TO EXAMINE THE FREE VIBRATION OF NON-HOMOGENEOUS VISCO-ELASTIC SKEW PLATE WITH THICKNESS AND THERMAL EFFECT

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ABSTRACT
An analytical approach for free mechanical vibration examination of four edges simply supported skew plates is presented. The classical (thin) plate theory is used to study vibration of plates in the present study. In present paper a simple model is presented to study the effect of temperature with bi-linear thickness variation on a visco-elastic plate. An expected but suitable frequency equation is resulting by Rayleigh-Ritz method with two term deflection function. The frequencies corresponding to the first two modes of vibration has been calculated for a simple supported visco-elastic skew plate for various values of taper constant and thermal gradient with the help of MAPPLE software. (today's computational software).

Keywords: Vibration, skew plate, thickness, taper constant, thermal gradient, non-homogeneity constant.

INTRODUCTION
In the engineering, all machines and engineering structures experiences vibrations so we cannot move without considering the effect of vibration. With the advancement of technology, the necessity to know the effect of temperature on visco-elastic plates of variable thickness has become crucial. Tapered Plates with uniform and non-uniform thickness and temperature are widely used in marine structure, aeronautical field, power plants, automobile sector etc. Various researchers studied the vibration behavior of homogeneous or non-homogeneous plates with variable thickness, with or without consideration of temperature effects. An extensive review on linear vibration of plates has been given by Leissa [1] in his monograph and a series of review articles [2]. Tomar and Gupta [3] studied the effect of taper constants in two directions on elastic plates, but not on visco-elastic plates. Bhatnagar and Gupta [4] studied the effect of thermal gradient on vibration of a visco- elastic circular plate of variable thickness. Gupta and Khanna [5] studied the effect of linearly varying thickness in both directions on vibration of a visco- elastic rectangular plate. Gupta and Khanna [6] studied the Vibration of clamped visco-elastic rectangular plate with parabolic thickness variations. Gupta and Khanna [7] analyzed free vibration of clamped rectangular plate with bi-direction exponentially thickness variations. Khanna and Sharma [8] have been studied on Vibration Analysis of Visco-Elastic Square. Plate of Variable Thickness with Thermal Gradient. Khanna & Sharma [9] Studied natural vibration of visco-elastic plate of varying thickness with thermal effect. Khanna & Sharma [10] studied Analysis of free vibrations of visco-elastic square plate of variable thickness with temperature effect. Khanna & Sharma [11] analyzed a computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect. Sharma, Raghav & Sharma [12] presented the study of a Modeling on frequency of Rectangular Plate. Sharma, Raghav & Sharma [13] represented the Vibrational study of Square Plate with Thermal Effect and Circular Variation in density. In present paper, the authors have studied the thermal effect on the bi-linear vibration of visco-elastic skew plate whose thickness and thermal effect vary bi-linearly in x direction. Also, it is supposed that the plate is simply supported on all the four edges. Due to temperature deviation, we suppose that non homogeneity occurs in modulus of elasticity. Frequency for the first two modes of vibration is obtained for various numerical values of thermal gradient, tapering constant and non-homogenous constant. Results are presented in graphical form.

MATHEMATICAL ANALYSIS
The parallelogram (skew) plate is assumed to be non-uniform, thin and orthotropic and the plate R be defined by the three number a, b and θ.

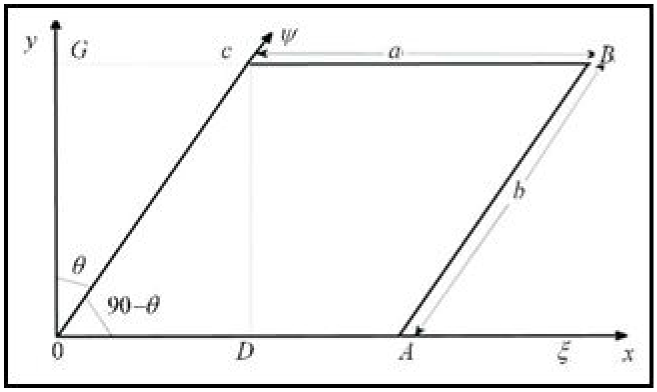

Figure a: The parallelogram plate with skew angle θ

The differential equation of motion and time function for visco elastic plate with thickness variation is given by

