Stochastic Modelling and Computational Sciences

SOME REMARKS ON FUZZY SEMI NORMALITY IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT
In this paper, several characterizations of fuzzy semi normal spaces are obtained. It is shown that fuzzy semi normal and fuzzy extremally disconnected spaces are fuzzy normal spaces. The conditions, for fuzzy Baire-separated and fuzzy extremally disconnected spaces to become fuzzy normal spaces and for fuzzy quasi regular spaces to become fuzzy semi normal spaces, are obtained. It is shown that fuzzy semi normal, fuzzy
Oz and fuzzy P-spaces are fuzzy normal spaces.

Keywords :Fuzzy regular open set, fuzzy δ-open set, fuzzy Baire set, fuzzy G_δ *-set, fuzzy normalspace,fuzzy Oz -space, fuzzy P-space,fuzzyquasi-regular space, fuzzy extremally disconnected space, fuzzy Baire-separated space.*

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1. INTRODUCTION

The potential of fuzzy notion introduced by **L.A.Zadeh**[**25**] in 1965 , as a new approach to a mathematical representation of vagueness , was realized by many researchers and it has been successfully applied in all branches of Mathematics. In 1968, **C. L. Chang**[**4**] introduced the concept of fuzzy topological spaces. The notion of semi normality in general topology was introduced and studied in[**3,5 , 9, 10, 24**]. **G. Palani Chetty and G.Balasubramanian**[**11**] introduced the concept of fuzzy semi- normality in fuzzy topological spaces and studied some of the characterizations and basic properties of fuzzy semi normal spaces.

The purpose of this paper is to study more deeply the notion of fuzzy semi normality in fuzzy topological spaces. Several characterizations of fuzzy semi normal spaces are obtained. It is obtained that fuzzy semi normal and fuzzy extremally disconnected spaces are fuzzy normal spaces. The conditions, for fuzzy Baire-separated and fuzzy extremally disconnected spaces to become fuzzy normal spaces and for fuzzy quasi regular spaces to become fuzzy semi normal spaces are obtained. It is shown that fuzzy semi normal , fuzzy Oz and fuzzy P-spaces are fuzzy normal spaces.

2 . PRELIMINARIES

Some basic notions and results used in the sequel, are given in order to make the exposition self – contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to **Chang** (1968). Let X be a non-empty set and I ,the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1[4] **:** The interior , the closure and the complement of a fuzzy set λ are defined respectively as follows :

(i). $\text{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$;

(ii). $\text{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$.

(iii). $\lambda'(x) = 1-\lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(iv). $\psi(x) = \sup_i\{\lambda_i(x) / x \in X\}$

(v). $\delta(x) = \inf_i\{\lambda_i(x) / x \in X\}$.

Lemma 2.1[**1**] **:**For a fuzzy set λ of a fuzzy topological space X,

(i). $1-\text{int}(\lambda) = \text{cl}(1-\lambda)$ and (ii). $1-\text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition 2.2 [**13**] : Two fuzzy sets μ and γ of X are said to be disjoint if they do not intersect at any point of X. That is, $\mu(x) + \gamma(x) \leq 1$, forall $x \in X$.

Definition2.3 : A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy regular-open in (X,T) if $\lambda = \text{int cl}(\lambda)$ and fuzzy regular-closed in (X,T) if $\lambda = \text{cl int}(\lambda)$ [**1**].

(ii). fuzzy G_δ –set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$;

Fuzzy F_σ -set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$ for $i \in I$ [**2**].

(iii). fuzzy δ-open set in (X,T) if $\lambda = \vee_i(\lambda_i)$, where (λ_i) is a fuzzy regular open

Set for each $i \in I$ [**12**].

(iv). fuzzy locally closed set (resp. fuzzy A-set) in (X,T) if $\lambda = \mu \wedge \delta$, where μ is

a fuzzy open set in X and δ is a fuzzy closed set(resp. Fuzzy regular closed)in X [**6**].

(v). fuzzy dense set in (X,T) if there exists no fuzzy closed set μ in (X,T) such

That $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [**15**] .

