Stochastic Modelling and Computational Sciences

ABSORBING MARKOV CHAIN MODEL IN STUDENTS' PROGRESSION OF HIGHER EDUCATION INSTITUTION

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ABSTRACT

Background: *Absorbing Markov chain model is an effective method to analysis the students' progression towards completing their higher education degrees. Such application will definitely help in the overall development of any educational institution.*

Objectives: *The main objective of this paper is to develop a stochastic model for estimation effectiveness indicators ofdifferent programs offered by higher education institution.*

Method: *A finite Markov chain with two absorbing and six transient stages is used to simulate the study plan. It is created to form the probability transition matrix. The quantitative properties of the absorbing Markov chain, including anticipated student time at each programme level, anticipated time till absorption, and anticipated probabilities of absorption, are utilised to choose indicators of various programmes.*

Results: *The model is used to examine the number of students and academic performance in an academic institution in India. The transition matrix was developed students intake data considering six consecutive semesters of academic years from 2018-19 until 2020-21. Estimates were made on the student's advancement towards the following level of the study curriculum. The expected time a student spends at each level of a program is calculated. The graduation and withdrawal probabilities were obtained. Data was analysed and the results were interpreted.*

Conclusion: *When students advance to higher semesters, the percentage of graduates increases and their probability of graduating increases significantly. It provides useful information to all stakeholders of educations institutions for informed decisions on investment in education.*

Keywords: Higher Education Institutes; absorbing Markov chain, transition probabilities; absorbing state

1 INTRODUCTION

The system of higher learning aids in the development of exceptional experts in many fields and facilitates the discovery of new information through research. In today's competitive society, the economic development and productive efficiency cannot be achieved without the high intellectual and professional individuals.The economic and social needs associated with higher educational institutions have attracted the attention of many segments of the Indian public to observe the performance of these organizations with renewed and increased interest (Al-Awadhi, S. &Konsowa, M. 2010). Increasing the responsibility of the institutions to maintain higher educational institutions efficiency may help to reduce financial burden on theIndian government. To promote higher education to the people of every part of the society, the government takes various initiative and reforms. By providing a variety of courses, several colleges and universities participate in this process. Autonomous colleges which are affiliated with universities are offering various skill oriented courses. Every university has a set timeframe for accomplishing a certain course of study.All affiliated colleges are generally following this definite time schedule.

A vital part of teaching learning process is the assessment of students which is done through examinations. However, for each college, students take different average time to complete a course. Students who join in a specific course may succeed and advance to the upper class in certain cases, drop out for a variety of reasons, or fail and have the choice of repeating the course or attempting the exam again.Each institution of higher learning can potentially be regarded as a hierarchical organisation in which a student stays in one academic year of a particular study stage before moving on to the next level, leaving the system as a graduate, or dropping out.

Due to continuous changing and the increasing amount of data the problem of understanding and assessing the students' progress through the educational system is very important (Mashat, Ragab&Khedra, 2012). The use of several resources is used in order to teach the students to specific levels. If a large number of students drop out or repeatedly take the same course, these colleges may lose a lot of revenue. Finally, this has an impact on the overall growth of an educational institution. As a result, it's important to estimate the amount of time it requires students to finish their academic work on average, as well as their probability of graduating and dropping out of a programme. This will definitely help in the overall development of any institution.

There are large numbers of educational institutions in the western region of India, especially in the state of Maharashtra. The colleges of Maharashtra are affiliated to different Universities. The affiliated colleges provide several courses in a variety of disciplines as graduate and postgraduate courses, in accordance with university regulations.The Maharashtra Public University Act of 2016 grants the university the authority to conduct examinations or evaluations, grant degrees, post-graduate diplomas, post-higher secondary diplomas and certificates, and other academic distinctions to students who have successfully completed the required coursework and obtained the necessary credits, marks, or grades imposed by the university. In accordance with any rules established by the State Government or University Grants Commission, the University may also designate a university department, conducted college, affiliated college, institution, or school as an autonomous university department, conducted college, affiliated college, institution, or school, as the case may be.

