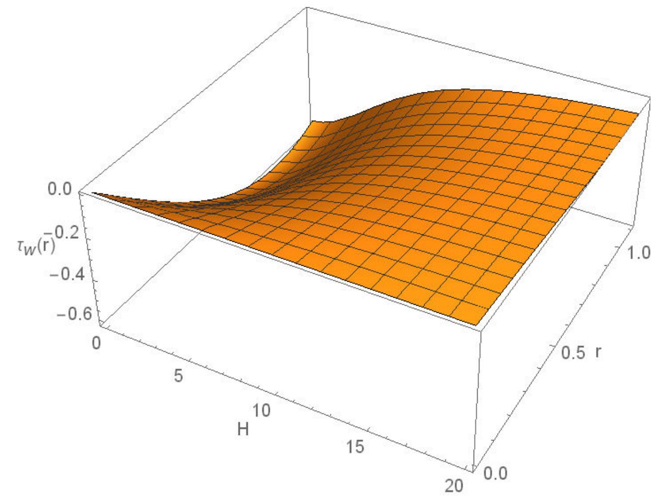

Fig. 7 Variation of wall shear stress with Hartmann number H:
$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, a_0 = 3, K = 2.5, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, \theta = 30^0, \mu = 1.5$

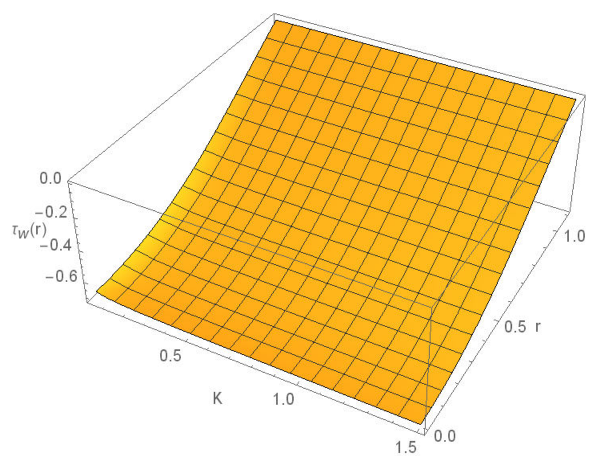

Fig. 8 Variation of wall shear stress with permeability of the porous media K:
$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, a_0 = 3, \theta = 30^0, \xi = 0.2, \epsilon = 0.9, \mathrm{r1} = 1.09861, \mu = 1.5$

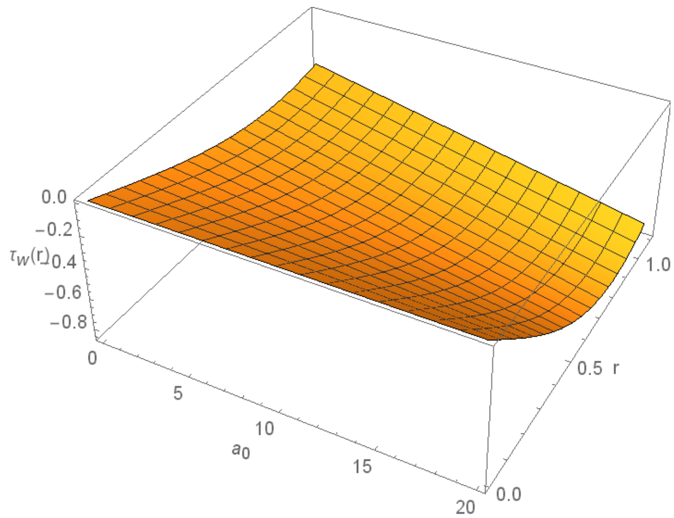

Fig. 9 Variation of wall shear stress with amplitude of body acceleration a_0:

$\bar{\alpha} = 1, \phi = 15^0, A_0 = 2, A_1 = 4, \theta = 30^0, \xi = 0.2, \epsilon = 0.9, r1 = 1.09861, \mu = 1.5$