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GOVERNING DYNAMICS FOR A MULTI-HOP NETWORK UNDER NOISY CONDITIONS –AN ANALOGY TO CHARGED PARTICLE MOVING THROUGH A FLUID UNDER TRANSVERSE VARIABLE ELECTROMAGNETIC FIELD

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ABSTRACT

Any network where a packet has to go through multiple nodes, where it would encounter queues, noise and other interferences. The governing dynamics for a packet like that needs to be derived. It can be analysed as an analogue to a charged particle moving through an electrohydrodynamic fluid under the influence of electric and magnetic fields. In the previous works, such an attempt has been made considering the effect of electric and magnetic fields exclusively. They have provided sable results with discernible accuracy. In this paper, the effect of electromagnetic field is considered as the cause for perturbation in the flow of a particle and an analogue has been drawn to a packet in a multi-hop network.

Keywords: *computer network, charged particle, electromagnetic field, electrohydrodynamics, multi-hop network.*

INTRODUCTION

Networking has changed the way of life forever in an irreversible manner. One cannot imagine life without connectivity in the current scenario. Analysis of networks and design to meet the requirements has always been dependent extensively on various parameters such as noise, distance, cost and even probability of loss of packets. Thus, the analysis would never be a complete one and always been prone to errors. Hence networks never offer a closed solution for their services. The best solutions offered are usually best effort services which offer a sufficient error rate of $1X10^{-6}$. This is sufficient for working conditions. However, analytics require closed form solutions to run accurate diagnostics and design procedures. Hence, it is imperative to find a closed system analogy for the same and provide clear solution. In this paper, an attempt has been made to analyse a network as a fluid with a packet being analogised to a charged particle moving through the fluid under the influence of electric and magnetic fields.

LITERATURE REVIEW

Viscous flow and Rayleigh equation in inviscid flow, has been used to derive Orr-Somerfield equation in literature by Hyde [1], Howard [2][3], and Kent [4][5].

The hydrodynamic stability of heterogeneous inviscid fluid studied by Lynn [6], Drazin [7], Taylor [8], Goldstein [9], Synge [10], Miles [11] and Howard [12] propose their use in physical fluidic systems.

Stuart [13] and Lock [14].used magnetic field to suppress the onset of instability in fluids. Later, Rudraiah [16][17][18] etal continued the work of Synge [10] by applying a transverse magnetic field on a particle travelling in an electrically charged fluid pushed by electrical fields. This work is extended in this research paper.

Kelvin- Helmholtz instability (KHI) and Richtmeyer-Meshkov instability (RMI) at the interfaces of two fluids have studied by Rudraiah [19]. Melcher and Taylor [8] and Lee et al [20], Lee [21], Roberts [22], Rudraiah etal [23] have further improved the work by studying the stress at the interface of two fluids for EHD fluid flow.

Baygents and Baldessare [24] investigated EHD stability in a thin fluid layer that causes a noiseless flow, which brought fopcus onto noiseless fluid flow.

Reynolds [25] and Orr [26] extensively studied the functional parameters proposed to assess EHD fluid flow. Rudraiah [27] continued this research and developed a system to compute the parameters accurately.

Harsha S and ShubhaNagaraj [28] have presented the energy requirement to neutralize a packet loss in a noisy network where the fluid is the network and longitudinal electrodes are the source of the packet and transverse electrodes are the source of the noise.

