

Stochastic Modelling and Computational Sciences

PSEUDO IRREGULAR FUZZY SOFT GRAPHS

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ABSTRACT

This paper deals with pseudo irregular fuzzy soft graphs. The definition of pseudo irregular graphs is introduced with some properties. The pseudo edge irregular fuzzy soft graphs are illustrated with examples. The properties of the defined graphs are studied. Highly, neighbourly, strongly pseudo irregular graphs are explained with examples. Also some pseudo edge irregular fuzzy soft graphs are illustrated. The relation between strongly pseudo irregular fuzzy soft graphs with highly and neighbourly pseudo irregular FSG is given. Results on total pseudo irregular FSG and total pseudo edge irregular FSG is examined.

1. INTRODUCTION

Results on soft set theory were presented in [5]. [4] dealt with soft sets in decision-making, which proved to be more effective. A study on fuzzy soft graphs was done by [1]. Regular fuzzy graphs were discussed in [2]. Santhi Maheswari and Sekar worked on a pseudo degree in fuzzy graphs and their properties in [6]. [7], [10] deal with various properties of pseudo irregular bipolar fuzzy and irregular intuitionistic fuzzy graphs. [3], [9] are about pseudo edge regular fuzzy graphs and edge pseudo regular fuzzy graphs respectively.

Here we extend the idea of pseudo degree and pseudo edge degree in fuzzy soft graphs. And bring out some of its characteristics and also provide some illustrations to prove them.

2. PRELIMINARIES

Definition 2.1. The pair (F, A) is soft set over the universal set, where $A \subseteq E$ and

$F : a \rightarrow \mathcal{P}(U)$. That is a soft set over U is parametered collection of subsets of U .

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Definition 2.2. An FSG G_α is a 4-tuple, such that:

1. G^* is crisp graph
2. A is the parameter set
3. (\tilde{F}, A) is fuzzy soft set over vertex set V
4. (\tilde{R}, A) is fuzzy soft set over edge set E .

Then $(\tilde{F}(\alpha), \tilde{R}(\alpha))$ is fuzzy (sub) graph of G^* , $\forall \alpha \in A$ and can be denoted as $\tilde{H}(\alpha)$.

The membership value of the edge in an FSG is given as

$$\tilde{R}(\alpha)(xy) \leq \min \{ \tilde{F}(\alpha)(x), \tilde{F}(\alpha)(y) \}.$$

Definition 2.3. If G is an FSG, then the vertex degree is $d_G(u) = \sum_{\alpha_i \in A} (\sum_{u \neq v} \tilde{R}(\alpha_i)(uv))$.

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Definition 2.4. The support (2-degree) of a vertex in an FSG \mathcal{G} is the addition of degrees of its adjacent vertices and denoted as $s_{\mathcal{G}}(u)$, whereas its total support is given as

$$ts_{\mathcal{G}}(u) = s_{\mathcal{G}}(u) + \sum_{a_i \in A} \hat{F}(a_i)(u).$$

Definition 2.5. The support (2-degree) of an edge in an FSG is the sum of edge degrees which are adjacent to given edges and can be defined as $s_{\mathcal{G}}(uv) = \sum_{e_i \in N(uv), a_i \in A} \hat{R}(a_i)(uv)$.

Definition 2.6. Let \mathcal{G} be a FSG, then pseudo degree of the vertex is $Pd_{\mathcal{G}}(x) = \frac{s_{\mathcal{G}}(x)}{D_{\mathcal{G}}(x)}$, where $s_{\mathcal{G}}$ is support of x and $D_{\mathcal{G}}(x)$ is the number of edges incident to x .

Definition 2.7. Let \mathcal{G} be a FSG, then the total pseudo degree of vertex is

$$TPd_{\mathcal{G}}(x) = Pd_{\mathcal{G}}(x) + \sum_{a_i \in A} \hat{F}(a_i)(x)$$

Definition 2.8. Let \mathcal{G} be a FSG. Then the pseudo degree of the edge is defined as $PEd_{\mathcal{G}}(v_i v_j) = \frac{s_{\mathcal{G}}(v_i v_j)}{ED_{\mathcal{G}}(v_i v_j)}$. Here $s_{\mathcal{G}}(v_i v_j)$ is support of the edge and $ED_{\mathcal{G}}(v_i v_j)$ is the number of edges incident with $v_i v_j$.

Definition 2.9. Let \mathcal{G} be a FSG, then the total pseudo edge degree is given by

$$TPEd_{\mathcal{G}}(v_i v_j) = PEd_{\mathcal{G}}(v_i v_j) + \sum_{v_i v_j \in E, a_i \in A} \hat{R}(a_i)(v_i v_j).$$

3 PSEUDO IRREGULAR FSG

Definition 3.1. A FSG, \mathcal{G} is pseudo irregular, if \exists atleast one vertex, whose adjacent vertices are with distinct pseudo degrees.

Definition 3.2. A FSG, \mathcal{G} is total pseudo irregular, if \exists atleast one vertex, whose adjacent vertices are with distinct total pseudo degree.

