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RAIN FORECASTING IN BANIHAL REGION OF JAMMU AND KASHMIR USING WAVELET TRANSFORM AND DEEP LEARNING

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ABSTRACT

Accurate rainfall forecasting is essential due to the significant impacts of heavy and irregular rainfall, such as crop destruction and property damage. A reliable forecasting model can provide early warnings to minimize risks to life and property while improving agricultural management. This benefits farmers by enabling more efficient use of water resources. Weather forecasting in the hilly territory in India has been always a challenging but needful task. Even after eternities of research and technological advancement in recent past, limitations exist particularly while data acquisition. Siting of the equipment in hilly terrains itself is a big challenge. While getting data using such equipment the resultant data may noisy because of inherit errors. In this paper, we have applied a technique using wavelet transform method at pre-processing stage to process the quality data for better forecasting. Several tabulated information and graphs are presented in this paper for validation of the proposed technique.

Keywords: wavelet transform, thresholding, rain data, forecasting.

1. INTRODUCTION

Rainfall forecasting is an important field of study as it helps in the effective management of water resources, agriculture, and disaster management. Traditional methods for rainfall prediction include statistical and numerical techniques that have their limitations, especially when dealing with highly complex and nonlinear phenomena. Wavelet analysis is a powerful mathematical tool that has the ability to analyse and extract information from complex signals. In recent years, wavelet-based techniques have been applied to rainfall prediction with promising results. Forecasting rainfall is essential for managing water resources effectively, including the operation of reservoirs and dams, and planning water supply and distribution. Accurate predictions allow for better decisions regarding water allocation, release, and conservation, which are critical in preventing and mitigating natural disasters such as floods, landslides, and droughts. These forecasts support strategic planning and resource management, ensuring the safety and sustainability of water resources. Early warning systems based on these forecasts can help communities prepare and reduce loss of life and property damage [1-3, 6, 12]. Additionally, accurate predictions aid in transportation planning, hydroelectric power generation, and the maintenance of other energy sources, thereby supporting sustainable development and improving community welfare.

Predicting rainfall is a complex and challenging task due to the nonlinear and intricate nature of the processes involved. Accurate forecasts are crucial because heavy and erratic rainfall can lead to severe consequences, such as crop damage and property destruction. A dependable forecasting model can issue early warnings, helping to mitigate risks to life and property and enhancing agricultural management practices. This benefits farmers by enabling more efficient use of water resources. Rainfall prediction is challenging and requires precise results. While there are various hardware devices that predict rainfall based on weather parameters like temperature, humidity, and pressure, employing machine learning algorithms can enhance accuracy over traditional approaches [19-21]. Traditional methods for rainfall prediction include statistical and numerical techniques, such as regression analysis, autoregressive models, and artificial neural networks. However, these methods have their limitations, especially when dealing with highly complex and nonlinear phenomena.

Wavelet analysis is a powerful mathematical tool that has been applied to a wide range of fields, including signal processing, image compression, and data analysis. In recent years, wavelet-based techniques have been used for rainfall prediction with promising results. Wavelet analysis has the ability to extract information from complex signals that may not be easily detected by other methods. Thus, it is a promising approach for rainfall prediction [9, 10]. Wavelet transform is a mathematical technique used for analysing and processing signals, and data. It is particularly useful in analysing data that has both high and low frequency components. The basic idea behind wavelet transform is to decompose a signal into its constituent wavelet functions, which are then analysed to extract relevant information [27].

In this paper, we propose to investigate the application of wavelet analysis for rain forecasting in Jammu & Kashmir. Specifically, we will examine the effectiveness of wavelet analysis in forecasting rainfall using historical data, and compare its performance with the data itself. The results of this study will provide insight into the potential of wavelet analysis for rainfall prediction and may have practical implications for water resources management and disaster preparedness. Jammu & Kashmir is a region that is particularly vulnerable to floods and landslides caused by heavy rainfall. Accurate rain forecasting is therefore critical for ensuring the safety and well-being of the region's inhabitants. Very recently, A. S. Raj et al. [1] proposed a wavelet-based analysis for rainfall in the Kanyakumari district of Tamil Nadu, India. In this paper they also analysed a short-term forecasting with some soft computing tools. For checking the performance of their model, they have used regression coefficients and mean absolute percentage error. Particularly Li Diao et al. [10] have proposed wavelet denoising of data and conducted to process part of the "learning" tasks in advance. This idea motivated us to use further analysis of wavelet transform in association with Recurrent Neural Network (RNN) to analyse rainfall data from Banihal region of Jammu and Kashmir and develop a model for better rain forecasting, which is mentioned in this paper.

2. Preliminaries

Let P represent the set of real numbers. The inner product of two functions $f, g \in L^2(\mathrm{P})$ is denoted by

$$\langle f, g \rangle = \int_{\mathrm{P}} f(x)\overline{g(x)}dx$$

where bar represents complex conjugations.

Wavelet transforms draws attention after Morlet and Grossman's development on signal processing. A wavelet is a function that can be used to decompose a signal into a set of coefficients at different scales and positions. After decomposing a signal using wavelet analysis into multiple layers, the signal's components at various frequencies are isolated. This technique converts a non-stationary signal into several stationary signals, allowing traditional forecasting methods to be applied. The Grey model is typically used for analyzing data with deterministic trends [25].

Mathematically, a wavelet is a function $\psi(t)$ that satisfies the following properties:

- Zero mean: $\int \psi(t)dt = 0$
- Finite energy: $\int |\psi(t)|^2 dt < \infty$
- Orthogonality: $\int \psi(t) * \psi(t-n)dt = \delta(n)$, where $\delta(n)$ is the Kronecker delta function.