University of Mumbai, Maharashtra Public University has 916 affiliated colleges. Some of these colleges are autonomous educational institutions. Students migrate between different institutions and engage in a programme of study that interests them in order to pursue higher education. The semester system, which requires students to appear for two examinations each year with a six-month gap between them, has been introduced by Mumbai University as of the 2011–12 academic years. Students must finish six academic sessions within three years in order to graduate. The first, third, and fifth semesters begin in July and end in October, respectively, with examinations, while the second, fourth, and sixth semesters continue from November to May with examinations.

Students who are unable to appear in any semester examinations or who fail to advance to the upcoming semester have the opportunity to complete their backlog along with to their regular semester examination, according university policy. If a student, for any reason, is unable to pass his first semester examination, he will have to pass the first semester with the regular second semester exam.In case of second semester if any student incapable to clear all but 2 courses and incapable to clear all but 2 course of earlier semester I, will be promoted to next the semester (Allowed to keep term-ATKT). In case student failed in more than 2 course in any of semester I or II, will not be promoted to next the semester unless he or she clears all but 2 course of earlier semester I and II. Likewise, in case of backlog of earlier semesters, students will have to clear with regular semester examination. This procedure is to followed every year.Students in the first, second, third, and fourth semesters may be permitted to retake the theoretical paper examination in order to obtain higher grade points. The student must successfully complete all semester examinations, retakes, and opportunities for improvement within five years of the date of admission to the first semester course in order to graduate.

According to findings from earlier studies, students' progress towards obtaining higher education degrees possesses all the necessary stochastic characteristics, and can therefore be modelled as a Markov chain (see, for example, Crippa, Mazzoleni&Zenga, 2016; Mashat et al., 2012; Rahim, Ibrahim, Kasim& Adnan, 2013; Symeonaki&Kalamatianou, 2011). The most important family of stochastic processes is the Markov chain.The process's future probability behaviour is unaffected by the process's past behaviour and only depends on its current state. This is called the Markovian property (Tijms, 2003).

With particular reference to Nagindas Khandwala College ((HEI), an autonomous college affiliated with Mumbai University, the current study used an absorbing Markov chain model to calculate the probabilities of graduation, withdrawal, and three-year degree course completion for Mumbai University students.

Following the introduction and literature review sections, we present the basic theoretical properties of absorbing Markov chains. The development of the model is the primary focus of the study. The idea is used to the bachelor's degree study programmes at Nagindas Khandwala College, an autonomous college affiliated with Mumbai University, to illustrate its relevance. The acquired results are analysed and discussed. We conclude by summarising our findings and providing a few possibilities for future investigation.

2 LITERATURE REVIEW

Absorbing Markov chains is a special Markov chain with the finite states. This method of analysing the many social and economic issues is extremely valuable. It is used in various kinds of transit processes (Wainwright, 2007; Nichols, 2008; Al-Awadhi &Konsowa, 2007; Bessent&Bessent, 1980; Kolesat, 1970; Kwak, et al., 1985; Merddith, 1976; McNamara, 1974; Dalvi, M. 2023). In order to prevent the unfavourable future of dissertation overload for supervising professors, Bessent and Bessent (1980) investigated the progression process of doctorate students in a university department. Moody &DuClouy (2014) have applied the Markov chain to analyse and predict the mathematical achievement gap between African American and white American students. Shah and Burk (1999) used the Markov Chain to model the flow of undergraduate students of higher education system of Australia and tried to find out the probability completion, mean time takes to complete the course, and the time they spend in their study.Alawadhi and Konsowa (2010) applied absorbing Markov Chain to study the students flow at Kuwait University and discussed different aspects such as students' mean life time in different levels, percentages of dropping out and make a comparative study of different colleges. Reynolds and Porath (2008) studied absorbing Markov chains to model the academic progress of students attending the University of Wisconsin-Eau Claire over a specific time period.