(vi).fuzzy nowhere dense set in (X,T) if there exists no non-zero fuzzy open

set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$, in (X,T) [**15**].

(vii). fuzzy first category set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$s are fuzzy

nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of

fuzzy second category[**15**] .

(viii) fuzzy residual set in (X,T)if $1-\lambda$ is a fuzzy first category set in (X,T) [**16**].

(ix). fuzzy Baire set in (X,T) if $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X,T) [**17**].

Definition 2.4 [18]: A fuzzy closed set λ in a topological space (X,T) is called a fuzzy P-set if $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in (X,T), implies that $\lambda \leq 1-\text{cl}(\mu)$, in (X,T).

Definition 2.5: A fuzzy topological space (X,T) is called a

(i). fuzzy extremally disconnected space if the closure of each fuzzy open set

of (X,T) is fuzzy open in (X,T) [**7**].

(ii).fuzzy normal space if for every fuzzy closed set K and fuzzy open set U such that K $\leq$U,there exists a fuzzy set V such that K$\leq$ int (V) $\leq$cl (V) $\leq$U [**3**].

(iii).fuzzy Baire-separated space if for each pair of fuzzy closed sets μ_1 and μ_2 in (X,T) such that $\mu_1 \leq 1-\mu_2$, there exists a fuzzy Baire set ηin (X,T) such that $\mu_1 \leq \eta \leq 1 - \mu_2$[**23**].

(iv). fuzzy Oz-space if each fuzzy regular closed set is a fuzzy G_δ -set in (X,T)[**20**].

(v). fuzzy quasi- regular space if for each fuzzy open set λ in (X,T), there exists a fuzzy regular closed set μ in (X,T) such that $\mu \leq \lambda$ [**21**].

(vi). fuzzy quasi-Oz-space if for a fuzzy regular closed set λ in (X,T), there exists a fuzzy G_δ-set μ in (X,T) such that λ = cl int (μ) [**22**].

(vii). fuzzy P-space if each fuzzy G_δ -set in (X,T) is fuzzy open in (X,T) [**14**].

Definition 2.6 [8]: A fuzzy topological space (X,τ) is said to be a normal space if for each pair of fuzzy closed sets C_1 and C_2such that $C_1 \leq 1 - C_2$, there exist fuzzy open sets $M_1$and M_2such that $C_i \subseteq M_i$(i =1,2) and $M_1 \leq 1 - M_2$.

Theorem 2.1 [1] : In a fuzzy topological space ,

(a). The closure of a fuzzy open set is a fuzzy regular closed set.

(b). The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.2 [19] : If λ is a fuzzy regular open set in a fuzzy extremally disconnected space (X,T) , then λ is a fuzzy closed F_σ -set in (X,T).

Theorem 2.3[19] : If μ is a fuzzy regular closed set in a fuzzy extremally disconnected space (X,T) , then μ is a fuzzy open G_δ -set in (X,T).

Theorem 2.4[23] : If (X,T) is a fuzzy Baire - separated space in which fuzzy Baire sets are fuzzy regular open sets, then (X,T) is a fuzzy semi normal space.

Theorem 2.5[20] : If λ is a fuzzy regular open set in a fuzzy Oz - space, then λ is a fuzzy F_σ -set in (X,T).

Theorem 2.6[21] : If δ is a fuzzy closed set in a fuzzy quasi-regular space (X,T), then there exists a fuzzy regular open set α in (X,T) such that $\delta \leq \alpha$.

Theorem 2.7[22] : If γ is a fuzzy regular open set in a fuzzy quasi - Oz - space (X,T), then there exists a fuzzy closed set η in (X,T) such that $\gamma \leq \eta$.

Theorem 2.8[22] : If λ is a fuzzy open set in a fuzzy quasi - Oz - space (X,T), then there exists a fuzzy G_δ-set μ in (X,T) such that $\lambda \leq$ cl int (μ).

Theorem 2.9[22] : If δ is a fuzzy closed set in a fuzzy quasi - Oz - space (X,T), then there exists a fuzzy F_σ-set η in (X,T) such that int cl (η) $\leq \delta$.