2.1 Scaling and Translation: A wavelet can be scaled and translated to obtain a family of functions that can be used to analyze signals at different scales and positions. Mathematically, the scaled and translated wavelets are given by:

$$\psi_{s,u}(t) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{t-u}{s}\right)$$

where s is the scale parameter, u is the translation parameter, and $\sqrt{s}$ is a normalization factor.

2.2 Definition [15]: A function $\psi : \mathbb{R} \to \mathbb{R}$ is called a mother wavelet of order m if the following properties are satisfied:

(i) If $m > 1$, then ψ is $(m-1)$ times differentiable.

(ii) $\psi \in L^\infty(\mathbb{R})$. If $m > 1$, for each $j \in \{1, ..., m-1\}$, $\psi^{(j)} \in L^\infty(\mathbb{R})$.

(iii) ψ and all its derivatives up to order $m-1$ decay rapidly.

(iv) For each $j \in \{0, ..., m\}$, we have $\int t^j \psi(t) dt = 0$. This property is also known as the vanishing moment property. It is highly beneficial because it enables efficient representation of the functions being analyzed.

(v) The set $\{\psi_{j,k} : j,k \in \mathbb{Z}\}$ is an orthonormal basis of $L^2(\mathbb{R})$, where $\psi_{j,k}$ are

derived from the mother wavelet by relationship $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$.

The expression for wavelet coefficients is given by

$$f_{j,k} = \int_{-\infty}^{+\infty} f(t)\psi_{j,k}(t)dt$$

2.3 MRA Definition [23]: A sequence of closed subspaces $\{V_j : j \in Z\}$ of $L^2(\mathbb{R})$ together with a function $\varphi \epsilon V_0$ is called a multiresolution analysis (MRA) if it satisfies the following conditions:

(a) (**Increasing**): $V_j \subset V_{j+1}, \quad \forall\, j \in \mathbb{Z}$

(b) (**Density**): cl $\bigcup_{j \in Z} V_j = L^2(\mathbb{R})$

(c) (**Separation**): $\bigcap_{j \in Z} V_j = \{0\}$

(d) (**Scaling**): $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}, \forall j \in \mathbb{Z}$

(e) (**Orthonormal Basis**): There exists a function $\varphi \in V_0$ such that $\{\phi(x-k) : k \in \mathbb{Z}\}$ forms a Riesz basis for V_0.

The function φ mentioned in (e) is called a scaling function of the given MRA [11, 16-17] or **father wavelet.**

Let W_0 be an orthogonal complement of V_0 in V_1, i.e. $V_1 = V_0 \oplus W_0$. Then, if we dilate the elements of W_0 by 2^j, we obtain a closed subspace W_j of V_{j+1} as

$$V_{j+1} = V_j \oplus W_j, \quad \forall\, j \in \mathbb{Z}$$

A function $\psi \in W_0$ whose translates $\{\psi(x-k) : k \in \mathbb{Z}\}$ form a Riesz basis of W_0 is called a **mother wavelet**.

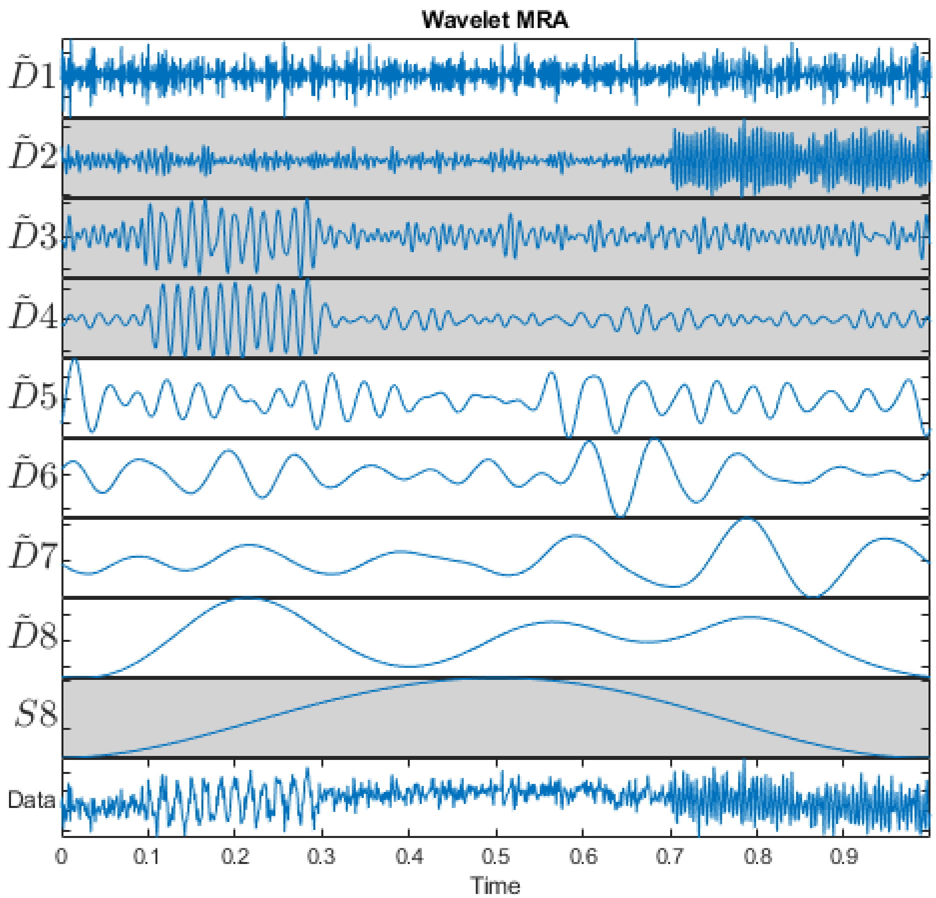

Fig1. Multiresolution Analysis of a Signal

We can represent any function (or signal) $f \in L^2(\mathrm{P})$ in terms of wavelet function $\psi_{j,k}$ and its scaling function $\varphi_{j,k}$ as