METHODS AND MATERIALS

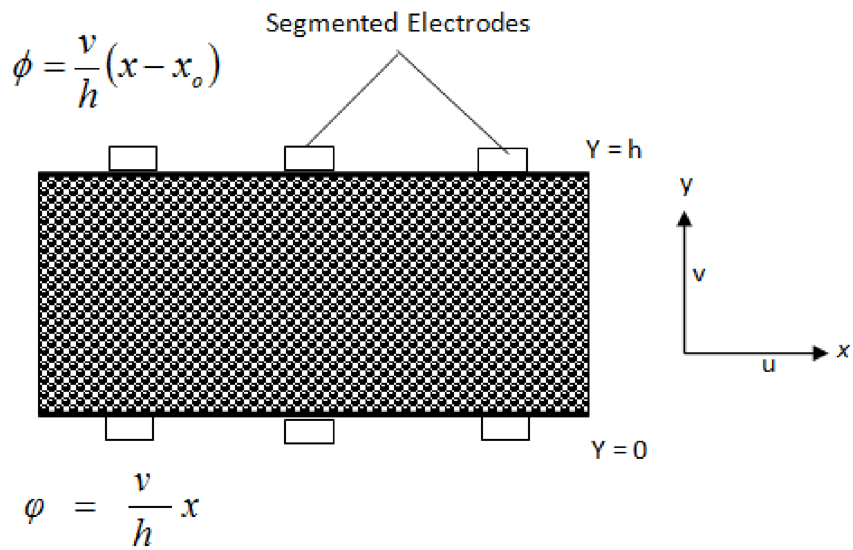

Figure 2. Physical Configuration

In this case Darcy-Lapwood-Brinkmann equation governing a poorly conducting fluid in the presence of electric and magnetic fields is utilized. For this physical configuration, the required basic equations i.e. conservation of mass and electric charges, considering the combined effect of electric and magnetic fields in a porous medium assuming Brinkmann viscosity $\bar{\mu} = \mu$, the viscosity of fluid, is:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_e}{\rho_0}\vec{E} + \frac{\mu}{\rho_0}\nabla^2\vec{q} + \frac{1}{\rho_0}\vec{J}\times\vec{B} - \frac{\mu}{k}\vec{q} \qquad (1)$$

These equations have to be supplemented with suitable boundary and initial conditions as follows.

Equation (1) using $\vec{J} = \sigma\vec{E} + \mu_m\sigma\,\vec{q}\times\vec{H}$ can be written as

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_e}{\rho_0}\vec{E} + \frac{\mu}{\rho_0}\nabla^2\vec{q} + \frac{\mu_m\sigma}{\rho_0}\vec{E}\times\vec{H} + \frac{\mu_m^2\sigma}{\rho_0}\vec{q}\times\vec{H}\times\vec{H} - \frac{\mu}{k}\vec{q} \qquad (2)$$

Assuming two-dimensional flow as in previous chapters, Under the approximations from (1) and (2) the basic equations along with Maxwell`s equation, for a poorly electrically conducting, viscous, incompressible two dimensional homogeneous fluid saturated porous layer in the presence of electric and magnetic fields, after making them dimensionless are the same for conservation of mass, conservation of electric charges and Maxwell`s equations. Conservation of Momentum takes the form

$$\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + W_1\rho_e E_x + \nabla^2 u - W_3\sigma E_y - M^2 u\sigma - A\sigma_p^2 u \qquad (3)$$

$$\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + W_1(\rho_e E_y) + \nabla^2 v - W_3\sigma E_x - M^2 v\sigma - A\sigma_p^2 v \qquad (4)$$

where $W_1 = \dfrac{\varepsilon_0 v^2}{\rho_0 U^2 h^2}$ is the Electric Number which has the same meaning as explained earlier. $M^2 = \dfrac{\mu_m^2 \sigma h^2 H_0^2}{\rho_0 U^2 h^2}$ is the Hartmann number which is a measure of the ratio of Lorentz force to Viscous force. $W_3 = \dfrac{v}{\mu H_0 U h}$ is the ratio of energies. $A.\sigma_p^2 = \dfrac{\mu}{\rho_0 \upsilon} . \dfrac{h^2}{k}$ is the porous parameter.

From the initial conditions, considering the usual basic state, substituting that basic state into (3), (4) and to Conservation of electric charge equation and then simplifying,

$$\frac{d^2 u_b}{dy^2} - M^2 u_b \sigma_b = \frac{\partial p_b}{\partial x} + W_1 \rho_{e_b} E_{bx} - W_3 \sigma_b E_{by} - A\sigma_p^2 u \tag{5}$$

$$\frac{\partial^2 \phi_b}{\partial y^2} + \alpha \frac{\partial \phi_b}{\partial y} = 0 \tag{6}$$

where $\rho_{eb} = -\dfrac{\partial^2 \phi_b}{\partial y^2}, W_1 = \dfrac{\varepsilon_0 v^2}{\rho_0 U^2 h^2}$ is the electric number which physically represents the ratio of electric energy to kinetic energy. Solution of (6) using the boundary conditions is,

$$\phi_b = x - \frac{x_0\left(1 - e^{-\alpha y}\right)}{\left(1 - e^{-\alpha}\right)} \tag{7}$$