Theorem 2.10[19] : If λ and μ are any two fuzzy sets defined on X in a fuzzy hereditarily extremally disconnected space (X,T) , then there exist a fuzzy closed set η and a fuzzy open set δ in (X,T) such that $[\lambda \wedge \text{int}(1-\mu)] \leq \eta \leq \delta \leq [\text{cl}(\lambda) \vee (1-\mu)]$,in (X,T).

Theorem 2.11[18] : If λ is a fuzzy P -set in a fuzzy topological space (X, T) such that $\lambda \leq 1-\mu$,where μ is a fuzzyF_σ- set in (X, T), then there exists a fuzzy open set δ in (X, T) such that $\lambda \leq \delta \leq 1-\text{int}(\mu)$.

Theorem 2.12[18] : If λ is a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda \leq \mu$,where μ is a fuzzy G_δ- set in (X,T), then $\lambda \leq \text{int}(\mu)$, in (X,T).

3. FUZZY SEMINORMAL SPACES

Definition 3.1 [10]: A fuzzy topological space (X,T) is called a fuzzy semi normal space if given a fuzzy closed set λ and a fuzzy open set μsuch that $\lambda \leq \mu$, then there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$.

Example 3.1: Let X = { a, b, c}. Let I = [0,1] and α, β and γ are the fuzzy sets defined on X as follows :

α: X → I is defined by α(a) =0.4 ; α (b) = 0.6 ; α (c) = 0.4,

β: X → I is defined by β(a) = 0.6 ; β (b) = 0.5 ; β (c) = 0.6,

γ: X → I is defined by γ(a) =0.6; γ (b) = 0.5; γ (c) = 0.7,

Then, T ={ 0, $\alpha,\beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \alpha \wedge \beta$,1} is a fuzzy topology on X. By computation, one can see that int$(1-\alpha) = 0$; int $(1-\beta) = \alpha \wedge \beta$; int $(1-\gamma) = 0$; int $(1-[\alpha\vee\beta]) = 0$; int $(1-[\alpha\vee\gamma]) = 0$; int $(1-[\alpha\wedge\beta]) = \beta$ andcl $(\beta) = 1-(\alpha\wedge\beta)$; cl $(\alpha\wedge\beta) = 1-\beta$. The fuzzy dense sets in (X,T) are α,γ, $\alpha\vee\beta$ and $\alpha\vee\gamma$. Now int cl $(\beta) =$ int $(1-[\alpha\wedge\beta]) = \beta$ and int cl $(\alpha\wedge\beta) =$ int $(1-\beta) = \alpha\wedge\beta$ and thus β and $\alpha\wedge\beta$ are the fuzzy regular open sets in (X,T). For each fuzzy closed set λ ($=1-\alpha$,$1-\beta, 1-\gamma, 1-[\alpha\vee\beta], 1-[\alpha\vee\gamma], 1-[\alpha\wedge\beta]$) and each fuzzy open set μ ($= \alpha,\beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \alpha\wedge\beta$) such that $\lambda \leq \mu$, then there exists a fuzzy regular open set σ ($= \beta$ or $\alpha\wedge\beta$) such that $\lambda \leq \sigma \leq \mu$, implies that the fuzzy topological space (X,T) is a fuzzy seminormal space.

Remark3.1 :It is to be noted that a fuzzy semi normal space need not be a fuzzy normal space. For, in example **3.1**, for the fuzzy closed sets $1-[\alpha\wedge\beta]$and$1-\gamma$, with $1-[\alpha\wedge\beta] \leq 1-(1-\gamma)$, there exist fuzzy open sets $\alpha\vee\gamma$ and$\alpha\vee\beta$ such that $1-[\alpha\wedge\beta] \leq \alpha\vee\gamma$and $1-\gamma \leq \alpha\vee\beta$ in (X,T).But $\alpha\vee\gamma \leq 1-[\alpha\vee\beta]$, shows that (X,T) is not a fuzzy normal space.

