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MOVING FLOW OF MICROPOLAR FLUID IN POROUS MEMBRANE CYLINDRICAL CELLS-ANALYTICAL STUDY

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ABSTRACT
Applying a cell concept technique, the transport of micropolar liquids over a covering is studied. This membrane is represented as an ensemble of solid cylinders pieces with a pore layer. In this instance, the flow is not parallel to the chambers' axis. On the exterior of an imaginary cell, there are spin-free, couple-stress-free, and peacefully circumstances relating to the boundaries value dilemma. Analytical methods are developed after the impact of micropolar and porosity media attributes on the physicochemical porosity of screens is examined.

Keywords: "Micropolar flow; Cell model; Porous medium; Hydrodynamic permeability"

INTRODUCTION
Oil enthusiasts flow in gravel mattresses and film filtering, etc., are all examples of implementations that include particle aggregation at random. To put this plan into action, we'll pick a single swarm particle and put it in a made-up cell. The outer layer of cells may be subjected to an appropriate boundary impairment to replicate the impact of nearby atoms. This reduces the complexity of the problem to a single cell flow, typically with a spherical or cylindrical cross section. Modern models account for spheroid cell morphologies and material changes in particles **(Yadav, 2020).** Modeling the flow of different substances requires the use of solid, porous, and liquid particles surrounded by a liquid envelope. An unbounded incompressible liquid containing partially porous particles is described as having a hard core surrounded by a porous, non-deformable hydrodynamically uniform covering **(Khanukaeva, 2020).**

Most cellular models use a Newtonian fluids assumption, where liquid elements are represented by points of material with three degrees of freedom (translation, rotation, and shear) Hence there is a linear relationship in the stress-deformation rate tensor. Many solvents have characteristics that the Neoclassical model is ill-suited to represent, prompting the development of non-Newtonian models. Review of the literature on non-Newtonian modeling approaches for flows in porous media **(Hauswirth et al., 2021).** Although both the stress and deformation tensors are symmetric, the term "non-Newtonian" implies a non-linear relationship between them. There are no universally applicable analytical solutions to the flow description problem because of the non-linear rheology at play. They are great at describing some niche flows, but as of yet, no comprehensive model of non-Newtonian flows has been developed.

Meanwhile, a completely different approach considers fluids to be indistinguishable from solids with no internal volume. Particles with three rotational degrees of freedom and three translational degrees of freedom can experience pair stresses. This class of substances is known as micropolar liquids. Despite the fact that simple microfluidics may be described by linear rheology, the stress and coupling stress tensors that characterize their deformation rates are not symmetrical. Liquids with such a structure are commonly referred to as Polar beverages are as opposed to non-polar Hydrodynamic solutions, and the non-symmetrical model of stiffness is used to characterise solids having this sort of structure. Since the orientation of liquid crystal molecules is critical to their behavior and a symmetrical description is insufficient, liquid crystals serve as the simplest example of a material where micropolar theory is applicable. As micropolar theory has developed, it has been used to the modeling of a variety of microstructured media. Fluids such as blood and synovial fluids in the joints are just a few examples.

Micropolar liquids have a wide range of potential applications and investigations because of the ease with which free and filter analytical solutions may be obtained. We would like to draw attention to the fact that the momentum equation for micropolar flows in porous media is wrongly formulated in certain publications by the inclusion of a “Darcy-type member” in a form matching to the Newtonian liquid.

Despite the widespread availability of commercially available membranes, researchers continue to conduct experiments and develop models to better understand membrane processes and features. Fewer yet have addressed the application of micropolar theory to cellular models, and no one, to the authors' knowledge, has considered the part played by a mixed solid-porous particle within the cell. We have already solved and investigated the flow in a cylindrical cell along its axis. In this research, we consider micropolar flow through a collection of spherical particles, which is perpendicular to the cell axis. The problem of cylinders aligning themselves at random is also addressed.

Following the resolution of the issue, a membrane's fluid permeability is computed by simulating its surface as a collection of impenetrable particles coated in a porous layer. All relevant parametric research, including studies examining the impact of boundary circumstances, make use of this fundamental aspect of the flow.

