REGULAR 2-EQUITABLE DOMINATION IN FUZZY GRAPHS

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ABSTRACT

A sub set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma,\mu)$ is said to be equitable dominating set if each $v \in V-D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u)-\deg_s(v)| \leq 1$. An equitable dominating set D of $V(G)$ is said to be 2-equitable dominating set in a fuzzy graphs $G(\sigma,\mu)$, if every vertex $v \in V-D$ there exists a vertex $u \in D$ or v is equitable dominated by at least two vertices in D. A 2-equitable dominating set $D \subseteq V(G)$ is said to be regular 2-equitable dominating set if for each vertex $u \in D$ has same strong degree. In this study, regular 2-equitable equitable dominating set, its number in fuzzy graphs are introduced. Bounds and some theorems related to regular 2-equitable equitable domination numbers are stated and proved.

Keywords: Fuzzy graph, equitable dominating set, equitable domination number, 2 - equitable dominating set, regular equitable dominating set and its number, regular 2 - equitable dominating set, regular 2 - equitable dominating number.

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1. INTRODUCTION

Applications of fuzzy graph are include data mining, clustering, image capturing, networking, communications, planning, etc., L.A Zadeh [1] introduced fuzzy sets in 1965. Fuzzy graph theory was initiated by A. Rosenfeld [2] in 1975. Revathi et.al., [3] introduced the concept regular equitable domination number in fuzzy graphs in 2018. Gurubaran et.,all[4] initiated the concept 2- equitable domination in fuzzy graphs in 2018. Complementary nil g-eccentric domination fuzzy graphs concepts introduced by Mohamed Ismayil and Muthupandiyan[5] in 2020. S. Muthupandiyan and A. Mohamed Ismayil [6] introduced the concept perfect g-eccentric domination in fuzzy graph in 2021. Broumi S and et.al.,[7] introduced the concept Equitable domination in Neutrosophic Graphs in 2022. John JC, Xavier P, Priyanka GB.[8] Divisor 2-equitable domination in fuzzy graphs in 2023.

2. BASIC DEFINITIONS

Definition 2.1[5]: A fuzzy graph $G=(\sigma,\mu)$is characterized with two functions ρ on V and μ on $E \subseteq V \times V$, where $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x,y) \leq \rho(x) \wedge \rho(y)\ \forall x,y \in V$. We expect that V is finite and non-empty, μ is reflexive and symmetric. We indicate the crisp graph $G^*=(\sigma^*,\mu^*)$ of the fuzzy graph $G(\sigma,\mu)$ where $\sigma^*=\{x \in V: \rho(x)>0\}$ *and* $\mu^*=\{(x,y) \in E: \mu(x,y)>0\}$. The fuzzy graph $G=(\sigma,\mu)$ is called *trivial in this case* $|\rho *|=1$.

Definition 2.2[5]: *A path* P *of length* n *is a sequence of distinct nodes* $u_0,u_1,\dots,u_n$ *such that* $\mu(u_{i-1},u_i)>0, i=1,2,\dots,n$ *and the degree of membership of a weakest arc is defined as its strength.*

Definition 2.3[5]: *An edge is said to be strong if its weight is equal to the strength of connectedness of its end nodes. Symbolically,* $\mu(u,v) \geq CONN_{G-(u,v)}(u,v)$.

Definition 2.4[5]: *The order and size of a fuzzy graph* $G(\sigma, \mu)$ *are defined by* $p = \sum_{u \in V} \sigma(u)$ *and* $q = \sum_{uv \in E} \mu(u, v)$ *respectively.*

Definition 2.5[5]: *Let* $G(\sigma, \mu)$ *be a fuzzy graph. The strong degree of a vertex* $v \in \sigma^*$ *is defined as the sum of membership values of all strong arcs incident at* v *and it is denoted by* $d_s(v)$*. Also, it is defined by* $d_s(v) = \sum_{u \in N_s(v)} \mu(u, v)$ *where* $N_s(v)$ *denotes the set of all strong neighbors of* v*.*

Definition 2.6[6]: *A fuzzy graph* $G(\sigma, \mu)$ *is connected if* $CONNG(u,v) > 0$ *where* $CoNNG(u,v)$ *is strength of connectedness between two vertices* u, v *in* $G(\sigma, \mu)$*.*

Definition 2.7[6]: *In a fuzzy graph* $G(\sigma, \mu)$*, strength of connectedness between two vertices* $u, v \in V(G)$ *is maximum strength off all paths between* u, v *in* $V(G)$*.*

Definition 2.8[4]: A subset D of V is called a dominating set (DS) in G if for every $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by the symbol γ. The maximum scalar cardinality of a minimal dominating set is called upper domination number and is denoted by the symbol Γ.