$[D_1 (w_{,xxxx} + 2 w_{,xxyy} + w_{,yyyy}) + 2 D_{1,x} (w_{,xxx} + w_{,xyy}) + 2 w_{,xxxx} + 2 D_{1,y}(w_{,yyy} + w_{,yxx}) + 2 D_{1,xx}(w_{,yyy} + w_{,yxx}) + D_{1,xx} (w_{,xx} + \nu w_{,yy}) + D_{1,yy} (w_{,yy} + \nu w_{,xx}) + 2 (1-\nu)D_{1,xy} w_{,xy}] - \rho k^2 l w = 0$ (1)

$\ddot{T} + k^2 D\dot{T} = 0$ (2)

Here, comma followed by suffix is known as partial derivative of W with respect to independent variable and double do represent the second derivative with respect to t. Also $D_1 = \frac{yl^3}{12(1-\nu)^2}$ is called flexural rigidity of the plate.

Now the expression for the kinetic energy (M_E) and the strain energy (N_E) is given by:

$M_E = \frac{1}{2}\, \omega^2\, \rho \iint l\, W^2\, dydx$ (3)

and

$N_E = \frac{1}{2} \iint D_1 \{(W_{,xx})^2 + (W_{,yy})^2 + 2\nu W_{,xx} W_{,yy} + 2(1-\nu)(W_{,xy})^2\}\, dydx$ (4)

The parallelogram (skew) plate is assumed to be non-uniform, thin and orthotropic and the plate R be defined by the three number a, b and θ.

The skew coordinates of the plate are:

$\xi = x - y \tan\theta,\ \varphi = y \sec\theta$ (5)

The boundary condition of the plate in skew coordinates are :

$\xi = 0\ , \xi = a\ \ and\ \ \varphi = 0\ , \varphi = b$ (6)

Using eqn. (5), the equation of K.E. (3) and Strain energy (4) will become:

$M_E = \frac{1}{2}\, k^2\, \rho \cos\theta \int_0^b \int_0^a l\, W^2\, d\xi d\varphi$ (7)

$N_E = \frac{1}{2} \int_0^b \int_0^a D_1 [\ (W_{,\xi\xi})^2 - 4\sin\theta\, (W_{,\xi\xi})(W_{,\xi\varphi}) + 2\, (\sin^2\theta + \nu \cos^2\theta)(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\, (1 + \sin^2\theta - \nu \cos^2\theta)\, (W_{,\xi\varphi})^2 - 4\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + (W_{,\varphi\varphi})^2] d\xi d\varphi$

(8)

ASSUMPTIONS

The thickness of the plate is assumed to be bi-linear in two dimensions.

$$g = g_0\,[1 + \beta_1 (1- \sqrt{1-\frac{\xi}{a}})]\,[1 + \beta_2 (1- \sqrt{1-\frac{\varphi}{b}})] \qquad (9)$$

Where β_1, β_2 is tapering constant. Thickness of the plate becomes constant at $\xi = 0$, $\varphi = 0$.

We consider plate's material to be non-homogeneous. Therefore, either density or Poisson's ratio varies circularly in one dimensions as :

$$\nu = \nu_0\,[1 - m\,(1- \sqrt{1-\frac{\xi}{a}})] \qquad (10)$$

Where m is known as non-homogeneity constant. Poisson's ratio becomes constant i.e. $\nu = \nu_0$ at $\xi = 0$, $\varphi = 0$.

The temperature variation on the plate is considered to be to bi-linear in ξ direction and bi- linear in φ direction as :

$$\eta = \eta_0\,[(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}})] \qquad (11)$$

Where η and η_0 denotes the temperature excess above the reference temperature on the plate at any point and at the origin the temperature dependence modulus of elasticity for engineering structures is given by:

$$Y = Y_0\,(1 - \gamma\,\eta) \qquad (12)$$

Where Y_0 is the Young's Modulus at mentioned temperature (i.e. $\eta = 0$) and γ is called slope of variation.