Example 3.3. Demonstration of above definitions.

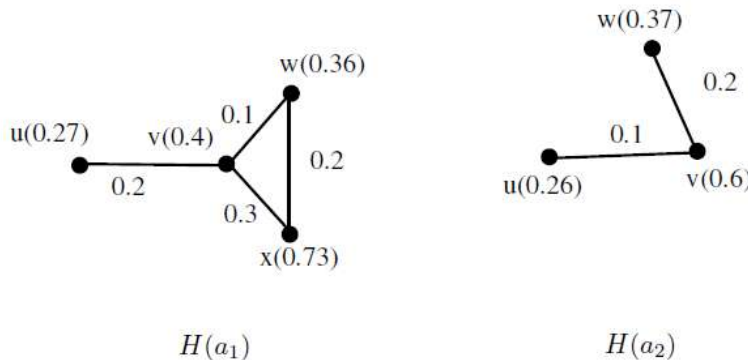


figure 3.3

Here the pseudo degree of all vertices are $Pd_{\mathcal{G}}(u) = 0.9$, $Pd_{\mathcal{G}}(v) = 0.433$, $Pd_{\mathcal{G}}(w) = 0.7$, $Pd_{\mathcal{G}}(x) = 0.7$. Here vertex x is adjacent to vertices v and w , which have distinct pseudo degrees.

Example 3.4. Consider the below graph.

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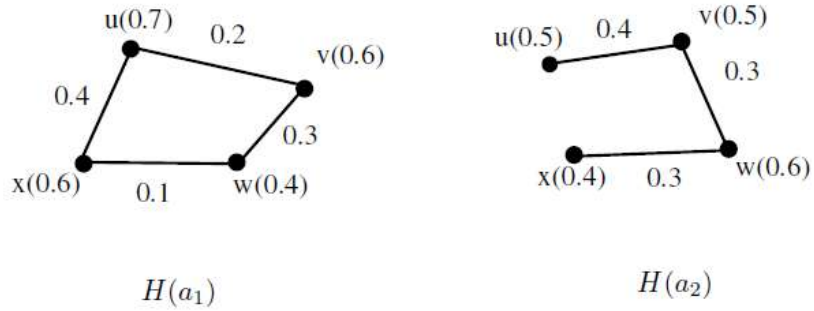


figure 3.4

The total pseudo degree of all vertices are $TPd_{\mathcal{G}}(u) = 2.2$, $TPd_{\mathcal{G}}(v) = 2.1$, $TPd_{\mathcal{G}}(w) = 2.0$, $TPd_{\mathcal{G}}(x) = 2.0$. The vertex u adjacent to vertices v and x with distinct total pseudo degrees.

Remark 3.5. A pseudo irregular FSG, need not be total pseudo irregular and vice versa.

Example 3.6. Consider the Example 3.3, in which the total pseudo degree of all vertices are same.

While considering Example 3.4, \mathcal{G} is total pseudo irregular but pseudo regular. Hence the remark

Definition 3.7. A FSG, \mathcal{G} is neighbourly pseudo irregular, if no two adjacent vertices have same pseudo degrees.

Example 3.8. Let us consider below fsg \mathcal{G} .

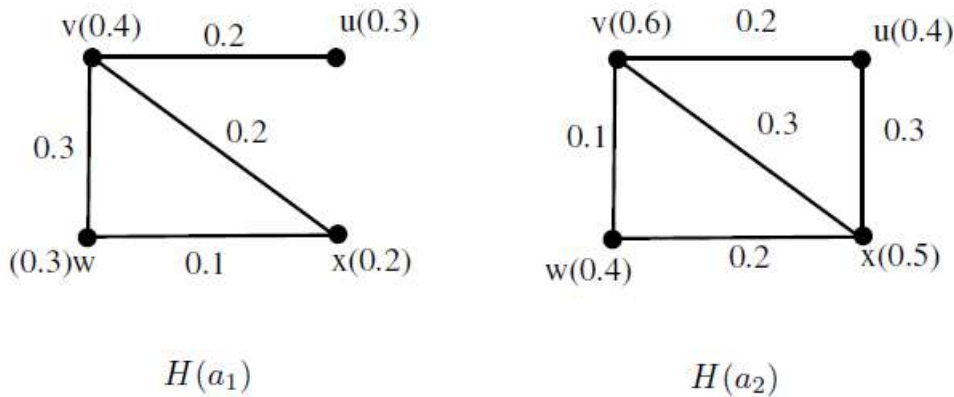


figure 3.8

The pseudo degree of the vertices are $Pd_{\mathcal{G}}(u) = 1.2$, $Pd_{\mathcal{G}}(v) = 0.833$, $Pd_{\mathcal{G}}(w) = 1.2$, $Pd_{\mathcal{G}}(x) = 0.9$. Here no pair of adjacent vertices have same pseudo degree, thus it is neighbourly pseudo irregular fsg. Here no pair of adjacent vertices have same pseudo degree, thus it is neighbourly pseudo irregular fsg.