The Conceptual Framework: Markov Chains

A Markov Chain is a stochastic process with a "one-step memory" that moves through a set of states in a series of steps (phases). This implies that the state that was occupied in the preceding step, and not in earlier stages, determines the probabilities of reaching a certain state in a particular step, also known as the transition probability between steps.This is known as the *Markov property.* A. Markov pioneered the fundamental ideas of Markov Chains while coding literal texts in 1907. A. Kolmogorov, W. Feller, and other notable mathematicians developed the Markov Chain theory after that. The significance of this theory to the natural, social, and the majority of the applied sciences, however, was not acknowledged until the 1960s (see Kemeny, J. G. & Snell, J. L., (1963), Suppes, p. & Atkinson, R.S., (1960)).According to the background details provided in Bairagi, A. &Kakaty, S. C. (2017), we refer to a finite Markov Chain when the set of states for a Markov Chain is a finite set.Markov chains usually represent first-order chains in which only the current time (n) and not any previous times (0, 1, 2,... n-1) have an impact on the chain's future time (n+1). The transition probability matrix is a matrix that may be used to categorise the transition probabilities. The states of the chain are the outcomes.

Some important definitions:

1. **Reachable State**: State j is referred to as reachable from state i if it is possible to travel there from state i.
2. **Communicate States:** If state j is approachable from state i and vice versa, then such states are referred to as Communicate States.
3. **Closed set in Markov Chain**: A closed set is one in which no state in a set of Markov Chain states S can be attained from any other state outside of S.
4. **Transient state**: A set If there is a state j that can be visited from i but not from j, then i is a transient state.
5. **Recurrent state:** State which is not transient is called recurrent state.

6. **Absorbing state:** If leaving a Markov Chain is not possible, the state is said to be absorbing.
7. **Absorbing Markov Chain:** If a Markov chain has at least one absorbing state with $p_{ii} = 1$, it is considered to be an absorbing chain.
8. **Absorption time from the state i:** Let u_i be the number of steps the chain required to absorb into any one of the absorbing states starting from any transient state, let's say i. The state i is referred as absorption time.

An absorbing Markov chain with s absorbing and t transient states has the following general form of the probability transition matrix:

$$P = \begin{pmatrix} I & O \\ R & Q \end{pmatrix} \text{---------} (1)$$

Where

I – $s \times s$ identity matrix

0 – $s \times t$ zero matrix

R – $t\times s$ matrix expressing transitions from the transient states to the absorbing states

Q – $t\times t$ matrix expressing transitions between the transient states.

In order to determine *the expected time until absorption* and *the probabilities of absorption* we need the *fundamental matrix* **N** which can be calculated as

$$N = (I - Q)^{-1} = I + Q + Q^2 \ldots\ldots \text{-------}(2)$$

Where I stands for the identity matrix of size t x t (as contrary to I in (1) where its size is r x r).

The n_{ij} elements of the matrix N express how frequently a Markov chain, on average, moves from the transient state i to the transient state j. This matrix plays very important role in absorbing Markov chain.

The ij^{th} element of any absorbing chain

$$P^n = \begin{pmatrix} I & O \\ R^n & Q^n \end{pmatrix}$$

gives the probability that after n steps starting from state i, it will be in state j.

Since only transient states are visited by the chain prior to absorption, the estimated absorption time is equal to the total expected number of visits to all transient states.

$$E(n_i) = \Sigma E(n_{ij}),$$

This represents the sum of i^{th} row of fundamental matrix **N**.

U = E (ni) or U = Ne in matrix form, where e is a column vector with all of its elements equal to one. The probabilities of absorbing a specific transient states are provided by the matrix B=NR.

The Model Development:

In the present study, Absorbing Markov Chain is used to model the higher education system of an educational institute affiliated to Mumbai University. This institute is offering Commerce, Arts, science and Management courses. In this study students from all streams are considered. For a three-year period, each of these students must appear for examination and successfully complete six semesters in order to receive their final degree.The students belong into the following categories:

1. **S1**: Students eligible to appear Semester I examination, according to University regulations after completing HSC (higher secondary School) examination entering in the system of college education.

2. **S2**: Students appearing for the semester II examination
3. **S3**: Students appearing for the semester III examination
4. **S4**: Students appearing for the semester IV examination
5. **S5**: Students appearing for the semester V examination
6. **S6**: Students appearing for the semester VI examination
7. **G**: Students who passed the VI Semester examination have the opportunity to attempt it, improve their results, and obtain the institution's graduate degree.
8. **W**: Students who dropped out of the course for various reasons.