Example 3.2: Let X = { a, b, c}. Let I = [0, 1] and α, β and γ are the fuzzy sets defined on X as follows :

α: X → I is defined by α(a) =0.4 ; α (b) = 0.4 ; α (c) = 0.5,

β: X → I is defined by β(a) = 0.5 ; β (b) = 0.5 ; β (c) = 0.6,

γ: X → I is defined by γ(a) =0.6 ; γ (b) = 0.4 ; γ (c) = 0.5,

Then, T={ 0, $\alpha,\beta, \gamma, \beta\vee\gamma, \beta\wedge\gamma$, 1} is a fuzzy topology on X. By computation, one can see that int $(1-\alpha) = \gamma$; int $(1-\beta) = 0$; int $(1-\gamma) = \alpha$; int $(1-[\beta\vee\gamma]) = 0$; int $(1-[\beta\wedge\gamma]) = \beta\wedge\gamma$; cl $(\alpha) = 1-\gamma$;cl $(\beta) = 1$; cl $(\gamma) = 1-\alpha$; cl $(\beta\vee\gamma) = 1$; cl $(\beta\wedge\gamma) = 1-(\beta\wedge\gamma)$. Now int cl $(\alpha) =$ int (1

$- \gamma) = \alpha$ and int cl $(\gamma) = $ int $(1 - \alpha) = \gamma$; int cl $(\beta \wedge \gamma) = $ int $(1 - [\beta \wedge \gamma]) = \beta \wedge \gamma$ and thus α, γ and $\beta \wedge \gamma$ are fuzzy regular open sets in (X,T). By computation, one can see that for the fuzzy closed set $1 - \beta$ and the fuzzy open set $\beta \vee \gamma$ with $1 - \beta \leq \beta \vee \gamma$, there are no fuzzy regular open sets σ ($= \alpha, \gamma, \beta \wedge \gamma$) such that $\lambda \leq \sigma \leq \mu$. Hence the fuzzy topological space (X,T) is not a fuzzy semi normal space.

Proposition 3.1: If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy semi normal space (X,T), then there exists a fuzzy regular open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Then, $\lambda_1 \leq 1 - \lambda_2$,in (X,T). Since (X,T) is a fuzzy semi normal space, for the fuzzy closed set λ_1 and fuzzy open set $1 - \lambda_2$ such that $\lambda_1 \leq 1 - \lambda_2$, there exists a fuzzy regular open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$.

Corollary 3.1 : If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy semi normal space (X,T), then there exists a fuzzy open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$.

Proposition3.2 : If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy seminormal space (X,T), then there exist fuzzy regular closed sets δ_1 and δ_2 in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1 - \delta_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Then, by Proposition**3.1,** there exists a fuzzy regular open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$. Now $\lambda_1 \leq \sigma$ implies that $1 - \lambda_1 \geq 1 - \sigma$, where $1 - \lambda_1$ is a fuzzy open set and $1 - \sigma$ is a fuzzy regular closed set in (X,T). Since a fuzzy regular closed set is a fuzzy closed set in a fuzzy topological space, $1 - \sigma$ is a fuzzy closed set in (X,T). Again since (X,T) is a fuzzy semi normal space, for the fuzzy closed set $1 - \sigma$ and the fuzzy open set $1 - \lambda_1$ such that $1 - \sigma \leq 1 - \lambda_1$, there exists a fuzzy regular open set γ in (X,T) such that $1 - \sigma \leq \gamma \leq 1 - \lambda_1$. This implies that $\lambda_1 \leq 1 - \gamma \leq \sigma$. Also $\sigma \leq 1 - \lambda_2$, implies that $\lambda_2 \leq 1 - \sigma$. Let $\delta_1 = 1 - \gamma$ and $\delta_2 = 1 - \sigma$. Then, δ_1 and δ_2 are fuzzy regular closed sets in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1 - \delta_2$.

Proposition 3.3: If $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in a fuzzy semi normal space (X,T), then there exists a fuzzy regular open set σ and a fuzzy regular closed set δ in (X,T) such that $\lambda \leq \sigma \leq \mu \leq \delta$.