Then the solution of (5) using (8.3.3) and the no-slip boundary conditions is

$$u_b = -\frac{P}{M^2 - A\sigma_p^2} y + \frac{x_0 W_1}{M^2 - A\sigma_p^2} + Ae^{My}\left[1 + \frac{y^2}{4}\left(M - \frac{1}{y}\right)\right] + Be^{-My}\left[1 - \frac{y}{4}\left(M - \frac{1}{y}\right)\right] \tag{8}$$

where

$$P = \frac{\partial pb}{\partial x} \quad A = \frac{4e^{-M}}{M+3}\left[\frac{e^{-M}(5-M)}{4}\left\{\frac{-4x_0 W_1\left\{e^M(M+3)-1\right\} + P}{M^2\left\{e^M(M+3) + e^{-M}(5-M)\right\}}\right\} - \frac{x_0 W_1}{\left[M^2 - A\sigma_p^2\right]} + \frac{P}{\left[M^2 - A\sigma_p^2\right]}\right]$$

$$B = \frac{-4x_0 W_1\left[e^M(M+3)-1\right] + P}{\left[M^2 - A\sigma_p^2\right]\left\{e^M(M+3) + e^{-M}(5-M)\right\}} .$$

To study the linear stability of the basic state, superimposing infinitesimal symmetrical disturbances and substituting these into. (3) and (4) and linearizing them by neglecting the product of higher order terms in perturbed quantities and for simplicity neglecting the primes,

$$\left[\frac{\partial u}{\partial t} + u_b \frac{\partial u}{\partial x} + v \frac{\partial u_b}{\partial x}\right] = -\frac{\partial p}{\partial x} + \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + W_1 \rho_{e_b} E_x + W_3 E_y - M^2 u - A\sigma_p^2 u \tag{9}$$

$$\left[\frac{\partial v}{\partial t} + u_b \frac{\partial v}{\partial x}\right] = -\frac{\partial p}{\partial y} + \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + W_1\left(\rho_{e_b} E_y + \rho_e E_{b_y}\right) - W_3 E_x + M^2 v - A\sigma_p^2 v \tag{10}$$

Eliminating the pressure between the eqns. (7) and (8) and using the stream function defined in eqn. (9) and using the normal mode solution of the form given by eqn. (10), and after some simplification the stability equation takes the following form

$$\left(D^2-l^2\right)^2\psi - il\left(u_b-C\right)\left(D^2-l^2\right)\psi + ilD^2u_b\psi + ilW_1x_0\left(D^2-l^2\right)\phi \\ -W_3\left(D^2-l^2\right)\phi - M^2\left(D^2-l^2\right)\psi + A\sigma_p^2\left(D^2-l^2\right)\psi = 0 \tag{11}$$

Similarly, the equation of continuity of charges takes the form

$$il\left(u_b-C\right)\left(D^2-l^2\right)\phi = -\alpha^2x_0\psi \tag{12}$$

Substituting $\left(D^2-l^2\right)\phi$ from (8.4.4) into (8.4.3) and after simplification,

$$\left(D^2-l^2\right)^2\psi - il\left(u_b-C\right)\left(D^2-l^2\right)\psi + ilD^2u_b\psi - il\frac{W_1x_0^2\alpha^2}{\left(u_b-C\right)}\psi + \frac{W_3x_0\alpha^2\psi}{\left(u_b-C\right)} \\ -M^2\left(D^2-l^2\right)\psi + A\sigma_p^2\left(D^2-l^2\right)\psi = 0 \tag{13}$$

Equation (13) is a modified form of Orr-Sommerfeld Equation, modified in the sense of incorporating the contribution from the electric force, $\rho_e\,\vec{E}$ and the effect of magnetic field.