$$f(x) = \sum_k c_{j_0}(k)\varphi_{j_0,k}(x) + \sum_{j=j_0}^{\infty}\sum_k d_j(k)\psi_{j,k}(x)$$

where j_0 is any arbitrary starting scale and the $c_{j_0}(k)$'s are normally called the approximation or scaling coefficients and $d_j(k)$'s are called the detailed or wavelet coefficients. The expansion coefficients are calculated as follows:

$$c_{j_0}(k) = \left\langle f, \tilde{\varphi}_{j_0,k} \right\rangle = \int f(x)\, \tilde{\varphi}_{j_0,k}(x)\, dx$$

and $$d_j(k) = \left\langle f, \tilde{\psi}_{j,k} \right\rangle = \int f(x)\tilde{\psi}_{j,k}(x)\, dx$$

The function can be represented as a sequence of numerical values, similar to sample points of a continuous function f. When the resulting coefficients are discrete, the series expansions are defined by specific equations.

$$W_{\varphi}(j_0, k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x) \tilde{\varphi}_{j_0, k}(x)$$

and

$$W_{\psi}(j, k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x) \tilde{\psi}_{j, k}(x)$$

For $j \geq j_0$, f can be expressed as follows

$$f(x) = \frac{1}{\sqrt{M}} \sum_{k} W_{\varphi}(j_0, k) \varphi_{j_0, k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_{k} W_{\psi}(j, k) \psi_{j, k}(x)$$

where f, $\varphi_{j_0,k}$ and $\psi_{j,k}$ are the functions of discrete variables x = 0, 1, 2, 3,…., M−1.

The wavelet transform decomposes the input signal into a set of coefficients, which represent the contribution of each wavelet function at different scales and positions [15-16]. These coefficients can be used to reconstruct the signal, or to analyse its properties in a multiscale manner.

Discrete Wavelet Transform: The discrete wavelet transform (DWT) is a variant of the wavelet transform that can be used to analyse digital signals. It uses a set of discrete wavelets instead of continuous wavelets. Mathematically, the DWT of a signal $\mathrm{x}[\mathrm{n}]$ using a discrete wavelet ψ[n] is given by:

$$W_{j,k} = \sum n \; x[n] \psi^*[n - 2^j k],$$

here $W_{j,k}$ is the wavelet coefficient at scale j and position k, and $\psi^*[n]$ is the complex conjugate of the discrete wavelet [23].

3. METHODOLOGY

In this study, rainfall data from Jammu and Kashmir were analyzed using wavelet transform to understand time-varying inputs. Wavelets are increasingly used in fields like communications, signal processing, and image processing due to their ability to provide precise localization of changes in input signals, unlike Fourier transforms which focus only on the frequency domain. Multiresolution wavelet analysis offered insights into the power spectrum of rainfall and water table depth data. Machine learning was then applied, using monthly rainfall data from 1972 to 2018 for training cum testing and rainfall data from July to August 2018 for the forecast.

3.1 Forecasting Tool

In recent years, many soft computing and deep learning tools have been used for forecasting data [4, 5, 7, 9-14, 18-22, 24, 26-28].

Some forecasting examples [Matlab Sources] can be seen as:

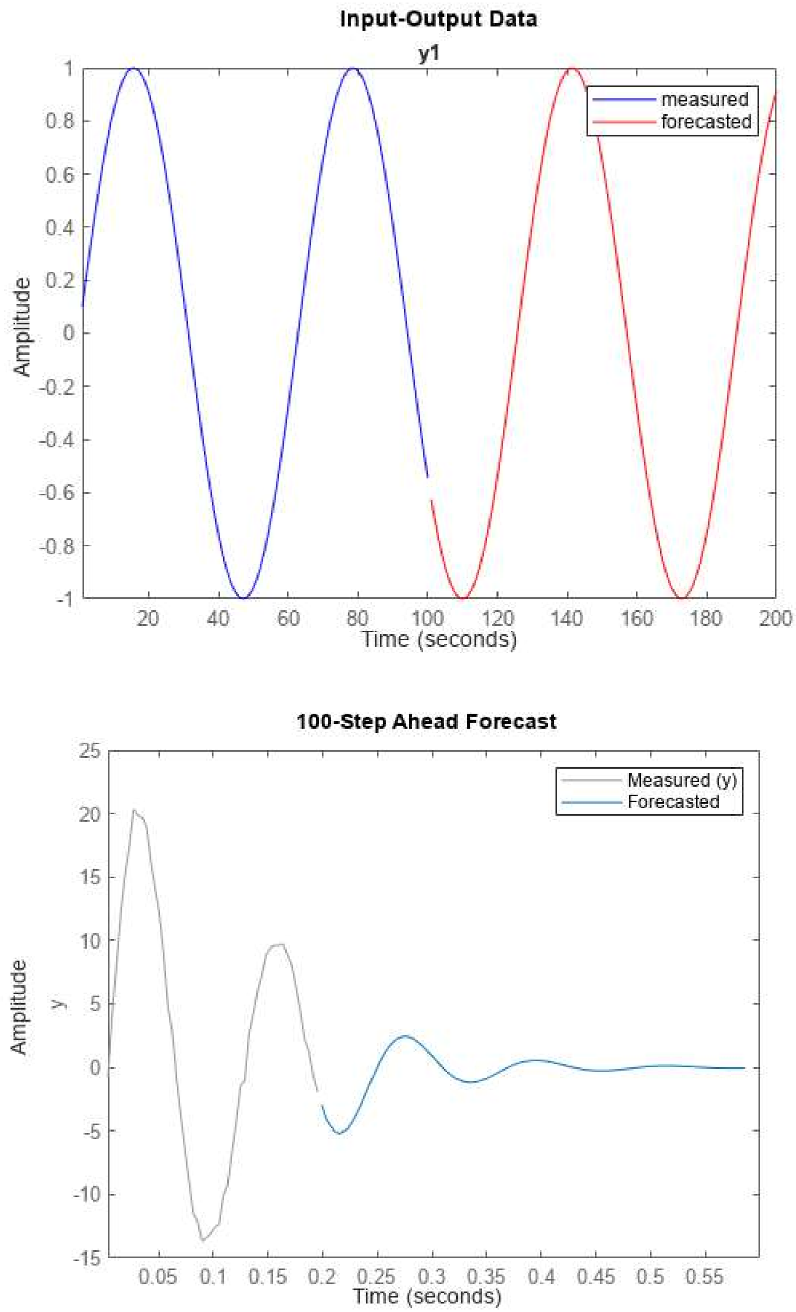

Fig 2: Basic forecasting examples

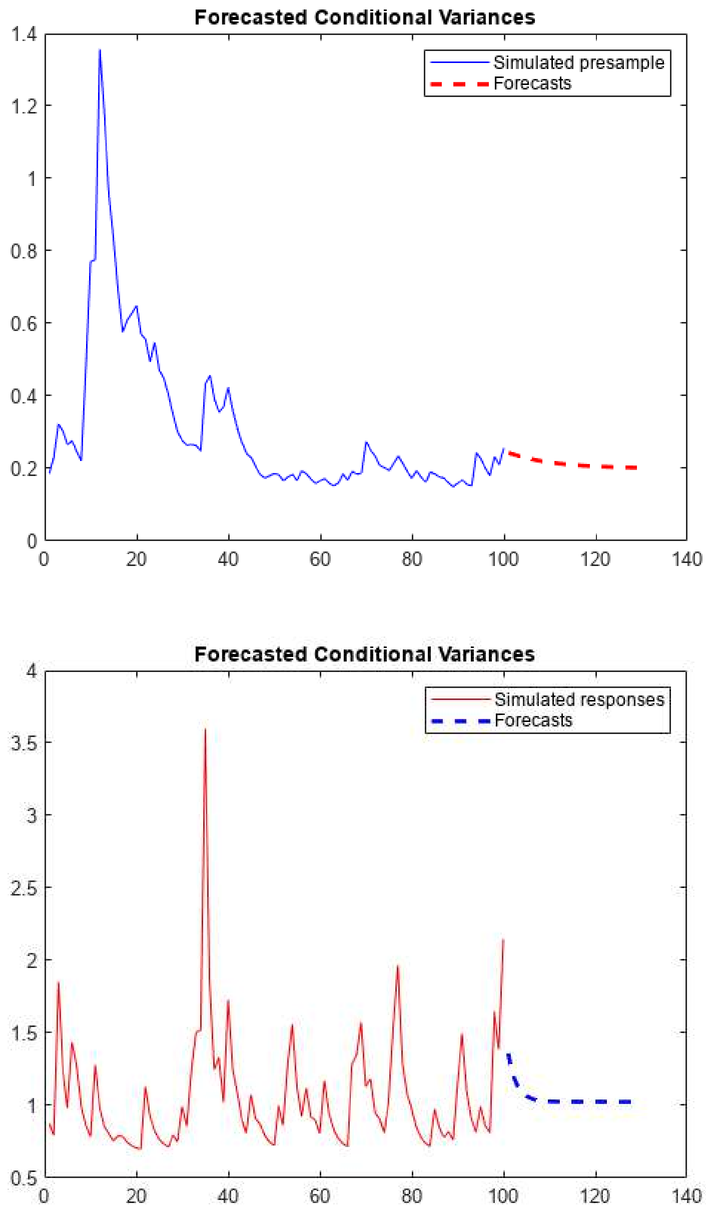

Fig 3: Forecasting examples with conditional variances

Particularly, in ANN the input data can be taken as:

$$I = \sum_{j=1}^{n} w_j x_j$$

To generate final output y, the sum needs to pass on to a nonlinear filter [7, 13], which releases the output. i.e. $y = \varphi(I)$. Now, a very important thresholding value is needed to minimise the noise power from the noisy signal. In this paper we are using universal thresholding, because white Gaussian noise is distributed throughout the signal.

3.2 Deep Learning Tool: RNN

LSTM stands out as a specialized form of deep learning tool designed to capture long-term dependencies [14]. Unlike traditional neural networks, LSTMs were specifically engineered to overcome challenges associated with retaining information over extended periods. In a deep learning system, the input layer comprises artificial input neurons responsible for delivering pre-processed weather data to subsequent layers for processing [14]. Despite their chain-like structure, LSTMs feature a distinct repeating module consisting of four neural network layers that interact in unique ways.