Proof : Suppose that $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy semi normal space, there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$. Then, $\lambda \leq \sigma \leq \mu \leq$ cl(μ). By Theorem **2.1,**cl(μ) is a fuzzy regular closed set in (X,T).Let $\delta = $ cl(μ). Then, it follows that, $\lambda \leq \sigma \leq \mu \leq \delta$.

Proposition 3.4: If $\lambda_i \leq \mu$, where each λ_i(i $=$ 1 to ∞) is a fuzzy closed set and μ is a fuzzy open set in a fuzzy semi normal space (X,T), then there exists a fuzzy F_σ -setγ in (X,T) and a fuzzy δ -open set θ such that $\lambda_i \leq \gamma \leq \theta \leq \mu$.

Proof : Suppose that $\lambda_i \leq \mu$, where each λ_i(i$=$ 1 to ∞) is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy semi normal space, there exist fuzzy regular open sets $(\sigma_i)'s$ in (X,T) such that $\lambda_i \leq \sigma_i \leq \mu$. This implies that $\vee_{i=1}^{\infty} (\lambda_i) \leq \vee_{i=1}^{\infty}(\sigma_i) \leq \vee(\mu)$. Let $\gamma = \vee_{i=1}^{\infty} (\lambda_i)$ and $\theta = \vee_{i=1}^{\infty}(\sigma_i)$. Then, γ is a fuzzy F_σ –set and θ is a fuzzy δ -open set in (X,T) and it follows that $\lambda_i \leq \gamma \leq \theta \leq \mu$.

Proposition 3.5: If $\lambda \leq \mu_i$, where λ is a fuzzy closed set and each μ_i(i = 1 to ∞) is a fuzzy open set in a fuzzy semi normal space (X,T), then there exists a fuzzy δ-open set θ such that $\lambda \leq \theta \leq \mu$, where $\mu \in T$.

Proof : Suppose that $\lambda \leq \mu_i$, where λ is a fuzzy closed set and each μ_i(i = 1 to ∞) is a fuzzy open set in (X,T). Since (X,T) is a fuzzy semi normal space, there exist fuzzy regular open sets $(\sigma_i)'s$ in (X,T) such that $\lambda \leq \sigma_i \leq \mu_i$. This implies that $\vee\lambda \leq \vee_{i=1}^{\infty}(\sigma_i) \leq \vee_{i=1}^{\infty}(\mu_i)$. Let $\mu = \vee_{i=1}^{\infty}(\mu_i)$ and $\theta = \vee_{i=1}^{\infty}(\sigma_i)$. Then, μ is a fuzzy open setand θ is a fuzzy δ-open set in (X,T) and it follows that $\lambda \leq \theta \leq \mu$.

Proposition 3.6 : If λ is a fuzzy P -set in a fuzzy semi normal space (X,T) such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in (X,T), then there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \leq 1 - \text{int}(\mu)$.

Proof: Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in (X,T). Then, by Theorem **2.11,** there exists a fuzzy open set δ in (X,T) such that $\lambda \leq \delta \leq 1 - \text{int}(\mu)$. Since (X,T) is a fuzzy semi normal space and the fuzzy P-set λ is a fuzzy closed set in (X,T) with $\lambda \leq \delta$, there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \delta$. Then, $\lambda \leq \sigma \leq 1 - \text{int}(\mu)$, in (X, T).

Corollary 3.2: If λ is a fuzzy P -set in a fuzzy semi normal space (X,T) such that $\lambda \leq \gamma$, where γ is a fuzzy G_δ- set in (X,T), then there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \leq \text{cl}(\gamma)$.

Proof: Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq \gamma$,where γ is a fuzzy G_δ-set in (X,T). Then, $\lambda \leq 1-(1-\gamma)$, where $1-\gamma$ is a fuzzy F_σ- set in (X,T). Since (X,T) is a fuzzy semi normal space, by Proposition **3.6,** there exists a fuzzy regular open set σ in (X,T) such that$\lambda \leq \sigma \leq 1 - \text{int}(1-\gamma)$. Then, it follows that $\lambda \leq \sigma \leq \text{cl}(\gamma)$.