Problem Statement

In Fig. 1, we see a three-coaxial-layer cell with uniform flow moving at a velocity U perpendicular to the symmetry axis. The first layer, with a radius of a, is a solid core; the second layer, from $a < r < b$, is porous; and the third and final layer, from $b < r < c$ is filled with free micropolar liquid. In order to make the direction of vector U coincide with =0, we introduce the cylindrical coordinate system (r, θ, z)

The remainder of the continuation calculation, energy the equation, and the mathematical model of micropolar substances, which describes the motion of the liquid in area 2, provide the Eringen force formulae of approaching flow.

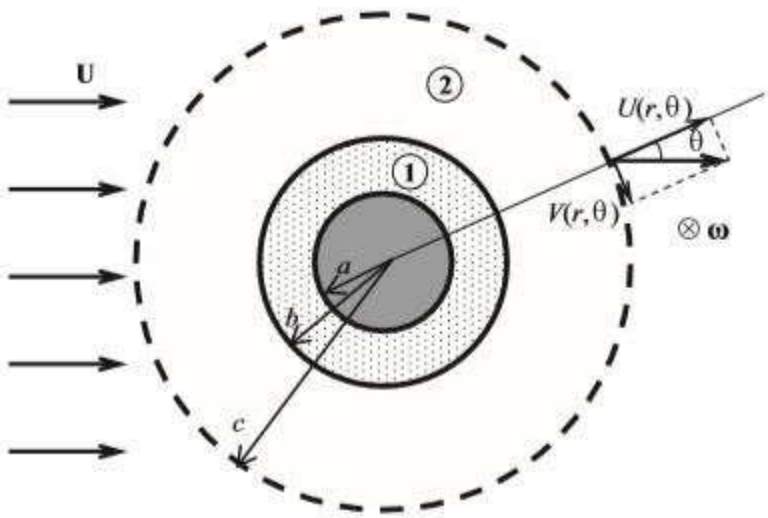

Fig. 1: Thescheme of the flow

$$\nabla\cdot\mathbf{v}=\mathbf{0},\quad =\rho\mathbf{F}-\nabla P+(\mu+\kappa)\Delta\mathbf{v}+2\kappa\nabla\times\boldsymbol{\omega},\mathbf{0}$$
$$=\rho\mathbf{L}+(\alpha+\delta-\varsigma)\nabla\nabla\cdot\boldsymbol{\omega}+(\delta+\varsigma)\Delta\boldsymbol{\omega}+2\kappa\nabla\times\mathbf{v}-\mathbf{4}\kappa\boldsymbol{\omega}$$

where v and $\boldsymbol{\omega}$ are vectors representing linear and rotational velocities, P stands for pressure equal to ρ for water density, F & L for force from the outside and linkage density values, and μ,κ,α,δ,ζ for the micropolar fluid's viscoelastic parameters. The value of the coefficient of resistance below are written in a somewhat modified form of the Eringen notation. It is chosen such as to guarantee the coefficient, which is calculated which represents the rapidly changing viscosity of a Hydrodynamic fluid equals unity. The skew-symmetric part of the deformed rate distribution γ ˆ is connected to the stress tensort ˆ by the rotational viscosity. The cyclical viscosity values are the constants α,δ, and s in the fundamental equation that connects the couple stress sheet m alongside the curvature-twist rate sheetχ ̂. The micropolar model for rigidity accepts the following form for the stress tensor, the couple stress matrices (m) ̂, distortion rates tensor, and curvature-twist rate scalar:

$$\hat{t} = (-P + \lambda \operatorname{tr} \hat{\gamma})\hat{G} + 2\mu\hat{\gamma}^{(S)} + 2\kappa\hat{\gamma}^{(A)},$$
$$\hat{m} = \alpha(\operatorname{tr} \hat{\chi})\hat{G} + 2\delta\hat{\chi}^{(S)} + 2\varsigma\hat{\chi}^{(A)},$$

where $(S)$and (A) are the symmetric and skew symmetric components of the tensor, and $\hat{G}$ is the metric tensor. The bending rate matrices as well as the curvature-twist rate scalar are made up of the variations of the straight-line and rotating speeds respectively, where $\hat{\gamma} = (\nabla\mathbf{v})^T - \hat{\varepsilon}\cdot\boldsymbol{\omega}, \hat{\chi} = (\nabla\boldsymbol{\omega})^T, \hat{\varepsilon}$, are the “Levi-Civita tensor”. As can be seen, the formulation of the deformation rate tensor allows the coefficient λ to be deleted for incompressible fluids.

