Stochastic Modelling and Computational Sciences

OPTIMIZATION OF PATIENT FLOW IN THE EMERGENCY DEPARTMENT OF A HEALTHCARE FACILITY USING QUEUING THEORY

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ABSTRACT
Emergency departments (EDs) are crucial healthcare settings where halting free movement of patients is the prime requirement. Triage is adopted for efficient patient categorization and provision of adequate treatment at the earliest. However long time elapsed between the emergency room and the adoption of triage generates a long queue of patients. A study has been carried out to determine the adequacy of queuing theory in handling patients for the triage process to provide effective patient care. The patient flow in the emergency department of AIIMS Delhi has been optimized using patient arrival and registration data. The study was conducted in August 2022 to observe the patient flow from the registration area, triage process and emergency department. The average expected patients in for emergency healthcare observe waiting of up to 2-3 mins and shift to triage for vitals immediately after the registration. The time spent by the patient in the process is evaluated using queuing theory and several interventions are determined to optimize the patient flow rate and reduce waiting time. The study suggested that continuous monitoring and data-driven decision-making are essential measures to be adopted in ED operations.

Keywords: Emergency department, queuing theory, patient flow, simulation

1. INTRODUCTION
Emergency departments (EDs) are critical healthcare settings where efficient patient flow is crucial for timely care and resource utilization requires zero waiting time for patients (Laskowski et al., 2009). The patients opt for the ED for different and severe health concerns that require immediate attention immediately. As ED has 24-hour access, nurses and other medical staff are expected to be available throughout the day to handle incoming patients without keeping patients in waiting lines (Kumar et al., 2019; Pak et al., 2021). Most of the delays that occur after receiving patients in healthcare facilities are due to transferring patients between different interfaces and starting the process of early examination (Jones, P., & Schimanski, 2010; Kumar et al., 2019). The patient categorization procedure is the initial step in the emergency department. The use of methods such as triage enables the examination, categorization, admission, and prioritisation of patients based on the severity of their symptoms rather than the order of arrival. However, this first stage of therapy is linked with a higher clinical risk, which may be avoided if patients were treated promptly (Napi et al., 2019; Fernandes et al., 2020). The high preference for ED has caused an increasing demand and saturation in institutions offering this service, which has prompted to examination and restructuring of the systems to optimize patient care.

Queuing theory, a mathematical framework for analyzing waiting lines, can provide insights into patient flow dynamics in EDs (Adele & Barry, 2005) and inform interventions for performance improvement. Emergency departments (ED) are critical towards handling patients who require immediate treatment for acute illness and accidents (Magnusson et al., 2020; Mittal and Sharma, 2020). Green et al. (2008) assessed the inflow of patients in the emergency department to identify the number of doctors and other resources required for serving the patient and the study found that half of the increase in the resources compared to the double patient volume can handle the greater number of patients attended in a year. Hulshof et al. (2012) propose the categorization criteria for the rapid classification of patients based on their symptoms in Germany. Other researchers also proposed methodologies for the identification of similar symptoms of illness in patients. Haghighinejad et al. (2016) simulated the waiting time and queue length in healthcare facilities to identify the resource utilization scenarios and varied the bed capacity and resource availability to determine the utilization of resources without any waiting time. Jáuregui et al. (2017) determined the services that require immediate attention and resources required to

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serve the patients without the formation of any queue. Hu et al. (2017) identify the demand and supply gap in emergency departments using queuing theory. And found that the application of queuing theory can simplify the operation and optimize resource allocation to minimize congestion (Haghighinejad et al., 2016).

Similarly, Chen et al. (2016) applied an algorithmic mobile application and a queuing system, an Apache Spark-based cloud, to a substantial database of patients from many hospitals to foresee the needed time at each point in the treatment process. The experimental results confirmed the application's effectiveness and reduction in waiting time. The study was performed in a tertiary hospital in Australia, patients with low acuity stayed in the waiting room, and high-acuity patients had priority in the no-waiting area. The study proposed machine learning algorithms and the mean squared to predict the waiting time (Pak et al., 2021). However, in another study, the autocorrelation coefficient and Pearson's correlation were used to forecast the crowding of patients in an emergency department. The results showed that, on average, the waiting time was approximately 13 min, the occupancy was 83%, and the length of stay was 6.4 hours (Hoot et al., 2008). Remarkably, the need to visit the ED is related to specific seasons or days of the year (Jones et al., 2008). Accordingly, Sun et al. (2012) disclosed that waiting times differ according to the days or weeks. They also recorded the times and dates of patients' treatments and categorized them into three categories, from the most critical to the least. Based on quantile regression and the absolute prediction error, the analysis revealed that strata (1, 2, and 3) composed 6.8%, 41.9%, and 51.3% of the total, respectively. However, after controlling for confounders, the median absolute prediction error for stratum 1 was insignificant for prediction waiting time because the queue size increased. As a result, the shorter the wait time, the faster the flow of patients, with categories 2 and 3 taking 9.2 and 12.9 min, respectively. Otherwise, Additionally, in analyzing the queuing theory in an Iranian ED, the aim was to minimize waiting time, so the suggestions were to increase the bed capacity and other required resources, as well as classify patients in terms of disease intensity, which would be better than the medical speciality (Haghighinejad et al., 2016; Mittal and Sharma, 2022).