Corollary 3.3 : If λ is a fuzzy P -set in a fuzzy semi normal space (X,T) such that $\lambda \leq \gamma$, where γ is a fuzzy G_δ- set in (X,T), then there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \wedge \text{int}(\gamma) \leq \text{cl}(\gamma)$.

Proof : The proof follows from Theorem**2.12** and Corollary **3.2.**

Proposition3.7: If λ is a fuzzy P-set in a fuzzy semi normal space (X,T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in (X, T), then δ is a fuzzy somewhere dense set in (X,T).

Proof: Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq \gamma$, where γ is a fuzzy G_δ-set in (X,T). Then, by Corollary 3.3, there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \wedge \text{int}(\gamma) \leq \text{cl}(\gamma)$. Let $\theta = \sigma \wedge \text{int}(\gamma)$. Then, θ is a fuzzy open set in (X,T) and $\lambda \leq \theta \leq \text{cl}(\gamma)$. This implies that int($\theta$) $\leq$ int cl(γ) and thus int cl(γ) $\neq$ 0, in (X,T). Hence δ is a fuzzy somewhere dense set in (X,T).

4. FUZZY SEMINORMAL SPACES ANDOTHER FUZZY TOPOLOGICAL SPACES

Proposition 4.1: If $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in a fuzzy semi normal and fuzzy extremally disconnected space (X,T), then there exists a fuzzy closed F_σ –set σ in (X,T) such that $\lambda \leq \sigma \leq \mu$.

Proof : Suppose that $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy semi normal space, by Proposition **3.1,** there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \leq \mu$. Also since (X,T) is a fuzzy extremally disconnected space, by Theorem **2.2,** the fuzzy regular open set σ is a fuzzy closed F_σ-set in (X,T) and thus $\lambda \leq \sigma \leq \mu$, where σ is a fuzzy closed F_σ-set in (X,T).

Proposition4.2: If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy semi normal and fuzzy extremally disconnected space (X,T), then there exists a fuzzy closed F_σ-setσ in (X,T) such that $\lambda_1 \leq \sigma \leq 1-\lambda_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy seminormal space, by Proposition **3.1,** there exists a fuzzy regular open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1-\lambda_2$. Also since(X,T) is a fuzzy extremally disconnected space, by Theorem **2.2,** the fuzzy regular open set σ is a fuzzy closed F_σ-set in (X,T). Hence , for the disjoint fuzzy closed sets $\lambda_1$and λ_2, there exists a fuzzy closed F_σ–set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1-\lambda_2$.

Proposition4.3: If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy seminormal and fuzzy extremally disconnected space (X,T), then there exist fuzzy open G_δ -sets $\delta_1$and $\delta_2$in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1-\delta_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy seminormal space, by Proposition **3.2,** there exist fuzzy regular closed sets δ_1 and δ_2 in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1-\delta_2$. Also since (X,T) is a fuzzy extremally disconnected space , by Theorem **2.3,** the fuzzy regular closed sets δ_1 and δ_2 in (X,T) are fuzzy open G_δ-sets in (X,T). Hence, for the disjoint fuzzy closed sets λ_1 and λ_2, there exist fuzzy open G_δ-sets δ_1 and δ_2 in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1-\delta_2$.

Corollary 4.1: If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy seminormaland fuzzy extremally disconnected space (X,T), then there exist fuzzy open sets δ_1 and δ_2 in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$with $\delta_1 \leq 1-\delta_2$.

The following proposition shows that fuzzy seminormal and fuzzy extremally disconnected spaces are fuzzy normal spaces.

Proposition4.4 : If a fuzzy topological space (X,T) is a fuzzy seminormal and fuzzy extremally disconnected space, then (X,T) is a fuzzy normal space.