Definition 3.9. A FSG, \mathcal{G} is highly pseudo irregular, if every vertex in the graph is adjacent to vertices with distinct pseudo degrees.

Example 3.10. Following is a FSG.

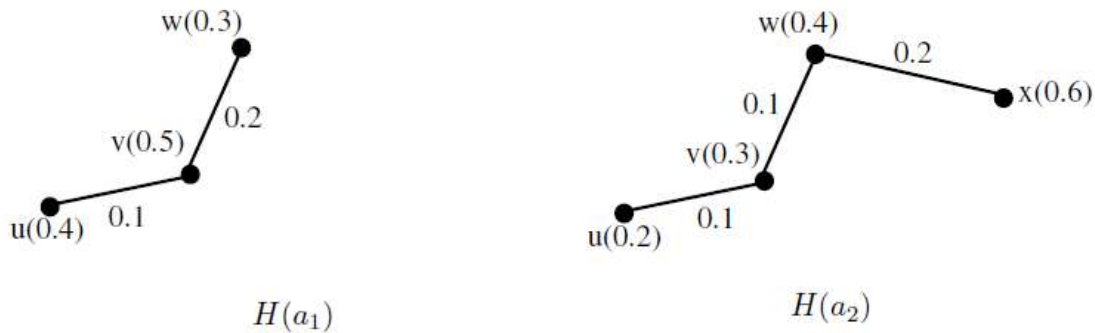


figure 3.10

Here $Pd_{\mathcal{G}}(u) = 0.5$, $Pd_{\mathcal{G}}(v) = 0.35$, $Pd_{\mathcal{G}}(w) = 0.35$, $Pd_{\mathcal{G}}(x) = 0.5$ and so the following is highly pseudo irregular FSG, since every vertex is adjacent to vertices with different pseudo degrees.

Definition 3.11. A FSG, \mathcal{G} is strongly pseudo irregular, if every pair of vertices have distinct pseudo degrees.

Example 3.12. Consider the below graph.

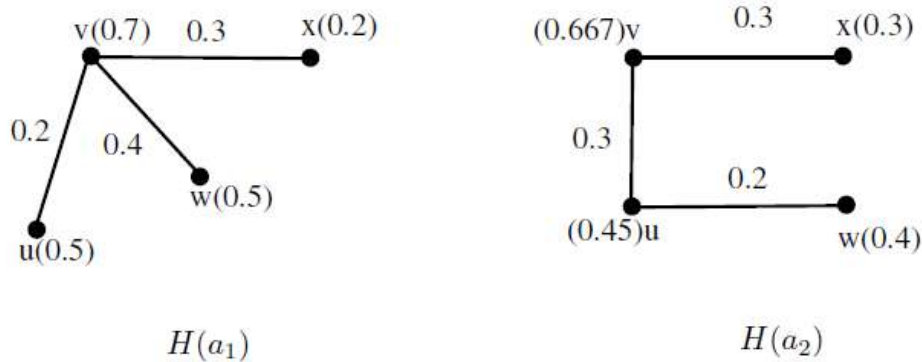


figure 3.12

The pseudo degree of all vertices $Pd_{\mathcal{G}}(u) = 0.35$, $Pd_{\mathcal{G}}(v) = 0.5$, $Pd_{\mathcal{G}}(w) = 0.3$, $Pd_{\mathcal{G}}(x) = 0.6$, which are all distinct, $\Rightarrow \mathcal{G}$ is strongly pseudo irregular.

Remark 3.13. A strongly pseudo irregular FSG, is both neighbourly and highly pseudo irregular.

Remark 3.14. A highly pseudo irregular may be neighbourly irregular, but the converse not necessarily be true.

Example 3.15. In figure 3.12, which is strongly pseudo irregular, it can be found that no pair of adjacent vertices have same $Pd_{\mathcal{G}} \Rightarrow$ it is neighbourly irregular, also no adjacent edges have same $Pd_{\mathcal{G}}$ and hence it is highly pseudo irregular.

Definition 3.16. An FSG is highly totally pseudo irregular, if every vertex is adjacent to vertices with distinct total pseudo degree.

Example 3.17. The below is an example.

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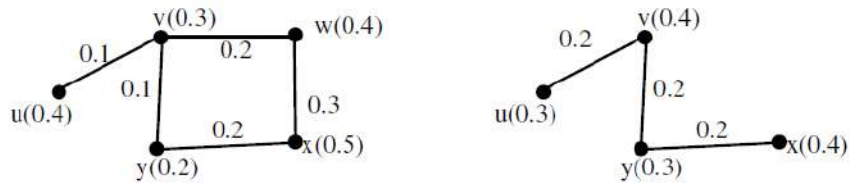


figure 3.17

Here $TPd_{\xi}(u) = 1.5$, $TPd_{\xi}(v) = 1.2$, $TPd_{\xi}(w) = 1.15$, $TPd_{\xi}(x) = 1.5$, $TPd_{\xi}(y) = 1.25$ and it is highly totally pseudo irregular fsg.

Definition 3.18. We call an FSG as neighbourly totally pseudo irregular, if no two adjacent vertices have same total pseudo degree.