Napi et al. (2019) reviewed the medical emergencies that require immediate care, triage scenarios were analyzed and patient prioritization was categorized and identified that automation of triage based on prioritization of patient illness can reduce the waiting time however, in case of disasters such as fire events, rail accidents, earthquake etc., the patient's load increases significantly and expert advice is required for assessment, categorization and timely treatment to patients. Elalouf, A., & Wachtel (2021) reviewed the studies published on queuing theory and resolving the complexity of queuing problems and found that priority identification, bed allocation and triage approach are the important approaches to solving the queuing issue.

In this case study, we assess the patient flow process in the ED of All India Institute of Medical Science (AIIMS) Delhi which involves the patient arrival, and triage according to AIIMS Triage Protocol (ATP) (Sahu et al., 2020). This paper aims to utilize the data obtained from the arrival and triage time on patient arrivals at the emergency department to analyze the arrival pattern of the patients which will help the management in scheduling the allocated resources for the efficient functioning and performance of measures which are determined with the help of this data to optimize the patient flow in ED.

2. MATERIAL AND METHODOLOGY

2.1. Study Area

The All India Institute of Medical Sciences (AIIMS), in New Delhi, India is one of the most prominent and largest governmental medical institutes of India consisting of different specialist centres and divided into 5 campuses. AIIMS Delhi receives an annual outpatient load of more than 43.5 lakh, inpatients of nearly 2.4 lakh and conducted more than 1.9 lakh surgeries in 2017-18. The ED catering to the load of adult patients handles both surgical and medical crises excluding the emergencies that need to be handled by specialized departments.

The ED follows the AIIMS Triage Protocol (ATP) managed by the two resident physicians, two nursing staff, four supporting staff for the movement of patients and four security staff. The ATP is performed by the physicians with the support of nurses, where physicians are usually junior residents. The physicians also get support from

trained specialists in emergency medicine. The uncontrollable incoming load of patients arriving in the adult ED of hospitals faces several challenges in managing the patient load.

The doctors that staff the ED are generally junior residents, with backup from emergency medicine (EM) speciality-trained specialists. AIIMS's general adult emergency department sees almost 400 patients every day. The unprecedented visit of patients from the entire nation makes the quick assessment and categorization of patients challenging and results in long queues of patients near the triage area.

2.2. Patient Flow Process

The patient flow process at the ED begins with patients arriving at the ED, and their arrival times are recorded. EMTs assess patients' vitals and assign them color-coded bands based on the condition of patients considering various parameters in account. Patients then move to triage for doctor assessment. After the doctor's assessment, patients register at the registration counter. Patients may be discharged or admitted for further treatment based on the doctor's recommendations. The description of patient flow processes is as follows and shown in Figure 1.

***Arrival at ED**:* The patient flow rate follows the poison ratio with the constant arrival rate which is also known as the exponential process. Triage is performed by 2 EMTs and bands are given depending upon the situation of the patient, where Yellow and Red bands proceed further into the Emergency Section

Triage by Doctors: Doctors check their vitals again to reassign the bands given to the patients.

Yellow(Y) & Red(R): The respective patients are treated until they become stable and the bands given to them are changed. The patients categorized under the red category are referred to other emergency departments for further patient care if become critically ill and need surgery. Whereas, treated patients whose category changes to yellow are referred to OPD.

Arterial blood gas analysis (ABG) room: The tests are done for the identified patients.

Operational desk: This desk is used to manage all the operations regarding the transfer of patients, maintaining results and keeping data and track of the patients.