Proof : Suppose that $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy seminormal space, by Proposition **3.1,** there exists a fuzzy regular open set σ in (X,T) such that $\lambda \leq \sigma \leq \mu$. Then, $\lambda \leq int\, cl(\sigma) \leq \mu$, in (X,T). Let $\delta = cl(\sigma)$ and then $\lambda \leq int(\delta) \leq \mu$. Now $int(\delta) \leq cl(\delta) = cl(cl(\sigma)) = cl(\sigma)$,in (X,T) and σ being a fuzzy regular open set in (X,T) , is a fuzzy open set in (X,T). Since (X,T) is a fuzzy extremally disconnected space, $cl(\sigma)$ is a fuzzy open set in (X,T) and thus $int(cl(\sigma)) = cl(\sigma)$. Then, it follows that $int(\delta) \leq cl(\delta) = cl(\sigma) = int(cl(\sigma)) = \sigma \leq \mu$,in (X,T). Thus, for a fuzzy closed set λ and a fuzzy open set μ with $\lambda \leq \mu$, there exists a fuzzy set δ in (X,T) such that $\lambda \leq int(\delta) \leq cl(\delta) \leq \mu$. Hence (X,T) is a fuzzy normal space.

The following proposition gives a condition for fuzzy Baire-separated and fuzzy extremally disconnected spaces to become fuzzy normal spaces.

Proposition4.5 : If fuzzy Baire sets are fuzzy regular open sets in a fuzzy Baire separated and fuzzy extremally disconnected space (X,T), then (X,T) is a fuzzy normal space.

Proof : The proof follows from Proposition **4.4** and Theorem **2.4**.

Proposition4.6 : If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy seminormal and fuzzy Oz -space (X,T), then there exists a fuzzy F_σ -setσ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy seminormal space, by Proposition **3.1,** there exists a fuzzy regular open set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$. Also since (X,T) is a fuzzy Oz -space, the fuzzy regular open set σ is a fuzzy F_σ -set in (X,T). Hence , for the disjoint fuzzy closed sets λ_1 and λ_2 in (X,T), there exists a fuzzy F_σ-set σ in (X,T) such that $\lambda_1 \leq \sigma \leq 1 - \lambda_2$.

The following proposition gives a condition under which fuzzy quasi regular spaces become fuzzy seminormal spaces.

Proposition4.7 : If fuzzy regular open sets are disjoint in a fuzzy quasi-regular space (X,T), then (X,T) is a fuzzy seminormal space.

Proof : Suppose that$\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in (X,T). Then, $\lambda \leq 1 - (1 - \mu)$,in (X,T). Since (X,T) is a fuzzy quasi-regular space, for the fuzzy closed sets $\lambda$and $1 - \mu$, by Theorem **2.6**, there exist fuzzy regular open sets α and β in (X,T) such that $\lambda \leq \alpha$ and $1 - \mu \leq \beta$. By hypothesis, the fuzzy regular open sets α and β are disjoint and thus $\alpha \leq 1 - \beta$, in (X,T).Now $1 - \mu \leq \beta$, implies that $1 - \beta \leq \mu$. Then, $\lambda \leq \alpha \leq 1 - \beta \leq \mu$and thus $\lambda \leq \alpha \leq \mu$,in (X,T). Hence the fuzzy quasi-regular space (X,T) is a fuzzy seminormal space.

Proposition 4.8 : If $\lambda \leq \mu$, where $\lambda$is a fuzzy closed set and μ is afuzzy open set in a fuzzy seminormal and fuzzy quasi - Oz - space (X,T), then there exists a fuzzy locally closed set θin (X,T)such that $\lambda \leq \theta \leq \delta$.

Proof : Suppose that$\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy seminormal space, there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$. Also since (X,T) is a fuzzy quasi - Oz -space, by Theorem 2.7, for the fuzzy regular open set σ in X,there exists a fuzzy closed set η in (X,T) such that $\sigma \leq \eta$. Then, $\sigma \leq \mu \wedge \eta$. Let$\theta = \mu \wedge \eta$. Then, θ is a fuzzy locally closed set in (X,T) and thus, there exists a fuzzy locally closed set θ in (X,T) such that $\lambda \leq \theta \leq \delta$.

Proposition 4.9 : If $\lambda \leq \mu$, where λ is a fuzzy closed set and μ is a fuzzy open set in a fuzzy seminormal and fuzzy quasi - Oz - space (X,T), then there exist a fuzzy G_δ-set δ, a fuzzy F_σ-set η and a fuzzy regular open set σ in (X,T) such that $\text{int cl}(\eta) \leq \lambda \leq \sigma \leq \mu \leq \text{cl int}(\delta)$.

