Stochastic Modelling and Computational Sciences

MODIFIED VOGEL'S APPROXIMATION METHOD – AN APPROACH TO FIND INITIAL BASIC FEASIBLE SOLUTION OF TRANSPORTATION PROBLEM

Dr. Madhukar Dalvi
Department of Mathematics and Statistics, Nagindas Khandwala College (Autonomous),
Malad, Mumbai, India
mhdalvi7@gmail.com

ABSTRACT

The goal of transportation problems (TP), a linear programming problem, is to apply the cost-minimizing principle to find the optimal solution for the distribution of commodities among many resources and destinations. To find the optimal solution to transportation problem, one must first identify the initial basic feasible solution (IBFS).Various methods have been used by researchers to calculate IBFS. The most effective method for determining an IBFS for transportation problems is Vogel's Approximation Method (VAM).Present study suggests DVAM, an innovative approach to solve modified VAM problems which utilizes use of a modified VAM algorithm that considers implied cost instead of to normal cost. Ten data sets from various secondary sources that were chosen at random were examined. IBFS has been calculated using a Python program. anddifferent methods, including NWCM, LCM, and VAM, as well as the proposed DVAM method. The Modified Distribution (MODI) approach was then used to find the optimal solution. In the majority of cases, DVAM is found to be the most efficient method for determining IBFS in terms of proximity to the optimal solution. This finding is therefore important in reducing transportation costs and optimizing transportation problemsto significantly improve an organization's performance in highly competitive market.

Keywords: Transportation Model, Initial Basic Feasible Solution (IBFS), Optimal Solution, VAM,DVAM

1. INTRODUCTION

One of the key areas of production management is the transportation model. In order to achieve optimal profitability, an organization must not only maximize profits but also reduce costs in an optimistic manner. Along with manufacturing costs, transportation costs make up the majority of a company's overall expenses. In a transportation model, a business typically consists of factories for product manufacturing and retail facilities for product distribution. These locations are referred to as sources and destinations, respectively.

The transportation problem's logical solution method was first proposed by Dantzig [6] in 1951. The North West Corner Method (NWCM), which takes the north-west-corner cost cell into consideration at every stage of allocation, was subsequently developed by Charnes et al. [5] in 1953. The Least Cost Method (LCM) [5] is the next established technique, which involves placing as much as possible in the Transportation Table's lowest cost cell through every step of allocation. In 1958, Vogel and Reinfeld [16] developed the Vogel's Approximation Method (VAM). The different versions of VAM by Shimshak et. al. [18], Goyal [9], Ramakrishnan [15], etc. Kirca and Satir [11] developed a heuristic known as the Total Opportunity-cost Method (TOM) to obtain efficient initial basic feasible solutions. Balakrishnan [4] suggested a modified version of VAM to address unbalanced transportation issues. These reputable methods are discussed in all Operation Research papers. Out of the available methods, the Vogel's Approximation Method (VAM) is the most efficient procedure to determine an initial basic feasible solution for transportation problems. This is because VAM provides a very effective initial basic feasible solution.The initial basic feasible solution is necessary for obtaining an optimal solution to transportation problems. In 2010, Kulkarni and Datar [13] introduced a new heuristic approach for obtaining an initial basic feasible solution for unbalanced transportation problems. Kulkarni and Datar argued that their suggested method reduces the number of iterations required to achieve optimality. In 2011, Koruko˘glu and Balli [12] proposed an improved version of the widely recognized VAM by considering the total opportunity cost. In 2012, Singh et al. [19] made modifications to the solution method of VAM by incorporating total opportunity cost and allocation costs. Hoque [10] in 2015, suggested a better efficient heuristic solution technique denoted by JHM

(Juman & Hoque Method) to obtain a more effective basic feasible solution for the transportation problems. Year 2014 saw the introduction of an innovative approach called the "Implied Cost Method (ICM)" by Babu et al. [3], wherein feasible solutions are having less cost than VAM and frequently coincide with the optimal solution. In 2016, Uddin et al. [20] introduced the Improved Least Cost Method (iLCM) as a solution to transportation problems. This improvement is essentially achieved by making modifications to the current conventional LCM solution method. Additionally, an enhanced initial basic feasible solution is being developed through this method.

In summary, multiple scholars have employed different approaches in their efforts to determine the Initial Basic Feasible Solution (IBFS). In this study, with the objective of identifying a more effective solution, we present DVAM, an innovative modified VAM solution method that uses the modified VAM algorithm which takes implied cost into account rather than ordinary cost.