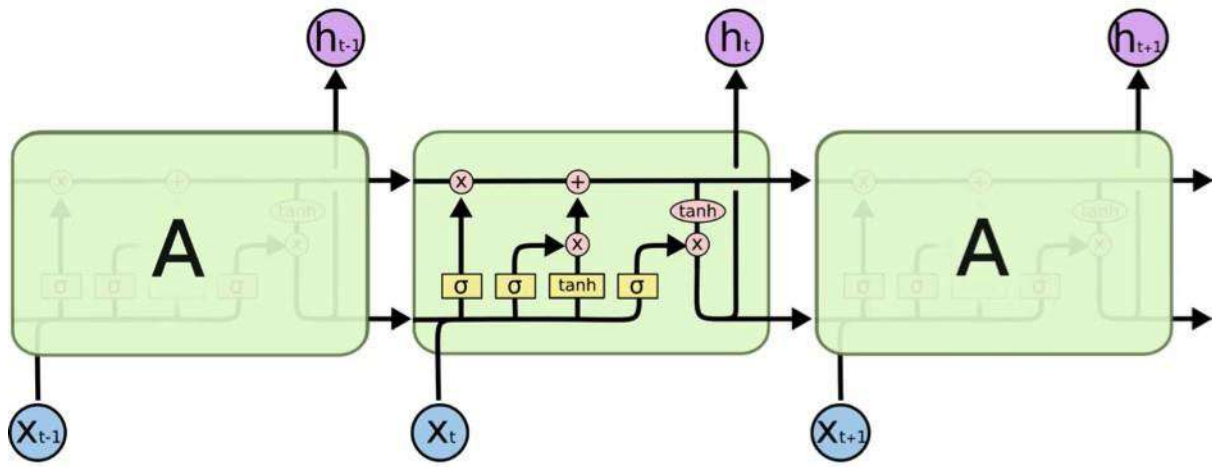

Fig. 4 LSTM Module and Interaction of Components

In this paper, we have proposed a method to investigate the effectiveness of wavelet analysis for rainfall prediction using historical data keeping in mind that the recorded data have some noise because of the inherent error of the recording machines. Specifically, we have used Haar wavelet to extract features from the rainfall time series and then used these features to train a machine learning model for rainfall prediction. We have compared the performance of wavelet analysis using the existing performance metrics technique i.e., mean square error (MSE) [8, 15].

3.3 Algorithm: We have used following algorithm for forecasting rain using wavelet transform:

- **Collect Rainfall Data:** The first step is to collect rainfall data of Jammu & Kashmir from 1974-2018, which is to be used to train the forecasting model.
- **Perform Wavelet Transform:** It is assumed that the collected rainfall data is corrupted with white noise due to inherent error of the system. We have used wavelet transform to decompose the signal into different scales. This is done using Haar wavelet.
- **Feature Extraction:** After performing the wavelet transform, statistical features such as mean, variance, skewness, and energy are extracted from the decomposed signals.
- **Data Refinement:** We have refined the data e.g. missing values and other kinds of artifacts to make data ready for further processing. We have used 4800 days data from refined dataset. LSTM was used to take some basic weather parameters and has been utilised to predict the rainfall based on the input parameter value.
- **Test and Validate the Model:** The trained model is tested and validated using the rainfall data from a different time period than the one used for training. The performance of the model is evaluated using metrics such as Mean Squared Error (MSE). It measures the average squared difference between the predicted values and the actual values. The formula for calculating the mean squared error is:

MSE = $\frac{1}{n}\sum_{i=1}^{n}(y_i - \tilde{y}_i)^2$, where y_i are true values and $\tilde{y}$ are approximated values.

Where n is the number of data points in the validation set, y_i is the actual value of the i^{th} data point and $\tilde{y}_i$ is the predicted value of the i^{th} data point.

- **Use the Model for Forecasting:** Once the model is validated, it can be used for forecasting rainfall for a future time period based on the weather conditions and other relevant factors.

By following above steps, an algorithm for rain forecasting using wavelet transform has been developed in this paper. This algorithm can be used by meteorologists and weather forecasting agencies to provide accurate and reliable rainfall forecasts, which can help in disaster management and resource allocation.

STATTION BANIHAL

DISTRICT RAMBAN

DIVISION SRINAGAR

DATE OF INSTALLATION 20-06-1956

HEIGHT A.M.S.L 1690 M

LATITUDE 33° 26' 11.29" N

LONGITUDE 75° 11' 48.62" E

Table 1:

STNID	Year	Month	Day	TempMax	Temp Min	Rain(mm)	RH 0830	RH 1730
9212	1977	7	1	34.2	14.7	0.0	84	85
9212	1977	7	2	26.3	19.2	0.0	84	85
9212	1977	7	3	26.6	17.2	11.8	95	80
9212	1977	7	4	27.3	19.5	0.0	88	78
9212	1977	7	5	26.6	20.2	0.0	77	64
9212	1977	7	6	26.7	21.2	0.0	86	81
9212	1977	7	7	25.1	20.4	0.0	99	78
9212	1977	7	8	26.1	20.2	17.6	88	95
9212	1977	7	9	22.4	18.5	2.2	91	72
9212	1977	7	10	26.0	20.2	0.0	81	70
9212	1977	7	11	26.6	19.4	13.3	96	80
9212	1977	7	12	24.6	19.5	0.0	85	77
9212	1977	7	13	26.3	19.8	4.0	93	80
9212	1977	7	14	26.0	19.6	4.6	98	92
9212	1977	7	15	24.3	19.0	60.4	96	98
9212	1977	7	16	20.1	18.0	23.0	96	98
9212	1977	7	17	20.7	17.9	40.3	98	77
9212	1977	7	18	23.9	17.7	2.4	83	78
9212	1977	7	19	26.1	17.2	4.8	86	84
9212	1977	7	20	26.0	20.2	0.0	84	77
9212	1977	7	21	27.4	20.2	0.0	88	77
9212	1977	7	22	25.7	21.2	0.0	96	77