Example 3.19. Consider the following

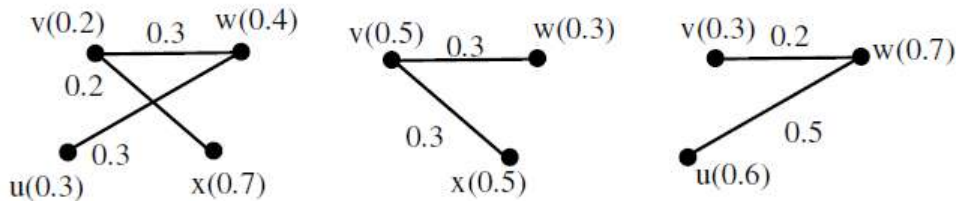


figure 3.19

The pseudo degrees are $TPd_{\xi}(u) = 2.5$, $TPd_{\xi}(v) = 2.05$, $TPd_{\xi}(w) = 1.45$, $TPd_{\xi}(x) = 2.5$

and it is totally neighbourly pseudo irregular fsg.

Definition 3.20. An FSG is strongly totally pseudo irregular, if the total pseudo degree of all vertices are distinct.

Example 3.21. Example for the above definition.

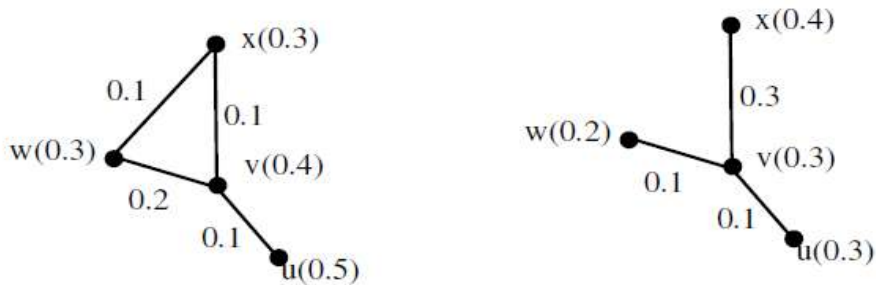


figure 3.21

Here, $TPd_{\xi}(u) = 1.7$, $TPd_{\xi}(v) = 1.06$, $TPd_{\xi}(w) = 1.2$, $TPd_{\xi}(x) = 1.35$ and all are distinct and so it is strongly totally pseudo irregular fsg.

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4 PSEUDO EDGE IRREGULAR FSG

Definition 4.1. A FSG, \mathcal{G} is pseudo edge irregular, if \exists atleast one edge whose adjacent edges are with distinct pseudo edge degree.

Definition 4.2. A FSG, \mathcal{G} is total pseudo edge irregular, if \exists atleast one edge with adjacent edges having distinct total pseudo edge degree.

Example 4.3. Consider the example.

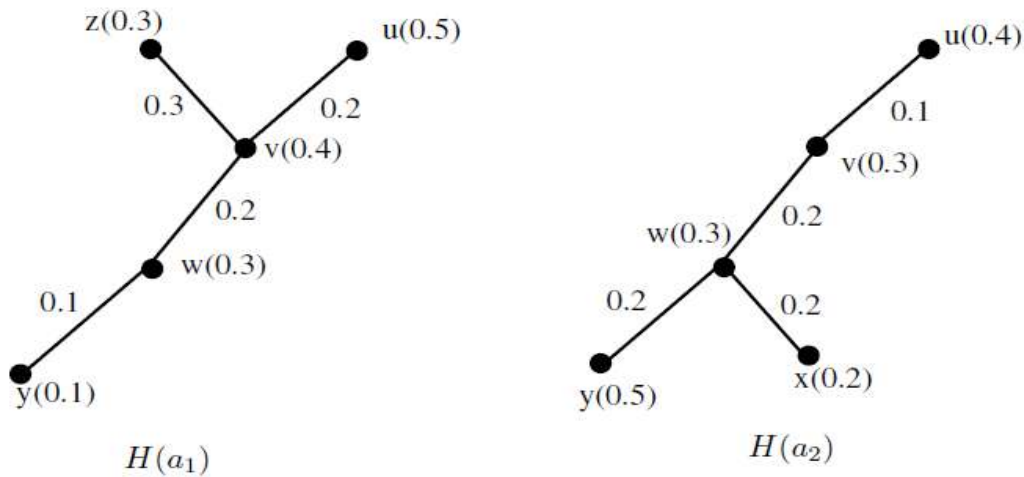


figure 4.3

There exist atleast one edge uv whose adjacent edges vz and vw are with different pseudo edge degrees. Thus it is pseudo edge irregular. Also the total pseudo edge degree of adjacent edges of atleast one edge is distinct, hence it is total pseudo edge irregular FSG.

Definition 4.4. A FSG, \mathcal{G} is neighbourly pseudo edge irregular, if no pair of adjacent edges are with same pseudo edge degrees.

Example 4.5. The following is an example.