The Stokes method can be applied because of the low velocities of the stationary filtering flows. The free micropolar fluid's governing equations are ($b < r < c$)when no external forces or couplings are present.

$$\nabla\cdot\mathbf{v}_2 = 0$$
$$-(\mu+\kappa)\nabla\times\nabla\times\mathbf{v}_2 + 2\kappa\nabla\times\boldsymbol{\omega}_2 = \nabla P_2,$$
$$-(\delta+\varsigma)\nabla\times\nabla\times\omega_2 + 2\kappa\nabla\times\mathbf{v}_2 - 4\kappa\omega_2 = 0 \ldots\ldots(1.1)$$

where we utilized the “continuity equation and the symmetry” of the geometry to get $\nabla\cdot\omega = 0$. from the identity $\nabla\times\nabla\times\mathbf{a} = \nabla\nabla\cdot\mathbf{a} - \Delta\mathbf{a}$,. For the free stream layer, all variables will have a subscript of 2, whereas the porous layer's subscripts will read 1.

Equations of motion for micropolar liquids at rest after filtering were developed using the intrinsic volume averaging method:

$$\nabla\cdot\mathbf{v} = 0$$
$$\nabla P = \left(\frac{\mu}{\varepsilon}+\frac{\kappa}{\varepsilon}\right)\Delta\mathbf{v} + \frac{2\kappa}{\varepsilon}\nabla\times\boldsymbol{\omega} - \frac{\mu+\kappa}{k}\mathbf{v},$$
$$0 = (\alpha+\delta-\varsigma)\nabla < \nabla\cdot\boldsymbol{\omega} > +(\delta+\varsigma)\Delta\boldsymbol{\omega} + 2\kappa\nabla\times\mathbf{v} - 4\kappa\boldsymbol{\omega},$$

“where ε is the porosity and k is the permeability of the porous medium and angle brackets mean the volume-averaged value”. The equations governing the behaviour for the porous area may be expressed as follows by using the diverge free condition of the magnetic environment for the circulation's relevant geometry: ($a < r < b$)

$$-\left(\frac{\mu}{\varepsilon}+\frac{\kappa}{\varepsilon}\right)\nabla\times\nabla\times\mathbf{v}_1 + \frac{2\kappa}{\varepsilon}\nabla\times\omega_1 - \frac{\mu+\kappa}{k}\mathbf{v}_1 = \nabla P_1,$$
$$-(\delta+\varsigma)\nabla\times\nabla\times\omega_1 + 2\kappa\nabla\times\mathbf{v}_1 - 4\kappa\omega_1 = 0 \ldots\ldots(1.2)$$

“If the non-dimensional variables and values are introduced as follows”

$$\tilde{r} = \frac{r}{b},\ \ell = \frac{a}{b},\ m = \frac{c}{b'},\ \tilde{\mathbf{v}} = \frac{\mathbf{v}}{U'},\ \tilde{\boldsymbol{\omega}} = \frac{\omega b}{U},\ \tilde{P} = \frac{Pb}{\mu U'} \ldots\ldots(1.3)$$

“the non-dimensional forms of systems Eqs. (1a)-(1c) and (2a)-(2c) are respectively”

$$\tilde{\nabla}\cdot\tilde{\mathbf{v}}_2 = \mathbf{0}$$
$$-(\mu+\kappa)\bar{\nabla}\times\bar{\nabla}\times\bar{\mathbf{v}}_2 + 2\kappa\bar{\nabla}\times\bar{\boldsymbol{\omega}}_2 = \mu\bar{\nabla}\bar{P}_2$$
$$\frac{\delta+\varsigma}{b^2}\bar{\nabla}\times\bar{\nabla}\times\bar{\boldsymbol{\omega}}_2 + 2\kappa\bar{\nabla}\times\bar{\mathbf{v}}_2 - 4\kappa\bar{\boldsymbol{\omega}}_2 = 0 \ldots\ldots(1.4),$$