9212	1977	7	23	26.3	20.2	6.6	84	74
9212	1977	7	24	26.6	20.2	0.0	98	88
9212	1977	7	25	26.1	19.2	14.6	100	88
9212	1977	7	26	24.1	18.2	5.8	91	77
9212	1977	7	27	25.1	19.2	0.0	91	77

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STNID	Year	Month	Day	TempMax	Temp Min	Rain(mm)	RH 0830	RH 1730
9212	2017	12	28	15.8	4.3	0.0	62	50
9212	2017	12	29	15.6	3.2	0.0	64	36
9212	2017	12	30	15.9	4.1	0.0	31	31
9212	2017	12	31	13.9	4.5	0.0	35	39
9212	2018	1	1	14.4	2.7	0.0	52	55
9212	2018	1	2	14.3	2.0	0.0	44	48
9212	2018	1	3	13.2	2.0	0.0	40	51
9212	2018	1	4	13.4	2.1	0.0	56	37
9212	2018	1	5	9.0	1.0	0.0	75	57
9212	2018	1	6	11.6	0.3	0.0	61	41
9212	2018	1	7	13.2	1.9	0.0	43	44
9212	2018	1	8	14.0	1.6	0.0	43	36
9212	2018	1	9	15.2	1.9	0.0	37	28
9212	2018	1	10	16.1	2.6	0.0	38	27
9212	2018	1	11	13.5	5.2	0.0	36	33
9212	2018	1	12	16.1	4.6	0.0	39	35
9212	2018	1	13	16.0	2.9	0.0	49	35
9212	2018	1	14	15.7	3.7	0.0	44	34
9212	2018	1	15	15.2	3.3	0.0	31	25
9212	2018	1	16	17.7	3.8	0.0	29	31
9212	2018	1	17	11.2	3.1	0.0	43	49
9212	2018	1	18	15.7	3.9	1.5	53	42
9212	2018	1	19	18.2	3.4	0.0	40	28
9212	2018	1	20	17.2	5.8	0.0	31	27
9212	2018	1	21	18.5	7.3	0.0	31	40
9212	2018	1	22	15.3	4.6	0.0	40	44
9212	2018	1	23	8.2	2.7	0.0	65	91
9212	2018	1	24	12.3	0.6	3.6	74	69
9212	2018	1	25	13.0	1.3	0.0	82	68
9212	2018	1	26	13.1	1.1	0.0	73	52
9212	2018	1	27	13.9	0.8	0.0	56	46
9212	2018	1	28	14.8	1.0	0.0	51	35
9212	2018	1	29	15.7	2.8	0.0	40	35
9212	2018	1	30	15.9	7.7	0.0	45	43
9212	2018	1	31	14.7	5.7	0.0	54	44

4. RESULTS AND DISCUSSION

We compared the performance of wavelet analysis with traditional methods for rainfall prediction. The threshold represents the estimated noise level. Values above this threshold are considered signal, while those below it is considered noise. When decomposing the noisy output using a selected basis, noise estimators can still be applied effectively, as white noise characteristics are preserved across different bases. For threshold selection we have used Universal Estimation method [15]. This method uses a fixed threshold chosen to yield better performance for mean square error. Donoho and Johnstone [8] proposed a universal threshold method, demonstrating that the risk associated with thresholding, whether it is hard or soft, remains minimal and meets the requirements of most applications. The level of decomposition was four. The results showed that wavelet analysis outperformed traditional methods in terms of accuracy and efficiency. For example, the MSE for wavelet analysis was 0.00392, while the MSE for regression analysis was 2.57490. Similarly, we can see other parameters for visualising errors in Table 2. These results demonstrate the potential of wavelet analysis in association with LSTM for rainfall prediction.

The following graphs gives the visual impact of the proposed methodology used in this paper.

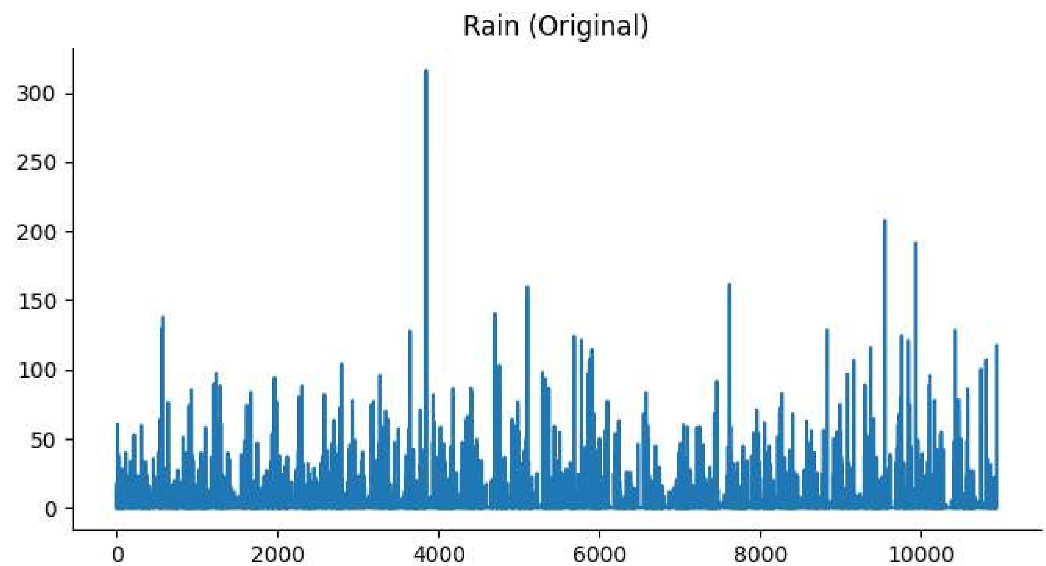

Fig 5: Rainfall data of Banihal region of Jammu & Kashmir from 1972-2018

Table 2: Comparative analysis of different tests with different models

Tests \ **Models**	Decision Tree	SVM	Linear Regression	WT+LSTM
MSE	0.03714	0.023038	2.57490	**0.00392**
MAE	0.08732	0.031222	1.27178	**0.00835**
R^2	0.977790	0.965140	1.01204	**0.94878**