Definition 2.9[4]: A sub set $D \subseteq V(G)$ of a fuzzy graphs $G(\sigma, \mu)$ is said to be equitable dominating set (EDS) if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u) - \deg_s(v)| \leq 1$.

Definition 2.10[4]: An equitable dominating set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is called 2 – equitable dominating set if for every vertex $v \in V - D$ there exist $v \in D$ or v is equitable dominated by at least two vertices in D.

Definition 2.11[3]: An equitable dominating set $D \subseteq V(G)$ of a fuzzy graph $G = (\sigma, \mu)$ is said to be regular equitable dominating set if for each vertex in $u \in D$ has the same degree.

3. MAIN RESULTS

Regular 2- Equitable Domination in Fuzzy Graphs

In this section discuss about regular 2- equitable dominating set and its number in fuzzy graphs. Bound and theorem related to regular 2- equitable domination number in fuzzy graphs are stated and proved.

Definition 3.1:

A 2-equitable dominating set $D \subseteq V(G)$ is said to be regular 2-equitable dominating set if for each vertex $u \in D$ has same 1degree. A regular 2 – equitable dominating set D is said to be minimal if no proper subset of D is regular 2 – equitable dominating set. The minimum scalar cardinality of a minimal regular 2 – equitable dominating set of G is called the regular 2 – equitable dominating number of G and is denoted by $\gamma_{r2eqd}(G)$. The maximum scalar cardinality of a minimal regular 2 – equitable dominating set of G is called the upper regular 2 – equitable dominating number of G and is denoted by $\Gamma_{r2eqd}(G)$.

Note 3.1: The minimum regular 2 - equitable dominating set is denoted by γ_{r2eqd}-set.

Example 3.1: Consider the fuzzy graph $G(\sigma, \mu)$.

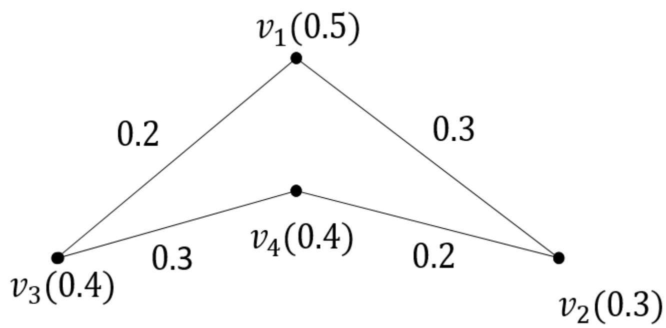

Figure: Regular 2-Equitable Dominating Set in a Fuzzy Graph

From the fuzzy graph given in example 3.1, the followings are observed.

1. The minimum regular 2- dominating set is $D_1 = \{ v_2, v_3, \}$, then $\gamma_{r2eqd}(G) = 0.7$.
2. The upper regular 2- equitable dominating set is, $D_2 = \{ v_1, v_4 \}$, then $\Gamma_{r2eqd}(G) = 0.9$.

Observation 3.1: For any connected fuzzy graphs $G(\sigma, \mu)$

1. $\gamma(G) \leq \gamma_{eqd}(G) \leq \gamma_{2eqd}(G) \leq \gamma_{r2eqd}(G)$
2. $\gamma_{r2eqd}(G) \leq \Gamma_{r2eqd}(G)$.

Observation 3.2: For any FG $G = (\sigma, \mu)$,

1. Every super set of a regular 2-equitable dominating set is also a regular 2-equitable dominating set.
2. Complement of a regular 2-equitable dominating set is not a need a regular 2-equitable dominating set
3. γ_{r2neqd} -set need not be unique.

Proposition 3.1: For any fuzzy graph G with order p, then $\sum_{\substack{v_i,v_j \in G \\ v_i \neq v_j}} \min\left(\sigma(v_i), \sigma(v_j)\right) \leq \gamma_{r2eqd}(G) \leq p$.