Here $PEd_{\mathcal{G}}(uv) = 0.6$, $PEd_{\mathcal{G}}(vw) = 0.633$, $PEd_{\mathcal{G}}(uw) = 0.85$, $PEd_{\mathcal{G}}(vx) = 0.85$ No pair of adjacent edges have same pseudo edge degree \Rightarrow given \mathcal{G} is neighbourly pseudo edge irregular. eudo edge degree \Rightarrow given \mathcal{G} is neighbourly pseudo edge irregular.

Definition 4.6. A FSG, \mathcal{G} is highly pseudo edge irregular, if every edge of the graph is adjacent to edges with different pseudo edge degrees.

Example 4.7. Illustration of above definition.

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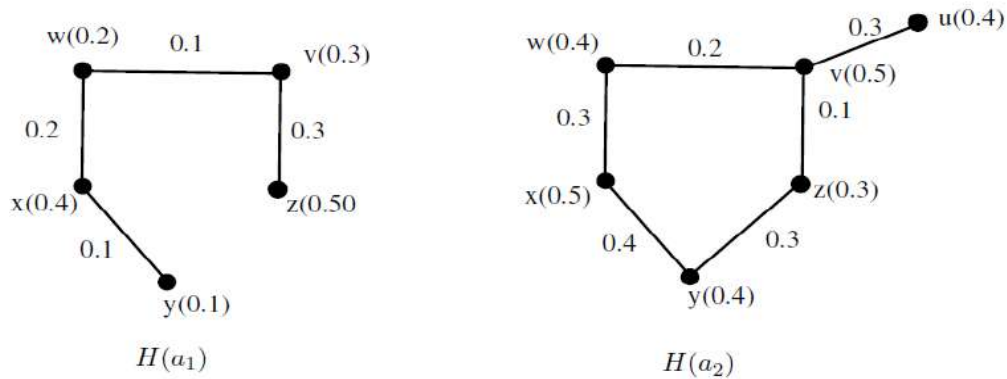


figure 4.7

Now, $PEd_{\xi}(uv) = 1.05$, $PEd_{\xi}(vw) = 0.8$, $PEd_{\xi}(wx) = 1.0$, $PEd_{\xi}(xy) = 0.85$, $PEd_{\xi}(yz) = 0.85$, $PEd_{\xi}(zv) = 0.933$ and no edge in this example is such, that its adjacent edges have same pseudo edge degrees, thus highly pseudo edge irregular.

Definition 4.8. A FSG is strongly pseudo edge irregular only when the pseudo edge degree of all edges are distinct.

Example 4.9. Illustration of above definition.

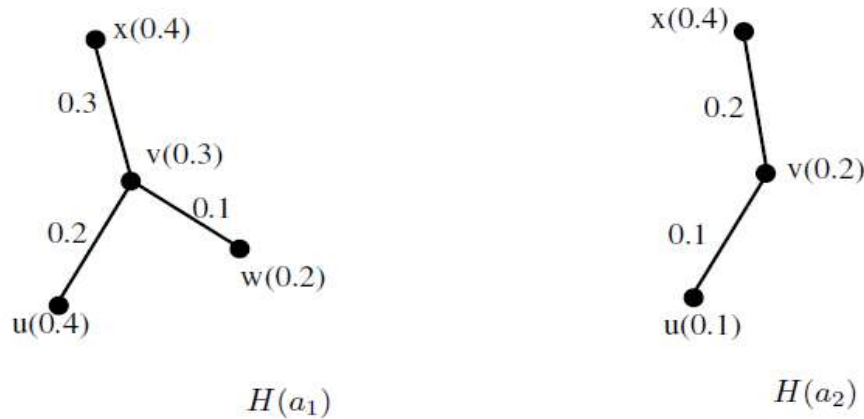


figure 4.9

The pseudo edge degree of the edges are $PEd_{\xi}(uv) = 0.6$, $PEd_{\xi}(vw) = 0.5$, $PEd_{\xi}(xv) = 0.7$, $\Rightarrow \mathcal{G}$ is strongly pseudo edge irregular FSG.

Remark 4.10. If a FSG \mathcal{G} is strongly pseudo edge irregular, then it is both highly and neighbourly pseudo edge irregular.

Definition 4.11. An FSG is highly totally pseudo irregular, if the total pseudo edge degrees of adjacent edges of each edge in the graph are not alike.

Example 4.12. Consider the below graph.