2. MATERIAL AND METHODS

2.1. General Transportation problem

The primary objective of the Transportation Problem (TP) is to facilitate the shipment of a single commodity from any collection of distribution points, known as sources, to any group of receiving facilities, known as destinations. Demand for a destination can originate from one or more sources. There is an allocated quantity of units available at each source, and these units must be distributed to all destinations. In the same way, every destination has a certain demand for units, and all of those demands require supply from the sources. Finding the unknowns that will minimise the total cost of transportation while meeting demand (requirement) and supply (capacity) constraints is the primary goal of the transportation model. Let us assume that there are m producers and n destinations. Then, si represents the supply at source 'i' and di represents the demand at destination 'j'. Let xij represent the quantity to be delivered from source i to destination j, and let cij represent the unit transportation cost from source 'i' to destination 'j'.

Then, transportation problem can mathematically be formulated as:

$$\text{Minimize } z = \sum_{i=0}^{m}\sum_{j=0}^{n} \text{cij xij} \, (\text{Objective Function})$$

$$\text{Subject to} \sum_{i=0}^{m} xij \leq \; si \;\; i = 1,2,3,\ldots\ldots\ldots, m \;\; (capacity\ constraits)$$

$$\sum_{i=0}^{n} xij \;\geq\; dj \;\; j = \; 1,2,3,\ldots\ldots\ldots, n \;\; (\text{Demand constraints})$$

$$xij \geq \; 0 \quad for\ all\ i\ and\ j \qquad (\text{Non} - \text{negative constraints})$$

There are two categories of transportation problems namely, balanced and unbalanced. When the total supply from all sources equals the total demand from all destinations, a transportation problem is said to be balanced. If this quantity is not equal, the problem is classified as an unbalanced transportation problem.

The following steps are used to find the optimal solution to the transportation problem after it has been mathematically formulated:

Step 1: Examine in order to determine if the transportation problem is balanced or not. First, balance the problem if it is not balance by adding dummy column (row)with all unit cost zero and remaining demand (supply).

Step 2: Find the initial basic feasible solution.

Step 3: Confirm that the initial basic feasible solution satisfies the optimality criteria.

Step 4: If the current solution is nonoptimal, make modifications to get the optimal solutions.

For a transportation problem with m sources and n destinations, a feasible solution with no more than (m+n-1) non-negative allocations represents the basic feasible solution. Moreover, even if a feasible solution isn't essentially feasible, it is considered optimal if it minimises the cost of transportation.

2.2. Initial Basic Feasible Solution (IBFS) - Proposed Method DVAM

This method is basically modified version of VAM with implied cost.

The proposed method can be applied to solve all types of Transportation Problem (TP). Procedure of finding an IBFS using this method is illustrated below.

- **Step-1:** Mathematical formulation of the transportation table.
- **Step 2:** If the total supply and total demand are not equal, then create a dummy row or dummy column with the surplus supply or demand and zero transportation costs, accordingly.
- **Step 3:** Compute the implied cost for each cell by multiplying the unit transportation cost by the maximum number of units of the commodity that can be produced in accordance with supply and demand.
- **Step 4:** Determine the difference between the least and the next least implied costs for each row and each column of the transportation table. Do not consider dummy column or row with cost 0 for calculation. This difference is called as penalty.
- **Step 5:** Choose the row or column that has the largest difference (penalty); if there is a tie, choose at random.
- **Step 6:** To meet demand, assign the maximum possible units to the row or column's least implied cost cell that was originally selected. In the event of a tie for least implied cost, choose the minimal cost cell. Choose the cell with the maximum unit allocation among them if the minimum cost and least implied cost cells are tied.
- **Step 7:** Eliminate all columns and rows where the assignment made in step 6 fully satisfies the demand or if there is no supply quantity available.
- **Step 8:** Recalculate the implied cost differences in rows and columns without considering the eliminated row/rows and column/columns.
- Without taking into account the eliminated row/row(s) and column/column(s), recalculate the implied cost differences in the rows and columns.
- **Step 9:** Proceed to step 4 and continue through steps 4–8 until a initial feasible solution is obtained.
- **Step 10:** Enter these assigned values in the corresponding cell of the original Transportation Table (TT).
- **Step 11:** Determine total transportation cost. This calculation is the result of multiplying the assigned quantity by the unit transportation cost.

3. RESULTS AND DISCUSSION:

The data used in this article are secondary data obtained from different published articles.

3.1. Mathematical Illustration 1: Balanced Transportation Problem

For illustration purpose, consider following transportation problem from Kirca, Ö., &Şatir, A. (1990).

Table No.1 Mathematical Model of a Transportation

	Destination				
Source	D1	D2	D3	D4	Supply (S)
S1	15	27	13	19	40
S2	18	21	24	14	40
S3	21	15	16	17	20

Demand(D)	30	20	30	20	100/100

Total demand = Total supply = 100. Hence TP is balanced type.