Proof:
Let D be a regular dominating set of a fuzzy graph G having atleast two vertices has minimum of V which is a sum of minimum value of vertices $v_i, v_j \in D, \gamma_{r2eqd}(G) \leq p$ it is obviously true.

Theorem 3.1: Let G be a fuzzy graph, $\gamma_{r2eqd}(G) = p$ iff the fuzzy graph G has adjacent to less than two vertices.

Proof:
Let G be a fuzzy graph then $\gamma_{r2eqd}(G) = p$ then definition of fuzzy graph has all vertices in dominating set D. which shows that every vertex in G has adjacent to less than two vertices. Conversely, G be a fuzzy graph has adjacent to less than two vertices then every vertex is in regular dominating set. Which is $\gamma_{r2\varepsilon qd}(G) = p$

Theorem 3.2: Let D is a minimal regular 2 - equitable dominating set then $V - S$ contains minimal regular 2 - equitable dominating set if every vertex of V in a fuzzy graph G adjacent to more than two vertices in V.

Proof:
Let D be a minimal regular 2 - equitable set of G suppose that $V - D$ is not regular 2 - equitable dominating set,

then there exists at least one vertex $v \in D$ which is not an equitable adjacent to any vertex in $V - D$. Therefore $V - D$ is 2 - equitable adjacent to at least two vertices in D then $D - \{v\}$ is a regular 2 - equitable dominating set which is a contradiction. Hence every vertex in D must be equitable adjacent to at least two vertices in $V - D$. Hence $V - D$ is regular 2 - equitable dominating set which contains minimal 2 - equitable dominating set.

Corollary 3.1: Every connected fuzzy graph has minimum regular 2 - equitable dominating set D then $V - D$ need not be regular $2 -$ equitable dominating set of G.

Proof:
Let D be a regular 2 - equitable dominating set of G satisfies the condition also $|d_s(u) - d_s(v)| \leq 1$, suppose $v \in V$, then v be in every 2 - equitable dominating set of a fuzzy graph G, since it has only one neighbor vertex. This one neighbor also strong neighbor of v. Which shows that every vertex in $V - D$ does not has two strong neighbors for v. This implies that $V - D$ is not a regular 2 - equitable dominating set of G.

Theorem 3.3: Let G be a connected fuzzy graph has no non - equitable edge and H is spanning subgraph of G then $\gamma_{r2eqd}(G) \leq \gamma_{r2eqd}(H)$.

Proof:
Let G be a connected fuzzy graph and H is the spanning subgraph of H. consider D is minimum regular 2 - equitable dominating set of G, D also a regular 2 - equitable dominate all the vertices in $V(H) - D$ that is D is an regular 2 - equitable dominating set in H. Hence $\gamma_{r2eqd}(G) \leq \gamma_{r2eqd}(H)$.

Theorem 3.4: For any fuzzy graph $G, \gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{r2eqd}(G)$, for $v_i \notin D$.

Proof:
Let D be regular 2 - equitable dominating set with minimum cardinality γ_{r2eqd}. for any vertex $v_i \in D, D - \{v_i\}$ is 2 - equitable dominating set. Hence $\gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{r2eqd}(G)$.

Theorem 3.5: For any fuzzy graph G without isolated vertices, a regular 2-equitable dominating set D is minimal if and only if for every $u \in D$, one of the following two properties holds.

(i) There exists a vertex $v \in V - D$ such that $N_s(v) \cap D = \{u\}$, $|\deg_s(u) - \deg_s(v)| \leqslant 1$.
(ii) $< D - \{u\} >$ contains no isolated vertices.

Proof:

Assume that D is a minimal regular 2- equitable dominating set and (i) and (ii) do not hold, then for some $u \in D$, there exists $v \in V - D$ such that $|\deg_s(u) - \deg_s(v)| \leqslant 1$ and for every $v \in V - D$, either $N_s(v) \cap D \neq \{u\}$ or $|\deg_s(u) - \deg_s(v)| \geqslant 2$.or both. Therefore $< D - \{u\} >$ contains an isolated vertex, which is contradiction to the minimality of D. Therefore (i) and (ii) holds.