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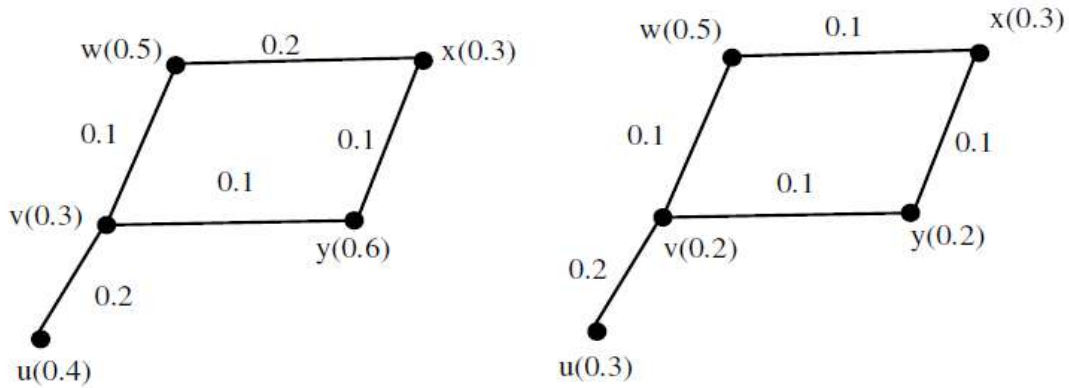


figure 4.12

Now, $TPEd_{\mathcal{G}}(uv) = 1.25$, $TPEd_{\mathcal{G}}(wv) = 0.733$, $TPEd_{\mathcal{G}}(wx) = 1.0$, $TPEd_{\mathcal{G}}(xy) = 0.8$, $TPEd_{\mathcal{G}}(yv) = 0.8$ and no edge is adjacent to edges with same total pseudo edge degree.

Definition 4.13. We call an FSG as neighbourly totally pseudo edge irregular, if no two adjacent edges in \mathcal{G} have same total pseudo edge degree.

Definition 4.14. An FSG is strongly totally pseudo edge irregular, if the total pseudo edge degree of all edges are distinct.

Example 4.15. The above is demonstrated using this example.

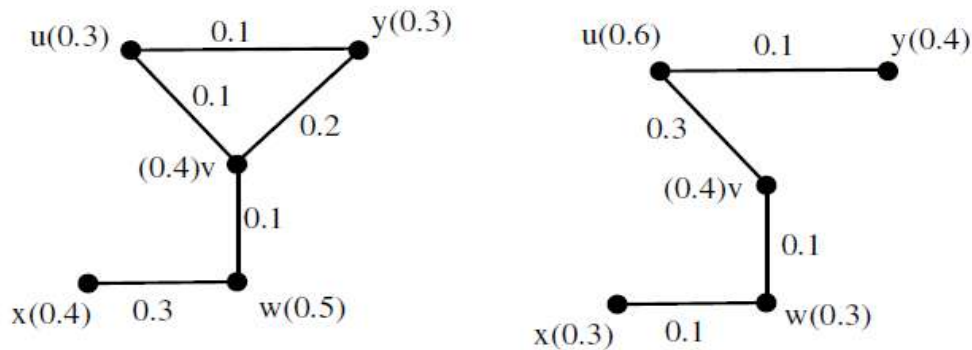


figure 4.15

Here, $TPEd_{\mathcal{G}}(uv) = 1.2$, $TPEd_{\mathcal{G}}(wv) = 0.733$, $TPEd_{\mathcal{G}}(wx) = 1.4$, $TPEd_{\mathcal{G}}(yv) = 0.933$, $TPEd_{\mathcal{G}}(uy) = 0.9$, all of them have distinct total pseudo edge degrees \Rightarrow strongly totally pseudo edge irregular FSG.

Remark 4.16. A pseudo irregular FSG not necessarily be pseudo edge irregular and vice versa.

Example 4.17. The following is an example for the remark.

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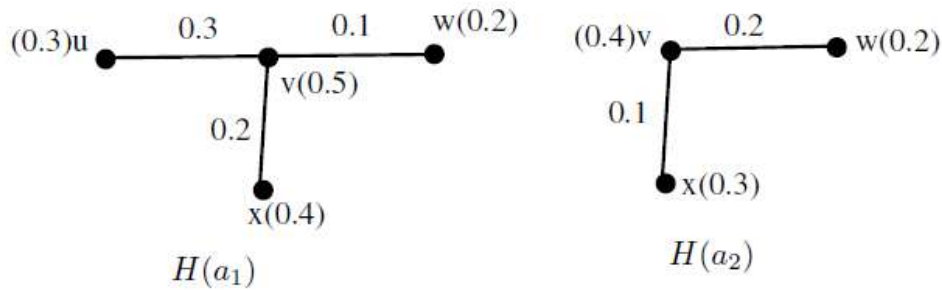


figure 4.17

Here $Pd_{\xi}(u) = 0.9, Pd_{\xi}(v) = 0.3, Pd_{\xi}(w) = 0.9, Pd_{\xi}(x) = 0.9 \Rightarrow$ pseudo irregular. While, $PEd_{\xi}(uv) = 0.6, PEd_{\xi}(wv) = 0.6, PEd_{\xi}(xv) = 0.6$ and hence not pseudo edge irregular.

Example 4.18. The below is pseudo edge irregular FSG but not pseudo irregular.