Iteration 1: Implied cost for each cell is calculated by multiplying the unit transportation cost by the maximum quantity of commodity (units) that can be produced in accordance with supply and demand.

Table No.2: Iteration 1-Determination of implied cost

	Destination				
Source	D1	D2	D3	D4	Supply (S)
S1	15 **450**	27 **540**	13 **390**	19 **380**	40
S2	18 **540**	21 **420**	24 **720**	14 **280**	40
S3	21 **420**	15 **300**	16 **320**	17 **340**	20
Demand(D)	30	20	30	20	100/100

Iteration 2:
Follow step 4 to 9 of proposed algorithm until we obtain initial basic feasible solution.

Table No.1.2: Iteration 2-Solution by DVAM

	Destination							
Source	D1	D2	D3	D4	Supply (S)	P1	P2	P3
S1	15 **450** **10**	27 **540**	13 **390** **30**	19 **380**	40	(10)	(160)*	(90)
S2	18 **540** **20**	21 **420**	24 **720**	14 **280** **20**	40	(140)*	(120)	(120)
S3	21 **420**	15 **300** **20**	16 **320** **ε**	17 **340**	20	(20)	(20)	(120)
Demand(D)	30	20	30	20	100/100			
P1	(30)	(120)	(70)	(40)				
P2	(30)	(120)	(70)	-				
P3	(30)	(120)*	-					

In the above solution obtained by DVAM,

No. of occupied cells = 5, but m+ n -1= 3 +4 -1= 6 indicates rim requirements are not satisfied. Hence, we assign very small amount of units ε (equivalent to '0') to the unoccupied cell S3-D3 with minimum implied cost in the table. Hence this initial basic solution obtained by DVAM is feasible with total transportation cost 1480.For comparison purpose, Initial solution obtained by VAM is 1480 and Optimum solution obtained by MODI method is also 1480.

3.2. Mathematical Illustration 2: Unbalanced Transportation Problem

For illustration purpose, consider following unbalanced transportation problem from Balakrishnan, N. (1990).

Table No. 3: Initial table

	D1	D2	D3	Supply(S)
S1	6	10	14	50
S2	12	19	21	50

S3	15	14	17	50
Demand(D)	30	40	55	125/150

Here, total demand is 125 and total supply is 150. Hence given transportation problem is unbalanced type. We add dummy column Dm with all costs zero and demand 25. We implied cost as below.

Iteration 1: Determination of **implied cost**same as in case of illustration 1

Table No. 4: Iteration 1- Determination of implied cost

	D1	D2	D3	Dm	Supply(S)
S1	6 **180**	10 **400**	14 **700**	0 **0**	50
S2	12 **360**	19 **760**	21 **1050**	0 **0**	50
S3	15 **450**	14 **560**	17 **850**	0 **0**	50
Demand(D)	30	40	55	25	150/150

Iteration 2: Solution by DVAM same as illustration 1

Note that for calculation of row penalty we do not consider dummy column element zero.

Table No. 5: Iteration 2-Solution by DVAM

	Destination							
Source	D1	D2	D3	D4	Supply (S)	P1	P2	P3
S1	6 **180** **5**	10 **400** **40**	14 **700** **5**	0 **0**	~~50455~~ 0	(220)	(220)	(300)*
S2	12 **360** **25**	19 **760**	21 **1050**	0 **0** **25**	~~50 25~~ 0	(400)*	-	-
S3	15 **450**	14 **560**	17 **850** **50**	0 **0**	50	(110)	(110)	(290)
Demand(D)	~~30 5~~ 0	~~40~~ 0	~~55~~ 50	~~25~~0	100/100			
P1	(180)	(160)	(150)	(0)				
P2	(270)*	(160)	(150)	-				
P3	-	(160)	(150)					

In the above solution obtained by DVAM,

No. of occupied cells = 6 and m+ n -1= 3 +4 -1= 6 indicates rim requirements are satisfied.

Hence this initial basic solution obtained by DVAM is feasible with total transportation cost 1650. Initial solution obtained by VAM is 1745 and Optimum solution obtained by MODI method is also 1650. This indicates that IBFS obtained by DVAM is far close to optimal solution than that of VAM solution. To support this claim, we now consider different secondary data sets obtained from available literature and find IBFS using differ methods like NWCM, LCM, VAM and finally optimum solution using Modified Distribution (MODI) Method.