$$\tilde{\nabla}\cdot\tilde{\mathbf{v}}_1 = \mathbf{0}$$

$$-\frac{\mu+\kappa}{\varepsilon}\bar{\nabla}\times\bar{\nabla}\times\tilde{\mathbf{v}}_1+\frac{2\kappa}{\varepsilon}\bar{\nabla}\times\tilde{\boldsymbol{\omega}}_1-\frac{\mu+\kappa}{k}b^2\tilde{\mathbf{v}}_1=\mu\bar{\nabla}\bar{P}_1,$$

$$-\frac{\delta+\varsigma}{b^2}\bar{\nabla}\times\bar{\nabla}\times\tilde{\omega}_1+2\kappa\bar{\nabla}\times\tilde{\mathbf{v}}_1-4\kappa\tilde{\omega}_1=0\ \ldots..(1.5)$$

Because the dimensions of viscosities μ and κ are the same, the number of micropolarity, $N^2=\kappa/(\mu+\kappa)$, is a nondimensional quantity. Since the dimension of the ratio $(\delta+\varsigma)/\mu$ is length squared, “the relation between the micro and macro scales” of the issue is represented by the combination of values $L^2=\frac{\delta+\varsigma}{4\mu b^2}$Using these two nondimensional parameters,

$$\nabla\cdot\mathbf{v}_2=\mathbf{0}$$

$$-\frac{1}{N^2}\nabla\times\nabla\times\mathbf{v}_2+2\nabla\times\omega_2=\left(\frac{1}{N^2}-1\right)\nabla P_2$$

$$-L^2\nabla\times\nabla\times\omega_2+\frac{1N^2}{21-N^2}\nabla\times\mathbf{v}_2-\frac{N^2}{1-N^2}\omega_2=0\ \ldots..(1.6)$$

and

$$\nabla\cdot\mathbf{v}_1=0$$

$$-\frac{1}{N^2}\nabla\times\nabla\times\mathbf{v}_1+2\nabla\times\omega_1-\frac{\varepsilon\sigma^2}{N^2}\mathbf{v}_1=\varepsilon\left(\frac{1}{N^2}-1\right)\nabla P_1$$

$$-L^2\nabla\times\nabla\times\boldsymbol{\omega}_1+\frac{1N^2}{21-N^2}\nabla\times\mathbf{v}_1-\frac{N^2}{1-N^2}\boldsymbol{\omega}_1=0,\ \ldots..(1.7)$$

“where the ratio of macro scale of the cell b to the micro scale of porous medium $\sqrt{k}$ is denoted as $\sigma=b/\sqrt{k}$”

Because of the stream's periodicity, the general solutions of systems (1.6) through (1.7) may be derived independently and stated in the form $\mathbf{v}_i(r,\theta)=\{u_i(r)\cos\theta;v_i(r)\sin\theta;0\},\omega_i(r,\theta)=\{0;0;\omega_i(r)\sin\theta\},P_i(r,\theta)=p_i(r)\cos\theta,i=1{,}2$. Since there are only Singular variable that is independent with four unidentified purposes, each system of Eqs. (1.6–1.7) simplifies to a set of four scalar equations, three of which are of the second order and one of which is of the first order. This means that there will be six completely random constants in each general solution. This signifies that the boundary value problem can't be solved without satisfying a set of 12 requirements.

Filtration Eqs (1.2) were mostly derived using the “no-slip and no-spin conditions on solid surfaces”. This implies that (1.2) the boundary conditions at $r=\ell$ are fixed. There

$$u_1(\ell)=0,\ v_1(\ell)=0,\ \omega_1(\ell)=0\ \ldots\ldots(1.8)$$

are the only permissible slip and spin operations

There is a greater range of possible circumstances at the liquid-porous interface, Continuity between all velocity components, i.e.

$$u_1(1-0)=u_2(1+0),\ v_1(1-0)=v_2(1+0),\ \omega_1(1-0)=\omega_2(1+0)\ \ldots\ldots(1.9)$$

and both normal and radial to the outer terrain, as well as continuity of each of the stress and combination sheet components.