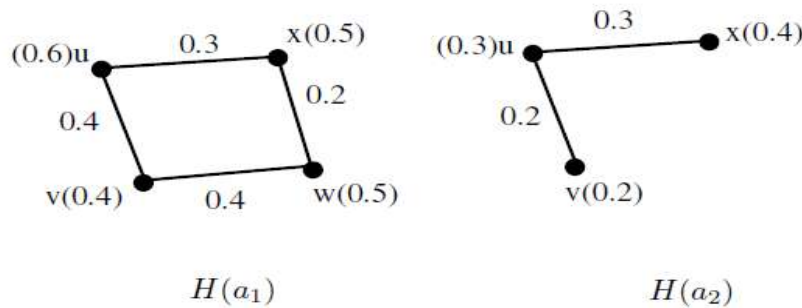


figure 4.18

Here, $Pd_{\xi}(u) = 0.9, Pd_{\xi}(v) = 0.9, Pd_{\xi}(w) = 0.9, Pd_{\xi}(x) = 0.9$. And $PEd_{\xi}(uv) = 0.8, PEd_{\xi}(wv) = 1.0, PEd_{\xi}(wx) = 0.8, PEd_{\xi}(ux) = 1.0$, thus it is pseudo edge irregular but not pseudo irregular FSG.

Result 4.19. The following results have been found using the definitions defined here.

1. A highly pseudo irregular may not be highly pseudo edge irregular FSG and other way around.

Example 4.20. Consider the figure 4.7 which is highly pseudo edge irregular. Observing pseudo degree of its vertices $Pd_{\xi}(u) = 1.0, Pd_{\xi}(v) = 0.6, Pd_{\xi}(w) = 1.0, Pd_{\xi}(x) = 0.8, Pd_{\xi}(y) = 0.85, Pd_{\xi}(z) = 0.9$, the vertex v is adjacent to vertices with same pseudo degree and hence it is not highly pseudo irregular.

The figure 3.9 is highly pseudo irregular. But $PEd_{\xi}(uv) = 0.4, PEd_{\xi}(wv) = 0.3, PEd_{\xi}(wx) = 0.4 \Rightarrow$ not highly pseudo edge irregular FSG.

2. A strongly pseudo irregular FSG may be strongly pseudo edge irregular and vice versa.

Example 4.21. Figure 3.4 is strongly pseudo irregular. And $PEd_{\xi}(uv) = 1.033, PEd_{\xi}(wv) = 1.0, PEd_{\xi}(xv) = 1.25, PEd_{\xi}(wu) = 1.25$, hence not strongly pseudo edge irregular.

Consider figure 4.5 which is strongly pseudo edge irregular. But $Pd_{\xi}(u) = 0.9, Pd_{\xi}(v) = 0.3, Pd_{\xi}(w) = 0.9, Pd_{\xi}(x) = 0.9$, implies it is not strongly pseudo irregular.

3. Neighbourly pseudo irregular FSG need not be neighbourly pseudo edge irregular FSG and vice versa.

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Example 4.22. Examine figure 3.8 which is neighbourly pseudo irregular. But we have, $PEd_{\xi}(uv) = 1.266$, $PEd_{\xi}(vw) = 1.266$, $PEd_{\xi}(wx) = 1.266$, $PEd_{\xi}(ux) =$

1.266 , $PEd_{\xi}(xv) = 1.2$, \Rightarrow it is not neighbourly pseudo edge irregular FSG. Look the below figure

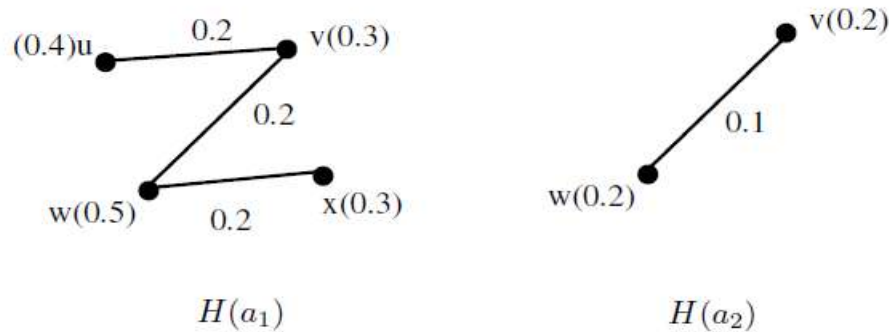


figure 4.22

Here, $PEd_{\xi}(uv) = 0.4$, $PEd_{\xi}(wv) = 0.3$, $PEd_{\xi}(wx) = 0.4$ and here no pair of adjacent edges have same pseudo edge degree, \Rightarrow it is neighbourly pseudo edge irregular. While $Pd_{\xi}(u) = 0.5$, $Pd_{\xi}(v) = 0.35$, $Pd_{\xi}(w) = 0.35$, $Pd_{\xi}(x) = 0.5$, \Rightarrow not neighbourly pseudo irregular FSG.

Result 4.23. The following are observed.

1. A strongly pseudo edge irregular FSG need not be strongly totally pseudo edge irregular and vice versa.

Example 4.24. An example is given below.