Based on equations (1) through (13), it can be theorised that,

A sufficient condition for Electromagnet hydrodynamic stability [EMHDS] of viscous homogeneous poorly conducting fluid saturated porous medium in the presence of electric field and magnetic field is

$$A \le \frac{\left[I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2\right]-lqI_0I_1}{I^2}$$

together with

$$lq < \frac{\left[I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2\right]}{I_0I_1}$$

Proof:

To prove the above theory, one needs to study the stability of a poorly conducting parallel flow. Complex conjugate method has been used for this purpose. Consider the modified Orr-Sommerfeld eqn.

$$\left(D^2-l^2\right)^2\psi = il\left[\left(u_b-C\right)\left(D^2-l^2\right)\psi - D^2u_b\psi + \frac{W_1x_0^2\alpha^2}{\left(u_b-C\right)}\psi\right] - \frac{W_3x_0\alpha^2\psi}{\left(u_b-C\right)} \\ +M^2\left(D^2-l^2\right)\psi - A\sigma_p^2\left(D^2-l^2\right)\psi = 0 \tag{14}$$

Multiplying (14) by ψ^*, the complex conjugate of ψ and integrating from 0 to 1 with respect to *y* using the boundary condition,

$$I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2 = -ilQ + ilC\left[I_1^2+l^2I_0^2\right] \tag{15}$$

where $I_n^2 = \int_0^1 \left|D^n\psi\right|^2 dy \qquad (n = 0, 1, 2)$

$$Q = \int_0^1 \left\{ u_b\left|D\psi\right|^2 + \left(l^2 u_b + u_b''\right)\left|\psi\right|^2 \right\} dy + \int_0^1 u_b'\left(D\psi\right)\psi^* dy - W_1 x_0^2 \alpha^2 \int_0^1 \frac{\left(u_b - C_r\right) + iC_i}{u_b - C} \left|\psi\right|^2 dy$$
$$- W_3 x_0 \alpha^2 \int_0^1 \frac{\left(u_b - C_r\right) + iC_i}{u_b - C} \left|\psi\right|^2 dy$$

(16)

From this equation $Re(Q)$ and $Im(Q)$ are obtained in the form

$$Re(Q) = \int_0^1 \left\{ u_b\left|D\psi\right|^2 + \left(l^2 u_b + u_b''\right)\left|\psi\right|^2 \right\} dy - \int_0^1 \frac{\left(W_1 x_0 + W_3\right)x_0\alpha^2\left(u_b - C_r\right)}{\left|u_b - C\right|^2} \left|\psi\right|^2 dy \tag{17}$$

and

$$Im(Q) = \frac{1}{2} i \int_0^1 u_b' \left\{\psi\left(D\psi^*\right) - \psi^*\left(D\psi\right)\right\} dy + \int_0^1 \frac{\left(W_1 x_0 + W_3\right)x_0\alpha^2 C_i}{\left|u_b - C\right|^2} \left|\psi\right|^2 dy \tag{18}$$

Equating the real parts and imaginary parts of the eqn. (15) using eqns. (16) and (19) and $C = C_r + iC_i$, from the real parts,

$$C_i = \frac{l\int_0^1 u_b'\left\{\psi\left(D\psi^*\right) - \psi^*\left(D\psi\right)\right\}dy - \left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{l\left[I_1^2 + l^2 I_0^2\right] + A_1 I^2} \tag{19}$$

where $A_1 = \left(W_1 x_0 + W_3\right)x_0\alpha^2$ and $I^2 = \int_0^1 \frac{\left|\psi\right|^2}{\left|u_b - C\right|^2} dy$

This eqn. (19) can be rewritten in the form

$$C_i = \frac{l\left|Im(Q)\right| - \left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{l\left[I_1^2 + l^2 I_0^2\right] + A_1 I^2} \tag{20}$$

By equating the imaginary parts we get,

$$C_r = \frac{\int_0^1 \left\{ u_b\left|D\psi\right|^2 + \left(\left(l^2 - \frac{\left(W_1 x_0 + W_3\right)x_0\alpha^2}{\left|u_b - C\right|^2}\right)u_b + u_b''\right)\left|\psi\right|^2 \right\} dy}{I_1^2 + l^2 I_0^2 - A_1 I^2} \tag{21}$$

This eqn. can be rewritten in the form

$$C_r = \frac{Re(Q)}{I_1^2 + l^2 I_0^2 - A_1 I^2} \qquad (22)$$

Eqn. (19) is the growth rate of the perturbations and physically it represents the energy equation for two-dimensional disturbances propagating in the direction of the basic flow. Similarly, C_r given in eqn. (22) represents the phase velocity of the disturbances.