The performance of students may be viewed as the Markov Chain's memory-less property because

a. Every student who appears a semester examination must either go on to a subsequent semester or remain in the current semester.

b. Students who don't pass the exam are permitted to repeat it with the regular semester exam.

c. The student has the freedom to discontinue the course at any stage.

d. A student cannot receive two promotions at once, and they cannot be demoted at any time.

e. The first semester is considered to be the only period that new admissions are permitted.

f. Exam results from previous semesters do not affect performance in subsequent semesters.

These different and non-overlapping student classes represent the states of the Markov chain in this particular case. 'Graduate' and 'Withdraw' are absorbing states, whereas the other six stages are transient states. Students have several opportunities to go from these transient states to an absorbing state. In absorbing states, students never leave out of the state and stay there forever. The movement of students between states is based on previous performance. The frequency of their success, failure, repeat of the course, withdrawal, and graduation represent the transition frequencies between the various phases. These frequencies may be transformed into transition probabilities, which provide the necessary Markov Chain transition probability matrix.

Data on student progression has been gathered for the study's purposes from the Nagindas Khandwala College (Autonomous) examination department. Since this institute achieved autonomous status in the academic year 2016–17, the students who enrolled in the degree programmes completed their studies in 2018–19. Hence the student progression data of three academic years namely 2018-19, 2019-20 and 2020-21 of ninedifferent programs offered by the college, two programs namely Bachelor of Commerce (BCOM), Bachelor of Arts (BA) under grant an aid system and remaining seven self-financing courses namely Bachelor of Commerce- Accounting and Finance (BAF), Bachelor of Commerce- Financial Markets (BFM), Bachelor of Commerce- Banking and Insurance (BBI), Bachelor of Management (BMS), Bachelor of Mass Media(BMM), Bachelor of Science- Information Technology(IT) and Bachelor of Science- Computer Science(CS)have been considered for the study. Each program's transition frequency and transition probability matrices are computed separately.

OBJECTIVES OF THE STUDY

The study's main objectives are

1. To estimate the expected time a student spends at each level of a program of Higher Education Institution (HEI)
2. To determine the average number of semesters needed to complete the two absorbing states of a Higher Education Institution (HEI) programme, namely graduation and withdrawal.

3. To evaluate the probabilities of attaining absorbing states namelyGraduation and Withdrawal from a program of Higher Education Institution (HEI).

Data Analysis

Initially on program (Bachelor of Commerce (B.Com) was selected and stepwise calculation of Markov Chain analysis is conducted considering progression data of three academic years namely 2018-19, 2019-20 and 2020-21. Detail of this stepwise analysis is presented below.

1. The transition frequency matrix of 695 students of Bachelor of Commerce (B.Com)Program:

In table 1, all together695 students admitted for first year of B.Com program. Si, i= 1,…..,6 represents semesters, W means students who left program for some reason and G denotes that the student passed the sixth semester examination, had the chance to attempt it and do better, and obtained the institution's graduate degree.

The cell S1-S1 indicates that after conduct of Semester I Examination out of total 695 students, 368 students remains (yet to complete all courses) in Semester I and remaining 327 are promoted to Semester II. At the end of academic year all 368 students who remains in Semester I were allowed to give their examination of all courses in which they failed along with all courses of Semester II. All 327 are promoted to Semester II are allowed to appear their Semester II examination. Other cells representtransition frequencies of remaining Semesters.536 students in total successfully passed the sixth Semester examination, were given a chance to attempt the examination and improve, and were awarded the institution's graduate degree.

Table 1: The transition frequency matrix of 536 students of Bachelor of Commerce (B.Com) Program

	S1	S2	S3	S4	S5	S6	W	G	Total
S1	368	327	0	0	0	0	0	0	695
S2	0	246	447	0	0	0	2	0	695
S3	0	0	385	202	0	0	0	0	587
S4	0	0	0	130	454	0	3	0	584
S5	0	0	0	0	86	536	0	0	578
S6	0	0	0	0	0	42	0	536	578
W	0	0	0	0	0	0	5	0	5
G	0	0	0	0	0	0	0	536	536

2. Transition probability matrix P for the students of Bachelor of Commerce (B.Com) Program:

From these frequencies the transition probabilities are calculated by dividing each frequency by its total class frequency.