Using equation (11) in equation (12), we get:

$$Y = Y_0\,[1 - \gamma\,(\eta_0 (\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}}))]$$

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Or $$Y = Y_0\,[1 - \alpha(\sqrt{1-\frac{\xi^2}{a^2}})\,(\sqrt{1-\frac{\varphi^2}{b^2}}))] \qquad (13)$$

Where α, $(0 \le \alpha < 1)$ is called temperature, which is the product of temperature at origin and γ slope of variation i.e. gradient $\alpha = \gamma\eta_0$

Using equation (9), (10) and (13), flexural rigidity i.e. $D_1 = \frac{yl^3}{(1-\nu)^2}$ of the plate becomes:

$$D_1 = \frac{Y0\ [1 - \alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))]l0\ \left[\left[\ 1+\beta1\left(1-\sqrt{1-\frac{\xi}{a}}\right)\right]\left[\ 1+\beta2\left(1-\sqrt{1-\frac{\varphi}{b}}\right)\right]\right]^3}{12(1-\nu_0{}^2\left[1-m\left(1-\sqrt{1-\frac{\xi}{a}}\ \right)\right]^2)} \qquad (14)$$

Using (9), (10) and (14), the eqn. of K.E. and Strain Energy becomes:

$M_E = \frac{1}{2} k^2 \rho l_0 \int_0^b \int_0^a (1+\beta_1 C1)(1+\beta_2 C2)\ W^2 d\xi d\varphi$ (15) N_E =

$$\frac{y_0 l_0}{24\cos^4\theta} \int_0^b \int_0^a \left[\frac{\left[\left[1-\alpha\left(\sqrt{1-\frac{\xi^2}{a^2}}\right)\left(\sqrt{1-\frac{\varphi^2}{b^2}}\right)\right]\right]\left[(1+\beta_1 C1)(1+\beta_2 C2)\right]^3}{(1-\nu_0{}^2\left(\left[1-m\,C1\right]^2\right)}\right]$$

$[(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta\,(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t[1 - mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2(1+\sin^2\theta - \nu_0[1 - m\,C1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]]d\xi d\varphi$

(16)

Where,

$C1 = (1-\sqrt{1-\frac{\xi}{a}})$, $C2 = (1-\sqrt{1-\frac{\varphi}{b}})$

In this paper, we are calculating first two mode of vibration on clamped boundary condition, therefore we have:

$W=W,_\xi = 0$ at $\xi = 0, a$

$W=W,_\varphi = 0$ at $\varphi = 0, b$ (17)

Hence, the two term deflection function, which satisfies eqn. (17), is:

$W(\xi,\varphi) = [B1\left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2 + B_2\left(\frac{\xi}{a}\right)^3\left(\frac{\varphi}{b}\right)^3\left(1-\frac{\xi}{a}\right)^3\left(1-\frac{\varphi}{b}\right)^3]$

$= \left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2[B1 + B_2\left(\frac{\xi}{a}\right)\left(\frac{\varphi}{b}\right)\left(1-\frac{\xi}{a}\right)(1-\frac{\varphi}{b})]$ (18)

Where B_1 and B_2 are arbitrary constant.

Solution for frequency equation by Rayleigh-Ritz method

We used Rayleigh-Ritz method to solve frequency equation and frequency mode i.e. in Rayleigh-Ritz method maximum kinetic energy must be equal to maximum strain energy.

Hence we have:

$\delta(N_E - M_E) = 0$ (19)

Using equation (15) and (16), we get:

$\delta(N_E{}^* - \lambda^2 M_E{}^*) = 0$ (20)

Where,

$M_E{}^* = \int_0^b \int_0^a (1+\beta_1 C1)(1+\beta_2 C2)\ W^2 d\xi d\varphi$ (21)

And

$$N_E^* = \frac{1}{\cos^4\theta}\int_0^b\int_0^a \left\{\left[\frac{\left[1-\alpha\left(1-\frac{\xi}{a}\right)\left(1-\frac{\varphi}{b}\right)\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0{}^2([1-mC1])^2}\right]\right.$$

$$[(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta\,(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t\,[1-mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) +$$

$$2\left(\frac{a}{b}\right)^2(1+\sin^2\theta - \nu_0\,[1-mC1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) +$$

$$\left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]\}d\xi d\varphi \qquad (22)$$

and $\lambda 2 = \frac{12\omega^2 a^4 \rho}{Y0\ell_0{}^2}$ is known as frequency parameter.

Equation (20) consists of two unknown constants which are obtained by the substitution of W and these constant can be evaluated by the following formula:

$$\frac{\partial}{\partial B_1}(N_E{}^* - \lambda^2 M_E{}^*) = 0\,,\ \frac{\partial}{\partial B_2}(N_E{}^* - \lambda^2 M_E{}^*) = 0 \qquad (23)$$

after solving equation (23) , we get,

$$d_{11}B + d_{12}B_2 = 0 \qquad (24)$$

$$d_{21}B_1 + d_{22}B_2 = 0 \qquad (25)$$

Where d_{11}, $d_{12} = d_{21}$ and d_{22} involve parametric constant and frequency parameter.