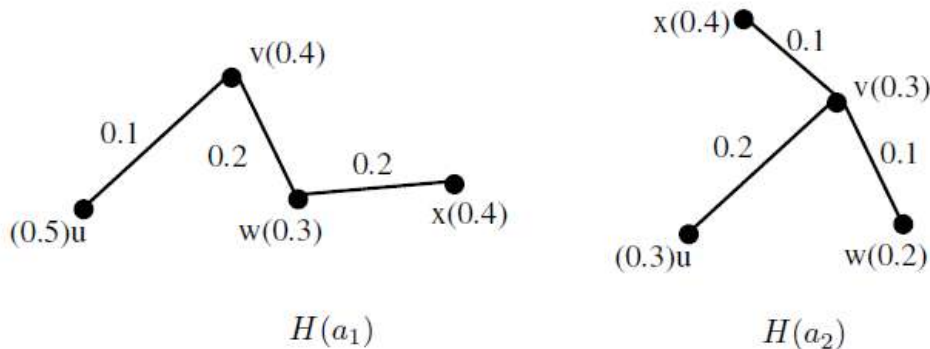


figure 4.24

Here $PEd_{\xi}(uv) = 0.6$, $PEd_{\xi}(wv) = 0.433$, $PEd_{\xi}(wx) = 0.6$, $PEd_{\xi}(yv) = 0.5$ and $TPEd_{\xi}(uv) = 0.9$, $TPEd_{\xi}(vw) = 0.733$, $TPEd_{\xi}(wx) = 0.8$, $TPEd_{\xi}(yv) = 0.6$. Thus we conclude that it is strongly totally pseudo edge irregular and not strongly pseudo edge irregular.

2. A highly pseudo edge irregular not necessarily be highly totally pseudo edge irregular FSG and vice versa.

Example 4.25. The below is an example.

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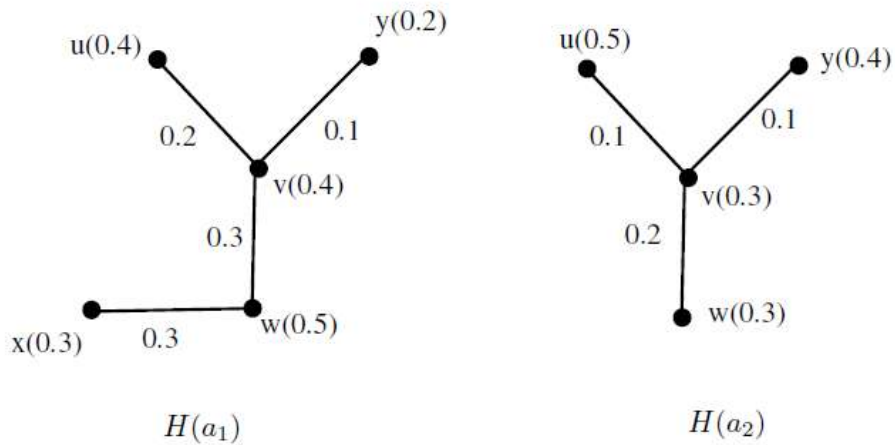


figure 4.25

3. Neighbourly pseudo edge irregular FSG not be neighbourly totally pseudo edge irregular FSG and vice versa.

Example 4.26. Below is an example

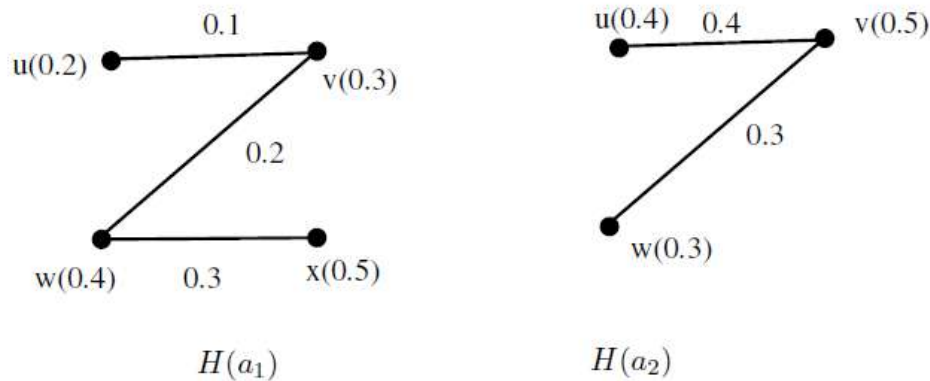


figure 4.26

Here $PEd_{\mathcal{G}}(uv) = 0.8$, $PEd_{\mathcal{G}}(wv) = 0.5$, $PEd_{\mathcal{G}}(wx) = 0.8$ and $TPEd_{\mathcal{G}}(uv) = 1.3$, $TPEd_{\mathcal{G}}(wv) = 1.0$, $TPEd_{\mathcal{G}}(wx) = 1.1$. Thus it is totally neighbourly pseudo edge irregular and not neighbourly pseudo edge irregular.

- 4. A strongly pseudo irregular need not be strongly totally pseudo irregular and vice versa.
- 5. A highly totally pseudo irregular not necessarily be highly pseudo irregular FSG and vice versa.
- 6. A FSG which is neighbourly pseudo irregular need not be neighbourly totally pseudo irregular and vice versa.

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