For example, for B.Com Program

p_{11} =368/695=0.5295 and p_{12}= 327/695=0.4705.

The Table 2 represents the transition probability matrix for the students B.Com Program of College in canonical form.

Table 2: Transition probability matrix P for the students of Bachelor of Commerce (B.Com) Program

	W	G	S1	S2	S3	S4	S5	S6
W	***1***	***0***	**0**	**0**	**0**	**0**	**0**	**0**
G	***0***	***1***	**0**	**0**	**0**	**0**	**0**	**0**
S1	**0**	**0**	0.5295	0.4705	0	0	0	0
S2	**0.0029**	**0**	0	0.3540	0.6431	0	0	0
S3	**0**	**0**	0	0	0.6559	0.3441	0	0
S4	**0.0051**	**0**	0	0	0	0.2226	0.7723	0
S5	**0**	**0**	0	0	0	0	0.1488	0.8512

S6	**0**	**0.9273**	0	0	0	0	0	0.0727

3. **The movement of students pursuing a Bachelor of Commerce (B.Com) (matrix Q) from one transient state to another**

The matrix Q illustrates the way students go from one transient state to another. The probability that a first-semester student will continue on to the second semester is 0.4705 and 0.5295, respectively. The other matrix gives the transition probabilities between the transient states.

Table 3: The movement of students from one transient state to another transient state of Bachelor of Commerce (B.Com) (matrix Q)

0.5295	0.4705	0	0	0	0
0	0.3540	0.6431	0	0	0
0	0	0.6559	0.3441	0	0
0	0	0	0.2226	0.7723	0
0	0	0	0	0.1488	0.8512
0	0	0	0	0	0.0727

4. **Probabilities of the Bachelor of Commerce (B.Com) program's transition from its six transient states to its absorbing state (R):**

In table 4, cell S2-W value 0.0029 is probability of students who left college from the course for different reasons whereas, cell S6-G value 0.9273 is probability of students who successfully passed the sixth Semester examination, were given a chance to attempt the examination and improve, and were awarded the institution's graduate degree and so on.

Table 4: Probabilities of Transition from six transient states to absorbing state of Bachelor of Commerce (B.Com) program (R)

	W	G
S1	0	0
S2	0.0029	0
S3	0	0
S4	0.0051	0
S5	0	0
S6	0	0.9273

5. **Fundamental matrix N= (I-Q) $^{-1}$ of Bachelor of Commerce (B.Com) program:**

For the absorbing Markov Chain, diagonal elements of fundamental matrix **N=(I-Q)** $^{-1}$ calculates the expected number of time a student will spend during each semester. From matrix it is clear that a student requires approximately 2 semesters to complete semester I, II and II course. To complete semester IV, V and VI student need approximately 2 semesters. The zeros in the N matrix stand for the impossibility of advancement to the lower semester.

Table 5: Fundamental matrix N= (I-Q) $^{-1}$ of Bachelor of Commerce (B.Com) program

2.125399	1.547988	2.893086	1.280564	1.161866	1.066056
0	**1.547988**	2.893086	1.280564	1.161866	1.066056
0	0	**2.906132**	1.286339	1.167105	1.070863
0	0	0	**1.286339**	1.167105	1.070863
0	0	0	0	**1.174812**	1.077935
0	0	0	0	0	**1.077935**

0.4705	-0.4705	0	0	0	0
0	0.6460	-0.6431	0	0	0
0	0	0.3441	-0.3441	0	0
0	0	0	0.7774	-0.7723	0
0	0	0	0	0.8512	-0.8512
0	0	0	0	0	0.9277

Where

I – Q =

6. The average number of semesters needed to reach the absorbing states U = Ne for Bachelor of Commerce (B.Com) program:

Column vector U estimates the expected total number of semesters needed required to move into or graduate from any one of the absorbing states from any other transient state.From table 6 it is seen that, semester I students require approximately 10 semesters to reach graduation state. To earn a graduate degree, students in semesters II, III, IV, V, and VI spend approximately 8, 6, 4, 2, and 1 semester. Note that equal importance is given to both absorbing states.