For a non- trivial solution the determinant of the coefficients of Equation (24) & (25) must be zero.

Therefore, we get the frequency equation,

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0 \qquad (26)$$

With the help of equation (26), we get quadratic equation in λ^2. We can obtain two roots of λ2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters.

RESULT AND DISCUSSION

The frequency (λ) for first and second mode of vibration of an orthotropic skew (parallelogram) plate has been determined for different values of thermal constant(α), tapering constant (β1 and β2), aspect ratio (a/b) and non-homogeneity constant (m) and skew angle(θ).Every one of the outcomes are acquired by utilizing MATLAB/MAPLE programming. All the results are shown with the help of Figures. Following boundaries are utilized for this estimation is: v_0=0.345, a/b=1.5.

In Fig I: Thickness (tapering parameter (β1) variation in plate v/s frequency (λ) with fixed value of θ= 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2= m=α = 0, 0.4, 0.8). From fig.1 that as value of taper constant (β1) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration also increases.

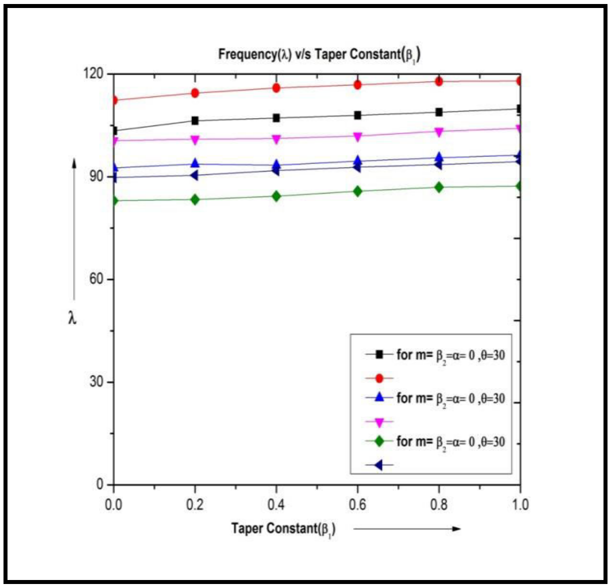

Figure -1 Taper Constant (β_1) v/s Frequency (λ)

In Fig II: Temperature (tapering parameter (β_2) variation in plate v/s frequency (λ) for $\theta = 30^0$ and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = m = \alpha = 0, 0.4, 0.8$). From Table-2 that as value of non-homogeneity (m) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

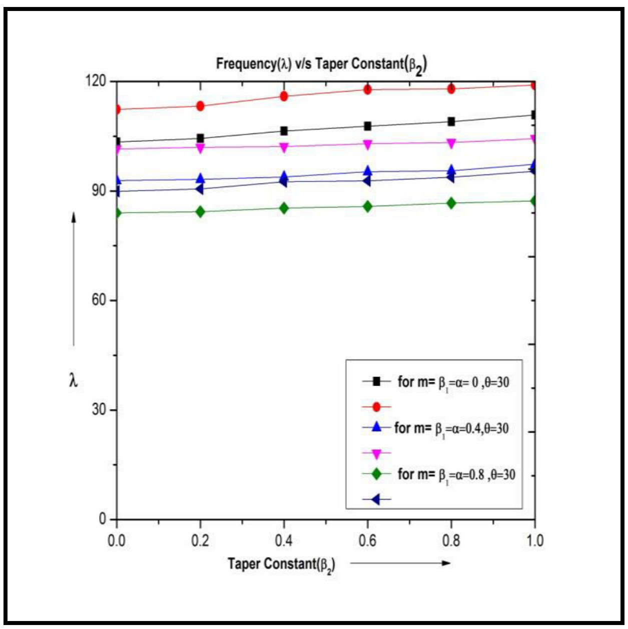

Figure -2 Taper Constant (β_2) v/s Frequency (λ)

In Fig III: non-homogeneity (m) variation in plates material v/s vibrational frequency (λ) for $\theta = 30^0$ and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta1 = \beta2 = m = \alpha = 0, 0.4, 0.8$).From fig.3 that as value of non-homogeneity (m) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration is decreases.

