Stochastic Modelling and Computational Sciences

PROBLEM ON THE SHORTEST PATH OF TRIANGULAR FUZZY NUMBER RANKING

Jayalakshmi S[1] and Kamali R[2]
[1]Research Scholar and [2]Assistant Professor
Department of Mathematics, VISTAS Chennai, India
[1]Jayasriraghav72@gmail.com and [2]Kamali_1883@yahoo.co.in

ABSTRACT

We emerge a new technique with a fuzzy shortest path problem in which each length (or edge) is constituted as a triangular fuzzy number in a network. Most of the problem involves some uncertainty in real life. This paper suggests a new fuzzy number result and gives the shortest path distance from the starting vertex to the end vertex using the ranking method. Finally, we included an illustrative example of the suggested algorithm.

Keywords—*Fuzzy sets, shortest path length, Hamming distance*

1. INTRODUCTION

A graph is a suitable way to involve the connection between objects. Fuzzy graphs are planned to represent the formation of a connection between an entity (vertex) and a connection between two things (edge). The shortest path problem is used in different areas like transportation, communication routing and scheduling. Any network path may be considered as time or cost. The distance between the vertex is deemed to be uncertain fuzzy numbers. Rosenfeld [8] described fuzzy graphs' concepts, connectedness, cycles, and paths. Mordeson and Peng [7] explained fuzzy graphs operation and introduced their properties. Chung and Kung [1] suggested a length of fuzzy shortest path procedure to find the length of fuzzy path among all viable paths in a network. K. Yadhav and R. Biswas [5] studied the length of the shortest path of the fuzzy network.S. Broumi and A. Bakali [9] described the concept of the interval-valued trapezoidal problem and interval-valued triangular problem. Problem on the shortest path of neutrosophic fuzzy sets. Okada and Soper [12] developed the shortest path problem in a network with fuzzy arc lengths. This paper suggests finding the fuzzy shortest path on a crisp graph of the fuzzy weight, and the fuzzy number is edge weight.

2. PRELIMINARIES

Definition 2.1[5]: Let X be a finite nonempty set, then the fuzzy set is defined as $A = \{(t, \mu_A(t))/t \in X\}$ where $\mu_A: X \to [0.\ 1]$ and denotes the membership degree of set A.

Definition 2.2[5]: Consider the two triangular fuzzy numbers are $\lambda_1 = (\alpha_1, \beta_1, \gamma_1)$ and $\lambda_2 = (\alpha_2, \beta_2, \gamma_2)$, then operations of these two are

$$\lambda_1 + \lambda_2 = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2)$$

$$\lambda_1 - \lambda_2 = (\alpha_1 - \alpha_2, \beta_1 - \beta_2, \gamma_1 - \gamma_2)$$

$$\lambda_1 * \lambda_2 = (\alpha_1 * \alpha_1,\ \beta_1 * \beta_2,\ \gamma_1 * \gamma_2)$$

Definition 2.3[11]: The edges are directed, and then the graph is a digraph. Hence there is no cycle of a digraph.

Definition 2.4[14]: The triangular fuzzy number represents a fuzzy membership

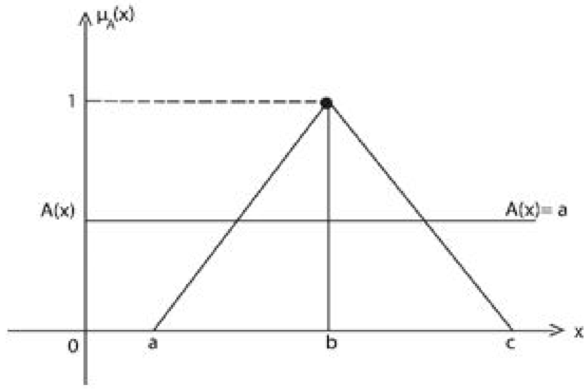

Fig.1 Triangular Fuzzy Number

If a is the lower bound value, c is the upper bound value, and b is the middle of the fuzzy number. Therefore, this function is described as the fuzzy membership

$$A(x) = \begin{cases} 0 & \text{if } x \le a_2 - a_1 \\ \dfrac{x - (a_2 - a_3)}{a_1} & \text{iff } - a_1 < x < m \\ 1 & \text{if } x = a_2 \\ \dfrac{(a_2 + a_3) - x}{a_3} & \text{iff } < x < a_2 - a_3 \\ 0 & \text{if } x \ge a_2 + a_3 \end{cases}$$