$$|Im(Q)| \leq \int_0^1 |u_b'| \, .|\psi| \, .|D\psi| dy + \int_0^1 A_1 |\psi|^2 \qquad (23)$$

Hence, by Schwarz's inequality,

$$|Im(Q)| \leq qI_0 I_1 + A_1 I^2 \qquad (24)$$

where $q = \max\limits_{0 \leq y \leq 1} |u_b'|$

This gives an upper bound for C_i

$$C_i \leq \frac{lqI_0 I_1 + A_1 I^2 - \left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{l\left[I_1^2 + l^2 I_0^2\right] + A_1 I^2} \qquad (25)$$

from which it follows that a sufficient condition for stability is

$$A_1 \leq \frac{\left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right] - lqI_0 I_1}{I^2} \qquad (26)$$

together with

$$lq < \frac{\left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{I_0 I_1} \qquad (27)$$

Where A_1 is the porous parameter and lq is the length of traversal.

INFERENCES

From the equations (26) and (27) it can be seen that, the hypothesis shown in the methods, is true and holds good for stable motion of a particle in an incompressible fluid under the influence of variable electromagnetic fields. Thus with the simulation for the equations (26) and (27)

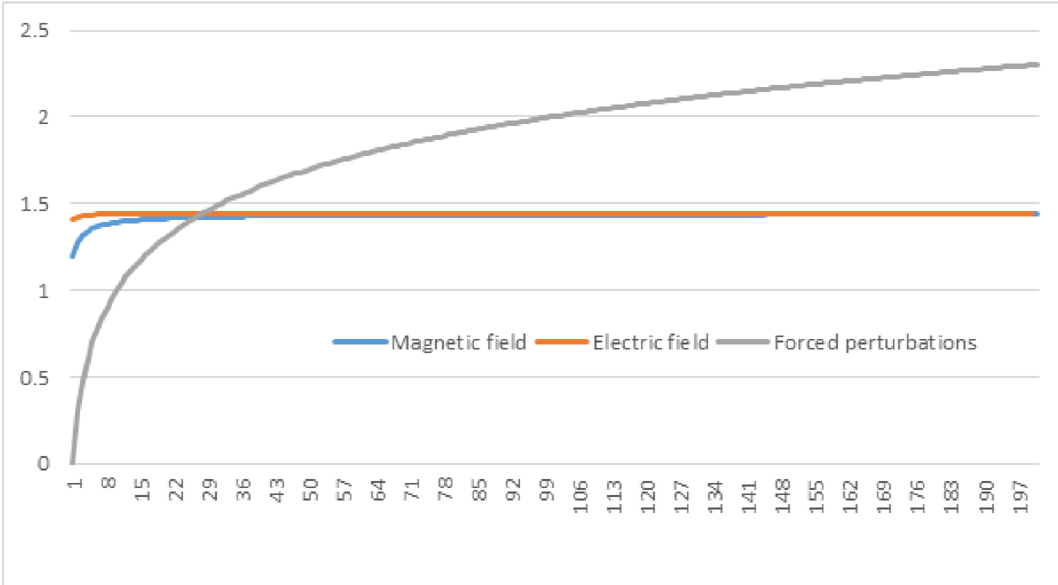

Figure 3. Graph from simulating the equations 26 and 27 for a network

From the graph in Figure 3. It can be observed that, when a particle moves through the fluid, the forced perturbations in the electric field as well as magnetic field, have little or no effect on the motion as the energy increases. The movement is stable over time. Thus, with a constant supply of energy to the routing system, a packet can traverse through a multi-hop network with multiple sources of noise. Thus, noise as per the analogy that has been hypothesised, has no effect on the packets if the packets are transmitted with increased energy.

CONCLUSION

To obtain stable particle movement, the sufficient conditions for the packet forwarding and the hop length are defined by equations (26) and (27) and the same have been simulated and plotted in figure3. The hypothesis is proven successfully and the analogy holds good for a multi hop network under the influence of noise. A packet that has been sufficiently energised will be able to survive the traversal through a network under noisy conditions.

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