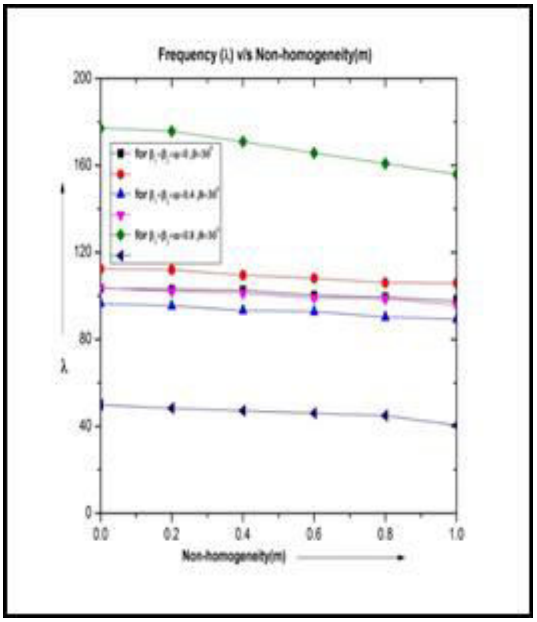

Figure -3 Non-Homogeneity (m) v/s Frequency (λ)

In Fig IV: Thermal gradient (α) variation in plates material v/s frequency (λ) for θ = 30^0 and a/b =1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = m = 0, 0.4,0.8). From fig.IV that frequency mode decreases as value of thermal gradient increases from 0 to 0.8i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration decreases.

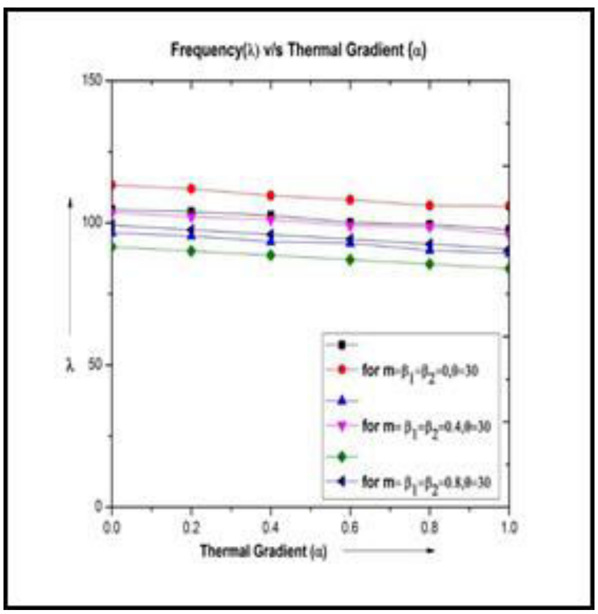

Figure - 4 Thermal Gradients (α) v/s Frequency (λ)

In Fig V: skew angle (θ) variation in plates material v/s frequency (λ) for a/b = 1.5 and different values of taper constants and non-homogeneity constant (β1 =β2 = α =0.4, m = 0, 0.4, 0.8). From fig.V clear that frequency mode increases sharply as value of skew angle increases from 0 to 75 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

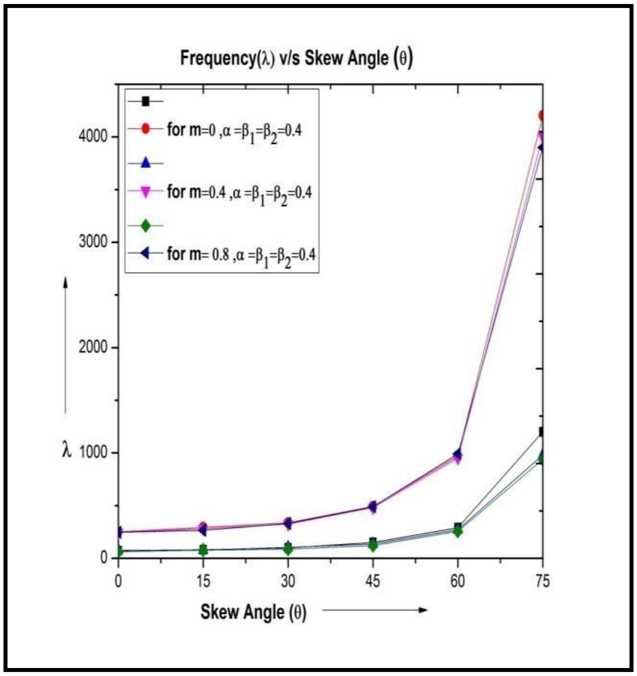

Figure -5 Skew Angle (θ) v/s Frequency (λ)

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