Proof : Suppose that $\lambda \leq \mu$, where $\lambda$is a fuzzy closed set and μ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy seminormal space, there exists a fuzzy regular open set σ such that $\lambda \leq \sigma \leq \mu$. Also since (X,T) is a fuzzy quasi - Oz -space, by Theorem**2.8**, for the fuzzy open set μ in X, there exists a fuzzy G_δ-set δ in (X,T) such that $\mu \leq \text{cl int}(\delta)$. By Theorem **2.9**, for the fuzzy closed set λ in X, there exists a fuzzy F_σ-set η in (X,T) such that $\text{int cl}(\eta) \leq \delta$. Hence it follows that $\text{int cl}(\eta) \leq \delta \leq \sigma \leq \mu \leq \text{cl int}(\delta)$, in (X,T).

Proposition 4.10: If λ and μ are any two fuzzy sets defined on X in a fuzzy hereditarily extremally disconnected and fuzzy seminormal space (X,T), then there exists a fuzzy regular open set σin (X,T) such that $[\lambda \wedge \text{int}(1 - \mu)] \leq \sigma \leq [\text{cl}(\lambda) \vee (1 - \mu)]$, in (X,T).

Proof : Let λ and μ be any two fuzzy sets defined on X in (X,T). Since (X,T) is a fuzzy hereditarily extremally disconnected space, by Theorem **2.10**, there exists a fuzzy closed set η and a fuzzy open set δ in (X,T) such that [λ∧int (1 − μ)]≤ η ≤ δ ≤ [cl(λ)∨(1 − μ)],in (X,T). Also since (X,T) is a fuzzy seminormal space and η ≤ δ , there exists a fuzzy regular open set σ in (X,T) such that η≤ σ ≤ δ . Then , it follows that[λ∧int (1 − μ)]≤ σ ≤ [cl(λ) ∨(1 − μ)] , in (X,T).

The following proposition gives a condition for fuzzy quasi regularand fuzzy extremally disconnected spaces to become fuzzy normal spaces.

Proposition4.11: If fuzzy regular open sets are disjoint in a fuzzy quasi-regular and fuzzy extremally disconnected space (X,T), then (X,T) is a fuzzy normal space.

Proof : The proof follows from Proposition**4.4** and Proposition 4.7.

Proposition4.12: If λ_1 and λ_2 are disjoint fuzzy closed sets in a fuzzy semi normal and fuzzy Oz-space (X,T), then there exist fuzzy G_δ -sets $\delta_1$and $\delta_2$in (X,T)such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1 - \delta_2$.

Proof : Suppose that λ_1 and λ_2 are disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy seminormal space, by Proposition **3.2 ,** there exist fuzzy regular closed sets δ_1 and δ_2 in (X,T) such that$\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with $\delta_1 \leq 1 - \delta_2$. Also since (X,T) is a fuzzy Oz- space , the fuzzy regular closed sets δ_1 and $\delta_2$are fuzzy G_δ -sets in (X,T). Hence for the disjoint fuzzy closed sets λ_1 and λ_2,there exist fuzzy G_δ -sets δ_1 and $\delta_2$in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with$\delta_1 \leq 1 - \delta_2$.

Proposition4.13: If a fuzzy topological space (X,T) is a fuzzy seminormal, fuzzy Oz and fuzzy P-space, then (X,T) is a fuzzy normal space.

Proof : Suppose that λ_1 and $\lambda_2$are disjoint fuzzy closed sets in (X,T). Since (X,T) is a fuzzy seminormal and fuzzy Oz- space , by Proposition **4.12,** there exist fuzzy G_δ -sets δ_1 and $\delta_2$in (X,T) such that $\lambda_1 \leq \delta_1$ and $\lambda_2 \leq \delta_2$ with$\delta_1 \leq 1 - \delta_2$. Since(X,T) is a fuzzy P-space, the fuzzy G_δ -sets δ_1 and δ_2 are fuzzy open in (X,T).

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