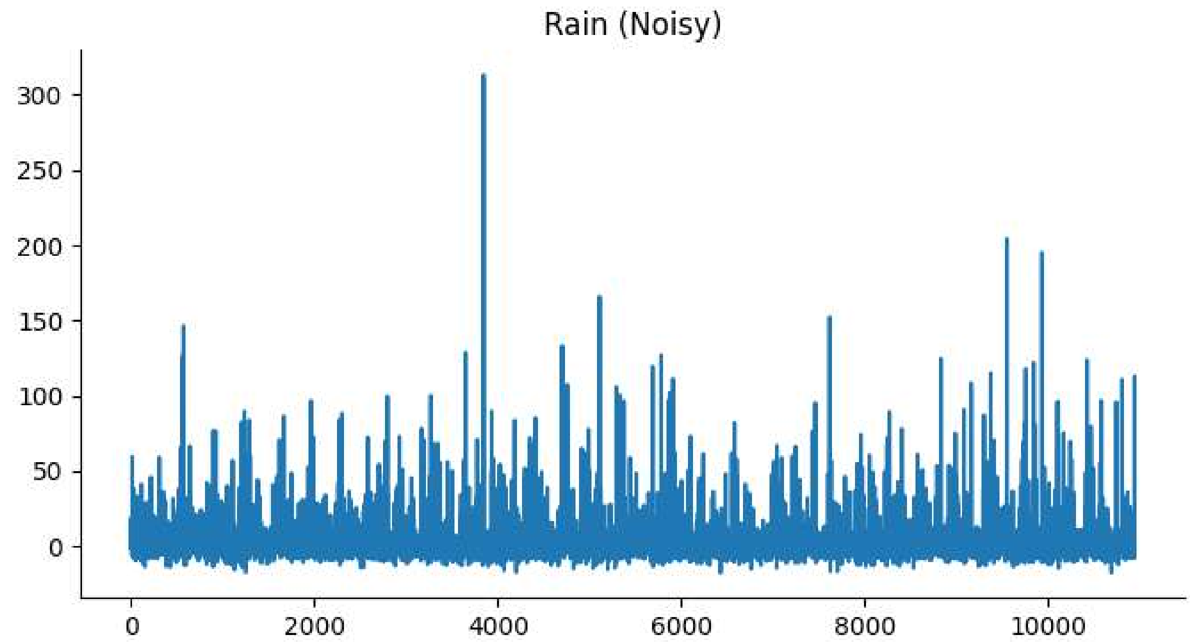

Fig 6: Noisy Rainfall data of Banihal region of Jammu & Kashmir from 1972-2018

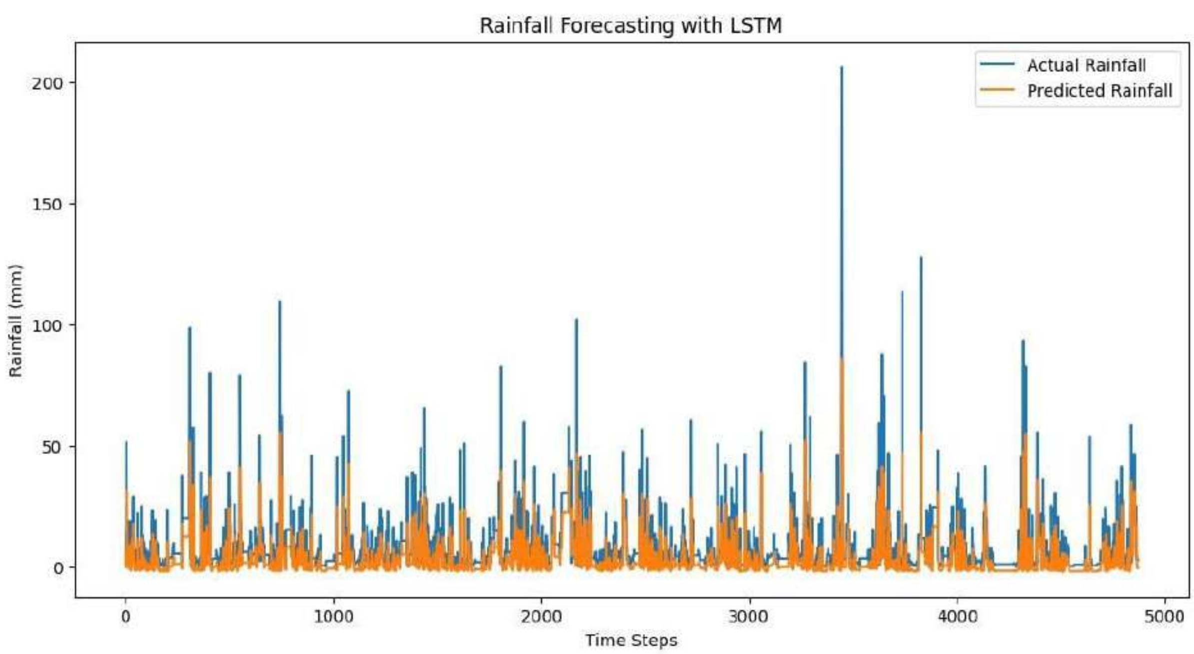

Fig 7: Prediction of Banihal region rainfall data of Jammu & Kashmir in noisy condition