Conversely, if for every vertex $u \in D$, the statement (i) or (ii) holds and D is not minimal. Then there exits $u \in D$, such that $D - \{u\}$ is a regular 2- equitable dominating set. Therefore, there exists $v \in D - \{u\}$ such that v 2-equitable dominates u. that is, $v \in N_s(u)$ and $|\deg_s(u) - \deg_s(v)| \leqslant 1$. Hence u does not satisfy (i). Then u must satisfy (ii) and there exists $v \in V - D$ such that $N_s(v) \cap D = \{u\}$, and $|\deg_s(u) - \deg_s(v)| \leqslant 1$. And also, there exists $w \in D - \{u\}$ such that w is adjacent to v. Therefore $w \in N_s(v) \cap D, |\deg_s(u) - \deg_s(v)| \leqslant 1$ and $w \neq v$, a contradiction to $N_s(v) \cap D = \{u\}$. Hence D is a minimal regular 2- equitable dominating set.

Theorem 3.6: Let G be a fuzzy graph without isolated vertices. Then $\gamma_r(G) \leqslant \gamma_{r2eqd}(G)$

Proof:

Every regular 2- equitable dominating set is a regular dominating set. Thus $\gamma_r(G) \leqslant \gamma_{r2eqd}(G)$.

Theorem 3.7 A regular 2 - equitable dominating set exists for any regular strong fuzzy graph G .

Proof.

Let $G = (\sigma, \mu)$ be a regular strong fuzzy graph. It is clear that $d_s(u_i) = R$ for every $u_i \in G$. Suppose a strong fuzzy graph G has a 2- equitable dominating set, obviously it contains the vertices with $d(u_i) = k$ for every $u \in S$. Therefore every regular strong fuzzy graph is a regular 2 - equitable Dominating set and it exists for strong fuzzy graph.

Theorem 3.8 Let G be a complete bipartite fuzzy graph with $\sigma(u_i) = R$ and $\sigma(v_i) = R$ for every $u_i \in V_1$ and $v_i \in V_2$ then $\gamma_{r2eqd}(G) = 2R$.

Proof.

Let G be a complete fuzzy graph with $\sigma(u_i) = R$ and $\sigma(v_i) = R$ for every $u_i \in V_1$ and $v_i \in V_2$. By the definition of regular 2 - equitable dominating set $= \{\min\ \sigma(u_i), \min\sigma(v_i): u_i \in V_1, v_i \in V_2,$ clearly $d_s(u) = d_s(v).\}$ Therefore regular 2-equitable dominating set exists. That is $\gamma_{r2eqd}(G) = 2R$.

Theorem 3.9 For a regular fuzzy graph $G\gamma_{f2eqd}(G) \leq \gamma_{r2eqd}(G)$.

Proof.

It is clear that every regular 2- equitable dominating set is a 2-equitable dominating set. we get $\gamma_{2eqd}(G) \leq \gamma_{r2eqd}(G)$.

Theorem 3.10 For a fuzzy graph $G = (\sigma, \mu)$ if γ_{r2eqd} is a regular 2-equitable dominating set then $V - D$ is a dominating set of a fuzzy graph G.

Proof

Let v be any vertex in D, D is a regular 2-equitable set in G . Since G has no isolated vertex $v \in N_s(u)$.It is clearly every regular 2-equitable dominating set is a equitable dominating set such that $v \in V - S$. Hence every vertex of D dominates some of the vertices in $V - S$. Therefore, $V - D$ is a dominating set of fuzzy graph G.

Theorem 3.11 Let G be a path fuzzy graph with $\sigma(u_i) = R$ for every $u_i \in V$ and having all edges (that is G be a strong fuzzy graph), then regular 2-equitable dominating set $\gamma_{r2eqd}(G)$ set exists.

Proof.

Let G be a path fuzzy graph, $\sigma(u_i) = R$ for every $u_i \in V$ and having all strong edges, the regular 2-equitable dominating set $\{u_i : i \neq 1\ or\ p\}$ such that D is a regular 2-equitable dominating set. Therefore γ_{r2eqd} set exists.

4. CONCLUSION

In this article, regular 2 - equitable dominating set, its number in fuzzy graphs are obtained. Theorems related to regular 2 - equitable dominating set and number are stated and proved. Bounds and some points are observed and discussed.

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