3. THE FUZZY SHORTEST PATH

We examine the graph G = (V, E) as a directed network involving a set of finite vertices V = {A, B, …., n}and a set of finite edges $E \subseteq V \times V$. A triad (a_i, b_i, c_i) is denoted by each edge. A path between the vertex u and v in a graph G = (V, E), where V is the vertices and E is the edge set in a sequence of vertices $(u, x_1, x_2, \ldots\ldots, x_n, v)$ such that $(u, x_1), (x_1, x_2), (x_2, x_3), \ldots\ldots, (x_n, y)$ are the edges of E. The path length is the sum of the edge length on the path. The distance between two vertices, u and v, is the length of the shortest path among all other paths. Now, each edge length is a triangular fuzzy number, and the path length between two vertices will be a number of the same aspect. Since the shortest path indicates a comparison between the length of the distances, comparing this number is required for a suitable ranking method. Here the shortest path represents the problem on the fuzzy shortest path. The fuzzy shortest path's formula on described path lengths uses the membership function.

3.1An approach of Fuzzy Number in shortest path length

Collect all attainable paths $P_i, i = 1, 2, \ldots., n$.from the starting vertex S to the end vertex E, and path length $L_j, j = 1, 2, \ldots., n$ of triangular fuzzy numbers, respectively. where $L_j = (a_j, b_j, c_j)$

$$L_{min} = (a, b, c) = (a', b', c')$$

$$a = \min(a', a_1)$$

$$b = \begin{cases} b' & \text{if } b' \le a_1 \\ \dfrac{(b' \times b_1) - (a' \times a_1)}{(b' + b_1) - (a' + a_1)} & \text{if } b' > a_1 \end{cases}$$

$c = \min(c', b_1)$

we defined $L_{min} = (a, b, c)$as follows:

$L_{min} = min\ (a_i, b_i, c_i), i\ =\ 1\ to\ n$

In a practical situation, how to analyze between two sets and groups. We need to utilize the degree of similarity between two fuzzy sets. A new method of similarity degrees between two triangular fuzzy numbers is introduced. The intersection region to compute the similarity degree between L_j and $L_{min}$of two triangular fuzzy numbers. Let jth path length $L_j\ =\ (a_j, b_j, c_j)$ and $L_{min} = (a, b, c)$, then the similarity degree (Hamming distance) can be calculated as

$$D\ (L_j, L_{min}) = \sum_{i=1}^{m} |\mu_j(x_i)\ -\ \mu_{min}(x_{min})|, j = 1, 2, \ldots, n$$
(3.1.1)

3.2 The Fuzzy shortest path Algorithm

Step 1: Observe all the viable paths from the starting vertex to the terminal vertex. Find the sum of all possible path P_k, $k\ =\ 1, 2, \ldots., n$

Step 2: To calculate L_{min} = min (a_i, b_i, c_i) = (a', b', c') by using the length of the shortest path approach

Step 3: Calculate the Hamming distance D (A, B) between all the possible paths L_j and L_{min}.

Therefore $D\ (L_{min}, L_j), j =\ 1, 2, \ldots, n$

Step 4: The distance which is minimum to mark the shortest path by ranking in ascending order.

3.3 Numerical Example

To illustrate the above procedure, consider six vertices and eight edges on a network, as given in fig. 3. The classical weighted graph with a triangular fuzzy number represents each arc length, as shown in Table 1

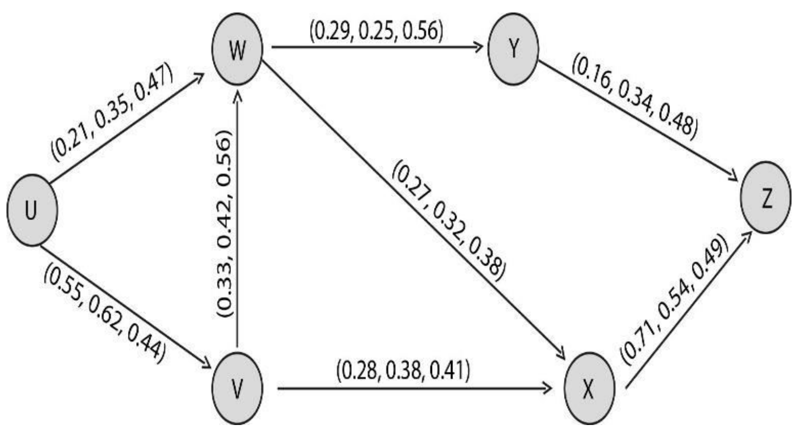

Fig. 3 Triangular Fuzzy Network

Table. 1 Weight of the Triangular Fuzzy Graph

Edges	Triangular Fuzzy Distance
U – V	(0.21, 0.35, 0.47)
U – W	(0.55, 0.62, 0.44)
V – W	(0.33, 0.42, 0.56)
V – X	(0.28, 0.38, 0.41)