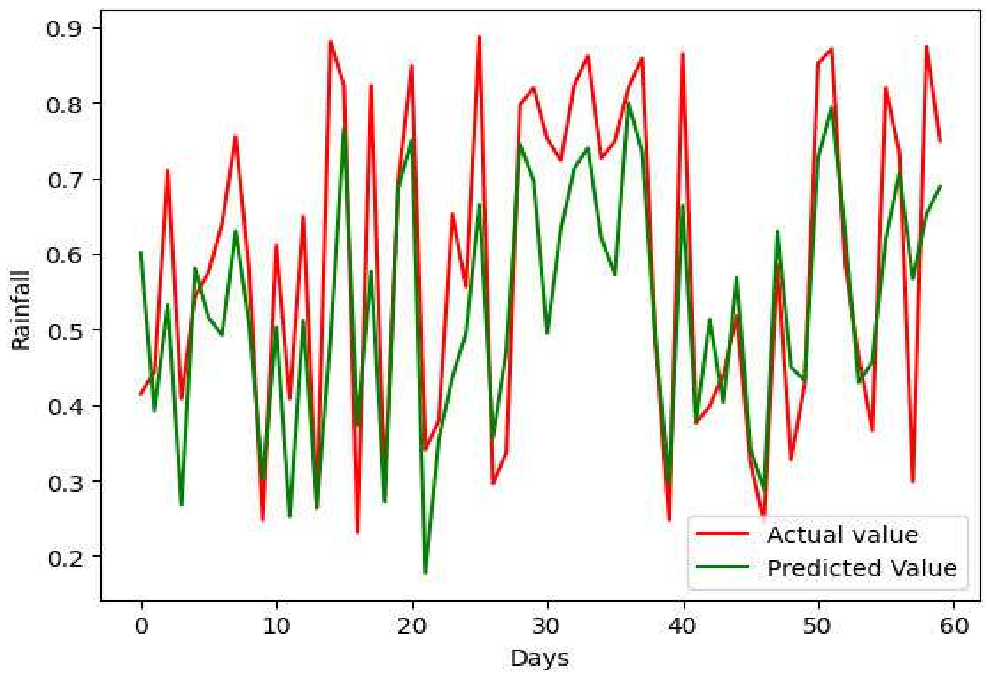

Fig 8: Forecasting rainfall in noisy environment in the months July-August, 2018

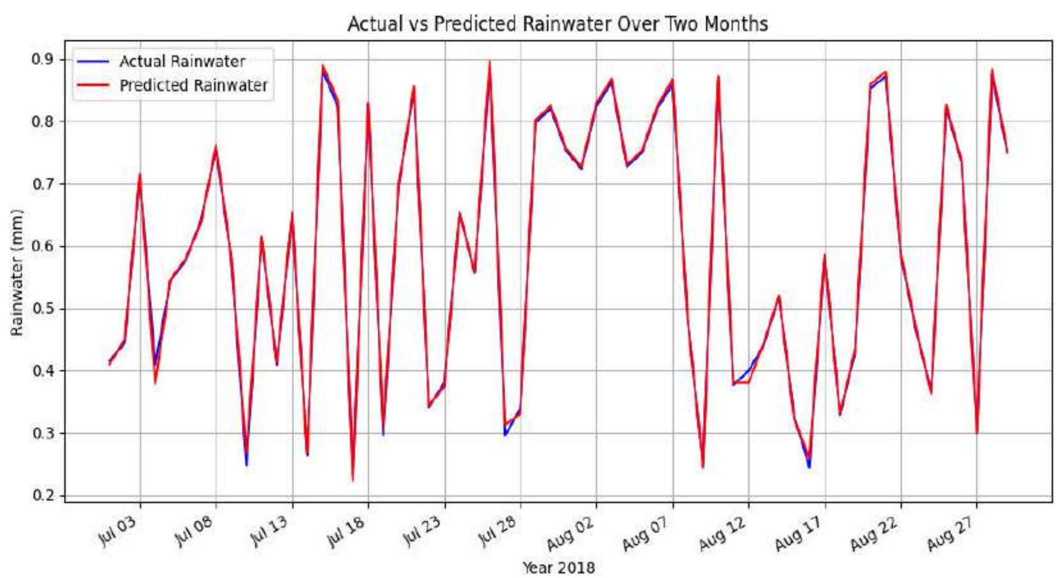

Fig 9: Forecasting rainfall after wavelet denoising for the months July-August, 2018

5. CONCLUSIONS

In this paper, we employ the LSTM and wavelet transform algorithms to forecast rainfall. Additionally, various other prediction techniques are employed for the purpose of comparative analysis. One metric used is the Mean Squared Error (MSE), which measures the average squared difference between the predicted and actual values. This metric is detailed in Table 2. The role of wavelet analysis is a promising method for prediction of rainfall for the Banihal region of Jammu and Kashmir. For simulation we have used Matlab and Python. The results in Fig 9 shows that it performs better than traditional methods in terms of accuracy and efficiency. The results of this study may have practical implications for water resources management and disaster preparedness.

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29. Figure 4 image source: colah.github.io.