Table 6: The average number of semesters needed to reach the absorbing states U = Ne for Bachelor of Commerce (B.Com) program

U=

S1	10.07496
S2	7.949559
S3	6.430439
S4	3.524307
S5	2.252747
S6	1.077935

Where **e =**

1
1
1
1
1
1

7. The probabilities of attaining absorbing states B = NR for Bachelor of Commerce (B.Com) program:

Next table 7 provides probability of attaining absorbing states W and G. It has been noted that the probability of a student in their first semester graduating is 0.988554, whereas the probability of them dropping out is 0.01102. The probability of graduating a student goes on increasing every year but the probability of withdrawing from the course is decreasing yearly.

Table 7: The probabilities of attaining absorbing states B = NR for Bachelor of Commerce (B.Com) program

	W	G
S1	0.01102	0.988554
S2	0.01102	0.988554
S3	0.00656	0.993011
S4	0.00656	0.993011
S5	0	0.999569
S6	0	0.999569

B = NR =

After presenting Markov Chain analysis in step-wise manner, we further represent consolidated details of all programs together. Table 8 depicts the probability of students transition and continuing in the same semester. For various colleges, the transition probabilities of students progressing onto higher semesters are represented in Table 9. Table 10 shows the probability of transitioning from various transient states to absorbing states.

Table 8: Probabilities of transition where students stay in the same state across all college programmes

	BCOM	BA	BMS	BAF	BBI	BFM	BMM	IT	CS
S1	0.5295	0.4112	0.2513	0.1926	0.5454	0.4627	0.5915	0.4667	0.5620
S2	0.3540	0.3462	0.2789	0.2148	0.5454	0.3077	0.3099	0.3777	0.4167
S3	0.6559	0.4545	0.3770	0.2042	0.5116	0.0448	0.0896	0.5379	0.5505
S4	0.2226	0.1328	0.0874	0.0141	0.0465	0.0154	0.0448	0.2803	0.3761
S5	0.1448	0.0547	0.0773	0.0426	0.1591	0.0769	0.0580	0.1462	0.1509
S6	0.0727	0.0492	0.0166	0.0284	0.0682	0.0615	0.0290	0.0615	0.0189

Table 9: Probabilities of student progression into higher states across all college programmes

	BCOM	BA	BMS	BAF	BBI	BFM	BMM	IT	CS
S1	0.4705	0.5878	0.7487	0.8074	0.4545	0.5373	0.4085	0.5333	0.4380
S2	0.6431	0.6461	0.7158	0.7852	0.4545	0.6923	0.6910	0.6223	0.5750
S3	0.3441	0.5454	0.6230	0.7958	0.4884	0.9552	0.9104	0.4621	0.4495
S4	0.7723	0.8672	0.9126	0.9859	0.9535	0.9846	0.9552	0.7197	0.6239
S5	0.8512	0.9453	0.9227	0.9474	0.8409	0.9231	0.9420	0.8538	0.8491
S6	0.9273	0.9951	0.9834	0.9716	0.9318	0.9385	0.9710	0.9385	0.9811

Table 10: Probabilities of Transition from Transient state to Absorbing state of all programs of college (R)

	BCOM		BA		BMS		BAF		BBI		BFM		BMM		IT		CS	
	W	G	W	G	W	G	W	G	W	G	W	G	W	G	W	G	W	G
S1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0.003	0	0.008	0	0.005	0	0	0	0	0	0	0	0	0	0	0	0.008	0
S3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S4	0.005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S6	0	0.927	0	0.995	0	0.983	0	0.972	0	0.932	0	0.937	0	0.971	0	0.939	0	0.981

The expected time a student spends at each level of program.

Table 11 demonstrates how students spend less time on average than they did in the earlier semesters as they advance to higher levels.