In the selected positional system, the appropriate elements of the stress tensor and relationship stress vectors are.

$$t_{rr} = (-p(r) + 2\mu u'(r))\cos\theta,$$
$$t_{r\theta} = \left((\mu+\kappa)v'(r) - (\mu-\kappa)\frac{u(r)+v(r)}{r} - 2\kappa\sigma(r)\right)\sin\theta$$
$$m_{rz} = (\delta+\varsigma)\omega'(r)\sin\theta.$$

The so-called effective viscosities of a liquid are the pure liquid viscosities divided by the porosity, which is shown by the averaging approach used to get the filtration equations. Porous area stress and couple stress expressions should make advantage of them. Boundary conditions for stresses and pair stresses can also be obtained in their non-dimensional forms by applying relations.

$$-p_1(1-0) + \frac{2}{\varepsilon}u_1'(1-0) = -p_2(1+0) + 2u_2'(1+0) \dots\dots (1.10)$$
$$\frac{1}{\varepsilon}v_1'(1-0) - \frac{1-2N^2}{\varepsilon}\frac{u_1(1-0)+v_1(1-0)}{1-0} - 2\frac{N^2}{\varepsilon}\omega_1(1-0) =$$
$$= v_2'(1+0) - (1-2N^2)\frac{u_2(1+0)+v_2(1+0)}{1+0} - 2N^2\omega_2(1+0) \dots\dots (1.11)$$
$$\omega_1'(1-0) = \varepsilon\omega_2'(1+0) \dots\dots (1.12)$$

At r=m, three additional boundary circumstances are needed. The continuing existence of the typical part associated with linear acceleration is one amongst it:

$$u_2(m) = 1 \dots\dots (1.13)$$

The second condition at the cell border in classical models for nonpolar liquids is one of four recognized variants. Both are viable options for the micropolar liquid. Happel's stress-free condition is used here:

$$v_2'(m-0) - (1-2N^2)\frac{u_2(m-0)+v_2(m-0)}{m-0} - 2N^2\omega_2(m-0) = 0 \dots\dots (1.14),$$

plus two additional conditions for investigating the impact of a single boundary condition change on the solutions. There are two possible scenarios, both of which include no-couple stress:

$$\omega_2'(m-0) = 0 \dots\dots (1.15)$$

and no-spin condition

$$\omega_2(m-0) = 0 \dots\dots (1.16).$$

Our study on parallel flow in a cylindrical cell utilized these circumstances. They are employed in this case to evaluate discrepancies between perpendicular and parallel flow measurements. Finally, a hybrid of the two flows will be generated to approximate the flow in the medium represented by the randomly “oriented cylindrical cells”.

Considered boundary conditions do not include gyro-viscosities in the stress or pair stress tensor components for perpendicular flow. Thus, it was not possible to summarize these viscosities with only two parameters, “*N* and *L”,* as in the case of a cell that is parallel to the flow direction. Therefore, unlike in the case of parallel flow, no additional parameters of the micropolar medium need to be introduced.

The Boundary Value Problems and Their Solutions

The related profiles of “linear velocities components (a,b) and angular velocities” (c) are presented graphically in Fig. 1.2 for the solutions obtained with conditions (1.8)-(1.14) and (1.15) or (1.16).

The data in Fig.1.2 is presented without dimension. The flow domain is an annulus with inner and outer radii of *l*=0.5 and *m*=1.5, respectively, indicating a relatively thick porous layer and a central location “(r=1) of the porous-liquid interface” within the cell, respectively. Fig. 1.2 is plotted using representative values of the parameters. Micropolarity is robust when *N*=0.5 and *L*=0.2; , ε ==0.75 and σ =3 define a porous media with features that are far from the limit. The parametric analyses to follow will make use of the parameter values that are presented. Each of the pictures below, beginning with Fig. 1.2, depicts two curves. The no-spin condition (Eq.1.16) corresponds to the dotted line, while the no-couple-stress condition (Eq.1.15) corresponds to the solid line. The layer's porosity and permeability are rather high, accounting for the almost linear relationships shown in Figures 2a and b. The nature of the curves is significantly altered for less permeable porous layers. Figure 2c shows that the average absolute value of has the same order as the average absolute value of linear velocity. While the boundary conditions change the curves for $\omega(r)$ the linear velocity components are virtually unaffected by the boundary conditions for $r = m$,