3.3. Numerical examples of transportation problems:

Table No. 6: Data sets

Numerical Example	Dimension of cost matrix	Source (Reference	Data

No.		)	
1	3x4	14	[cij]=[1 2 1 4; 4 2 5 9; 20 40 30 10] ; [si]=[30,50,20] ;[dj]=[20,40,30,10]
2	3x4	11	[cij]=[15 27 13 19; 18 21 24 14; 21 15 16 17] ; [si]=[40,40,20] ;[dj]=[30,30,30,20]
3	3x3	4	cij]=[6 10 14 ; 12 19 21; 14 14 17] ; [si]=[450,50,50] ;[dj]=[30,40,55] (Unbalanced)
4	5x5	7	[cij]=[8 8 2 10 2; 11 4 10 9 4; 5 2 2 11 10; 10 6 6 5 2;8 11 8 6 4]; [si]=[40,70,35,90,85] ; [dj]=[80,55,60,80,45]
5	4x4	3	[cij]=[7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3] ; [si]=[30,25,20,15]; [dj]=[30,30,20,10]
6	5x7	8	[[cij]=[12 7 3 8 10 6 6; 6 9 7 12 8 12 4;10 12 8 4 9 9 3; 8 5 11 6 7 9 3;7 6 8 11 9 5 6]; [si]=[60,80,70,100,90]; [dj]=[20,30,40,70,60,80,100]
7	3x3	1	[cij]=[15 7 25; 8 12 14; 17 19 21] ; [si]=[12,17,7];[dj]=[12,10,14]
8	5x5	17	[cij]=[73 40 9 79 20; 62 93 96 8 13; 96 65 80 50 65;57 58 29 12 87; 56 23 87 18 12] ; [si]=[8,7,9,3,5] ; [dj]=[6,8,10,4,4]
9	3x3	1	cij]=[4 3 5; 6 5 4; 8 10 7] ; [si]=[90,80,100]; [dj]=[70,120,80]
10	3x4	1	[cij]=[3 1 7 4; 2 6 5 9; 8 3 3 2] ; [si]=[300,400,500] ;[dj]=[250,350,400,200]

Comparison of IBFS with Optimum Solution (OS):

Initially we obtain IBFS from differ method and OS from MODI Method. Further we find % of deviation from OS.

Table No. 7: Comparison of IBFS with Optimum Solution (OS)

	IBFS (I_R)				OS(F_R)	% of deviation from OS			
Example No.	NWCM	LCM	VAM	Proposed DVAM	MODI	NWCM	LCM	VAM	Proposed DVAM
1	600	560	450*	450*	450	33.33	24.44	0.00	0.00
2	3160	1480*	1480*	1480*	1480	113.51	0.00	0.00	0.00
3	1815	1695	1745	1650*	1650	10.00	2.73	5.76	0.00
4	1870	1650	1505	1590	1475	26.78	11.86	2.03	7.80
5	540	435	470	490	410	31.71	6.10	14.63	19.51
6	3180	2080	1930	1900*	1900	67.37	9.47	1.58	0.00
7	545	433	425*	425*	425	28.24	1.88	0.00	0.00
8	1994	1123	1104	1127	1102	80.94	1.91	0.18	2.27
9	1500	1450	1500	1500	1390	7.91	4.32	7.91	7.91
10	4400	2850*	2850*	3400	2850	54.39	0.00	0.00	19.30

Ten randomly selected examples, as listed in Table 2.4, were solved in order to analyse and compare new method. To find % of deviation from OS, following formula is used.

% of deviation from OS (D)= $\frac{IR - FR}{FR}$.

This calculation is carried out to evaluate that how much nearer the $I_R$is to F_R. '*' in this table indicates IBFS equals to OS. It is observed that in five cases (50%) Proposed DVAM is closed to OS.

In 3 cases(30%) VAM is close to OS, 2 cases (20%) LCM is close to OS and in 0 cases NCWM is closed to OS. In most of the cases % of deviations from OS is also less. It means, for this randomly selected data set, Proposed DVAM is performing better than other standard methos of IBFS.

4. CONCLUSION

The main objective of TP is to find out optimal strategy to transport commodities from factories call source of supply and warehouse called destination with minimum cost. In the present study DVAM is proposed to obtain better IBFS. Ten randomly selected data sets from different secondary sources were analyzed. According to our research, DVAM is found to be the most effective technique for determining IBFS in terms of proximity to the optimal solution in the majority of the cases. If the IBFS is the closest to the optimal solution, it could be reasonable to assume that there cannot be as many iterations beyond that. As a result, this finding is crucial for minimizing cost of transportation and optimizing transportation methods, both of which may greatly enhance an organization's performance in the competitive environment. Limitation of the study is only ten randomly data sets are analysed to evaluate performance.

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