Table 11: The expected time a student spends at each level of program

	BCOM	BA	BMS	BAF	BBI	BFM	BMM	IT	CS
S1	2.1254	1.7013	1.3356	1.2385	2.2002	1.8612	2.4480	1.8751	2.2831
S2	1.5479	1.5295	1.3867	1.2735	2.2002	1.4444	1.4491	1.6069	1.7143
S3	2.9061	2.9061	1.6051	1.2566	2.0475	1.0469	1.0984	2.1640	2.2247
S4	1.2864	1.8331	1.0958	1.0143	1.0488	1.0156	1.0469	1.3895	1.6028
S5	1.1748	1.1531	1.0838	1.0555	1.1892	1.0833	1.0615	1.1712	1.1777
S6	1.0779	1.0517	1.1069	1.0292	1.0731	1.0655	1.0299	1.0655	1.0192

The average number of semesters needed for all programmes at a college to attain the absorbing states U.
The U matrix for various college study programmes is presented in Table 12 below. For the I semester students of B Com, BA BBI and CS Programs required approximately 10 semesters, BMS, BAF and BFM required approximately 7 semesters, BMM required approximately 8 semesters to complete their Graduation degree. It is seen that for last semester VI, all programs required almost equal that is approximately 8 semesters to complete their Graduation degree.

Table 12: The average number of semesters needed for all programmes at a college to attain the absorbing states U

	BCOM	BA	BMS	BAF	BBI	BFM	BMM	IT	CS
S1	10.075	10.093	7.4887	6.8678	9.7591	7.5170	8.1338	9.2723	9.9363
S2	7.9496	8.3915	6.1530	5.6292	7.5589	5.6558	5.6858	7.3972	7.6531
S3	6.4304	6.9438	4.8016	4.3556	5.3587	4.2114	4.2368	5.7903	6.0245
S4	3.5243	4.0377	3.1964	3.0991	3.3112	3.1645	3.1383	3.6262	3.7998
S5	2.2527	2.2049	2.1007	2.0848	2.26239	2.1488	2.0915	2.2368	2.19698
S6	1.0780	1.05175	1.0169	1.0292	1.07319	1.0655	1.0298	1.0655	1.01926

The probabilities of attaining absorbing states for different programs of a college:
Table 14 is the **B=NR** matrix of different programs of a college. It is seen that for all courses probability of graduating for semester I is in the range of 0.98 to 1, whereas when the students move to the higher levels this probability is comparatively more than that of early semesters.

The probability 1 implies that once these college students reach their sixth semester, they will certainly complete their graduation degree.

Table 13: The probabilities of attaining absorbing states for different programs of a college

	BCOM		BA		BMS		BAF		BBI		BFM		BMM		IT		CS	
	W	G	W	G	W	G	W	G	W	G	W	G	W	G	W	G	W	G
S1	0.01102	0.988554	0.0118	0.9880	0.00735	0.99265	0	1	0	1	0	1	0	1	0	1	0.014229	0.985771
S2	0.01102	0.988554	0.0118	0.9880	0.00735	0.99265	0	1	0	1	0	1	0	1	0	1	0.014229	0.985771
S3	0.00656	0.993011	0	0.9998	0	1	0	1	0	1	0	1	0	1	0	1	0	1
S4	0.00656	0.993011	0	0.9998	0	1	0	1	0	1	0	1	0	1	0	1	0	1
S5	0	0.999569	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
S6	0	0.999569	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

CONCLUSION

In order to assess students' performance at the degree level and throw some light on its peculiarities, an absorbing Markov chain model is developed. Nine programmes provided by an autonomous higher education institution from India have distinct performance characteristics that are estimated.To illustrate the usefulness Absorbing Markov Chain was applied tonine programsof a degree college. The research gives details on how long it takes students to complete a semester on average, how many semesters are needed to graduate, and the probability that a student will graduate or drop out of the course.Such information is also important for students and employers to help them make informed decisions on investment in education (Shah & Burke, 1999).It was found that students required approximately 1 to 3 semesters to complete one semester study. To complete their Graduation degree students required approximately 7 to 10 semesters. Probability of graduating from college ranges from 0.98 (98%) to 1(100%) whereas; probability of withdrawal from program ranges from 0 (0%) to 0.014(1.4%).Additionally, it has been observed that as students advance to higher semesters, the percentage of graduate's increases. As a result, the Markov Chain analysis of student performance offers an accurate representation of higher education institutions.This study is applicable for all higher education stakeholders. The main limitation of our study is this Markov chain model is based on the time-homogeneousness. Over short to medium term this is not an unreasonable assumption because student behaviour is unlikely to change dramatically over such time span (Shah &Burke, 1999). Further it is suggested to implement Absorbing Markov Chain model for other educational institutes providing post graduating programs or technical education.

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