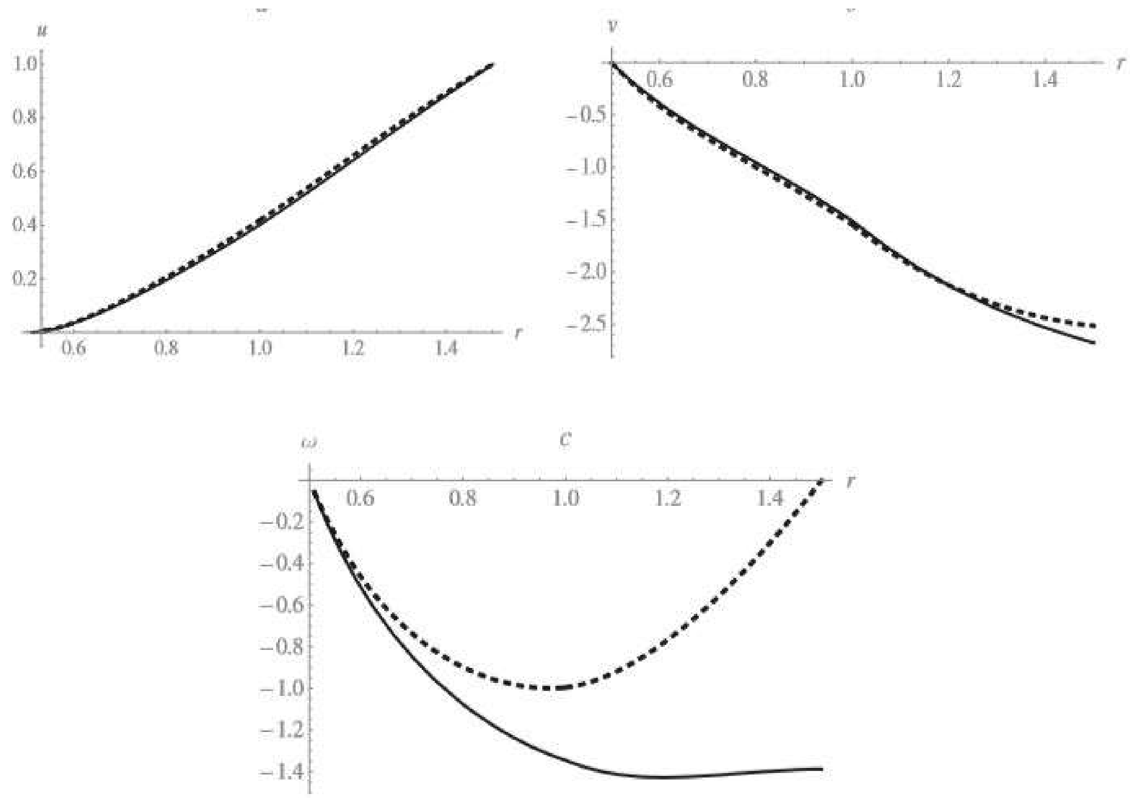

Fig.2: The variation of radial velocity componentu(r)(a),

Tangential velocity componentv(r)(b) and angular velocityω(r)(c) under theno-couple stress condition(solidlines)and the “no-spin condition (dashed lines) on the outer surface of the cell”.

CONCLUSION

This study is a continuation of previous efforts to use the cell model approach to simulating filtration fluxes for micropolar fluids. In order to examine the flow features parametrically, we solved the boundary value problems analytically and acquired the answers in an explicit manner. In this investigation, the hydrodynamic permeability was used to reflect the system's overall features. Although the hydrodynamic permeability is described in clear analytical form, studying its dependency upon any parameter is not possible due to the complexity of the description. In light of this, a series of computational parametric experiments was carried out. For all possible values of each variable, graphs showing how the hydrodynamic permeability changes with the variable were shown.

REFERENCE

Khanukaeva, D. (2020). Filtration of micropolar liquid through a membrane composed of spherical cells with porous layer. *Theoretical and Computational Fluid Dynamics*, *34*(3), 215-229.

Yadav, P. K. (2020). Influence of magnetic field on the Stokes flow through porous spheroid: hydrodynamic permeability of a membrane using cell model technique. *International Journal of Fluid Mechanics Research*, *47*(3).

Hauswirth, S. C., Bowers, C. A., Fowler, C. P., Schultz, P. B., Hauswirth, A. D., Weigand, T., & Miller, C. T. (2020). Modeling cross model non-Newtonian fluid flow in porous media. *Journal of Contaminant Hydrology*, *235*, 103708.