W – X	(0.27, 0.32, 0.38)
W – Y	(0.29, 0.25, 0.56)
X – Z	(0.71, 0.54, 0.49)
Y – Z	(0.16, 0.34, 0.48)

Iteration 1: Consider all the possible paths

P_1: U → V → X → Z : L_1 = (1.2, 1.27, 1.37)

P_2: U → V → W → Y → Z : L_2 = (0.99, 1.36, 2.07)

P_3: U → V → W → X → Z : L_3 =(1.52, 1.63, 1.9)

P_4: U → W → Y → Z : L_4 = (1.0, 1.21, 1.48)

P_5: U → W → X → Z : L_5 = (1.53, 1.48, 1.28)

Iteration 2: $L_{min} = (0.99, 1.21, 1.28)$

Iteration 3: Using the fuzzy Hamming distance, the absolute value of the corresponding membership values is used to determine the shortest fuzzy distance.

$$D(L_j, L_{min}) = \sum |\mu_j(x_i) - \mu_{min}(x_{min})|$$

$D(L_1, L_{min})$ = |1.2- 0.99|+|1.27-1.21|+|1.37-1.28|

= 0.21 + 0.06 + 0.09 = 0.36

$D(L_2, L_{min})$ =|0.99 - 0.99| + |1.36 - 1.21| + |2.07-1.28|

= 0.00 + 0.12 + 0.79 = 0.91

$D(L_3, L_{min})$ = |1.52 - 0.99| + |1.63 - 1.21| + |1.90-1.28|

= 0.53 + 0.42 + 0.62 = 1.57

$D(L_4, L_{min})$ =|1.0 - 0.99| + |1.21 - 1.21| + |1.48 - 1.28|

= 0.01 + 0.00 +0.20 = 0.21

$D(L_5, L_{min})$ =|1.53 - 0.99| + |1.48 - 1.21| + |1.28 - 1.28|

= 0.54 + 0.27 +0.00 = 0.81

Iteration 4: Available paths and its rank

Table 2. Triangular fuzzy distance, shortest path and rank

The proposed algorithm	Path length	Ranking
U → V → X → Z	0.36	2
U → V → W → Y → Z	0.91	4
U → V → W → X → Z	1.57	5
U → W → Y → Z	0.21	1
U → W → X → Z	0.81	3

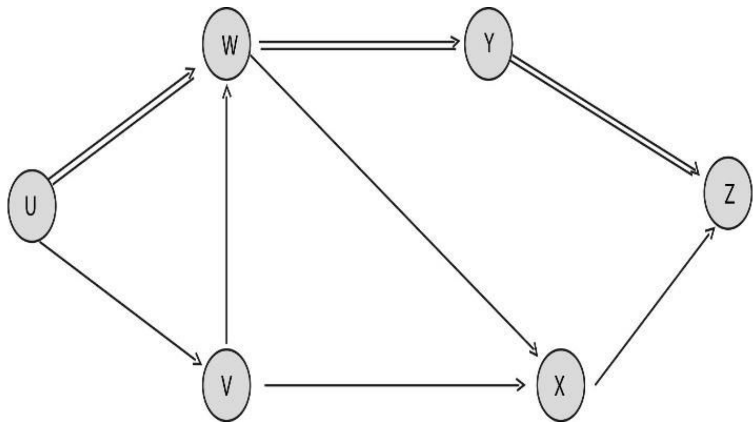

Fig. 4 Fuzzy Shortest Path

Now we need to express the shortest path by calculating the fuzzy Hamming distance between path length $L_j, j = 1, 2, \ldots, 5$ and L_{min}. By using the above calculation (3.1.1), we see that

$U \rightarrow W \rightarrow Y \rightarrow Z$ is the shortest path.

CONCLUSION

This problem is concluded to compute the shortest path among all other paths. An algorithm for putting a network on the shortest path with triangular fuzzy numbers as edges. The rules can assist decision-makers in making decisions. We presented a new approach to approximate numbers by this method, and an example is defined using theoretical data.

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