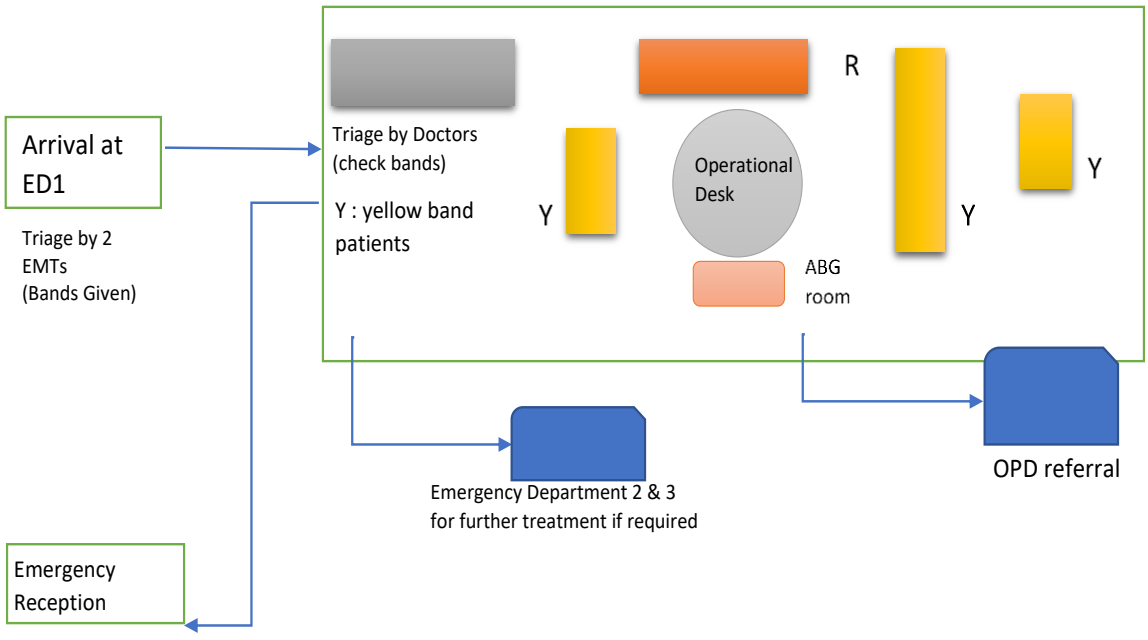

Figure 1: Emergency Department Flow Chart

2.3. Data Analysis

The patient arrival data, and registrations to understand the patient flow process were analyzed and areas were identified for improvement. We calculated basic arrival rate metrics and assessed variability in arrival times. We visualized patient arrival patterns over time and conducted a root-cause analysis to identify delays and waiting times. Our analysis revealed fluctuations in patient arrivals, with peak hours showing higher volumes. EMT assessment times varied based on patient severity, leading to wait time variability. Delays at the registration counter contributed to overall patient waiting time.

The patients enter in ED and triage is performed by 2 EMTs and then the band is assigned to the patient which is later on checked by the doctors at the entry counter in the Emergency Department. The emergency department is operational for 24 hours but this study is conducted between the time slot of 8 am - 5 pm for 7 days a week.

2.4. Queuing Model Formulation

The M/M/c priority queue model is most preferred for allocating patients in an emergency department because it realistically captures the dynamics of multiple servers and the prioritization of patients based on the severity of their medical conditions. By utilizing this model, EDs can effectively manage patient flow, reduce waiting time, and improve overall patient care. The M/M/c model is designed for a multi-server system handling K classes of patients. These patients are differentiated by their arrival rate (λ), processing requirements, and service rate (μ), which can lead to potentially long waiting times. The long waiting time of patients is attributed to the high patient load, time of arrival, age, number of available doctors, day of arrival, treatment-related diagnosis, and the roles of providers, record clerks, and clinicians (Tiwari et al., 2014; Ogaji et al., 2017). In our model, some patient classes have higher priority than others. All the patients arriving in the emergency department are considered urgent patients, where treatment is required without wasting time in the waiting room and proceeded with classification into high-priority patients. The various characteristics of the model are formulated

Patient Arrival: Patients arrive at the ED according to a stochastic process (often modelled as a Poisson process). Arrival rate (λ) represents the average number of patients arriving per unit of time.

Triage Classification: Patients are classified into different triage categories based on the severity of their conditions. Triage categories (e.g., Category 1: Immediate, Category 2: Urgent, Category 3: Less Urgent) have different service priorities and times.

Service Process: Patients receive treatment based on their triage category. Service rate (μ) represents the average number of patients that can be treated per unit of time.

Number of serves (c): The number of servers depicts the total staff of doctors/nurses available in the hospital to handle patient load.

Queue Structure: Model the ED as a multi-class queuing system with different priority levels (M/M/c queues for multiple servers). Each triage category forms a separate queue or sub-queue within a larger queue. A priority queuing system is implemented where higher triage categories are given precedence using priority queuing rules (e.g., preemptive priority, non-preemptive priority) to manage the flow.

The utilization factor (ρ) for each triage category is calculated using Equation 1:

$$\rho = \frac{\lambda}{c.\mu}$$

…1

The probability of occupancy of all the servers in the hospital is determined using Equation 2:

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!\,(1-\lambda/c\mu)}\right]^{-1}$$

…2

The average number of patients in the system (L) can be determined by:

$$L = \frac{\lambda\mu(\lambda/\mu)^c P_0}{c!\,(c\mu-\lambda)^2} + \frac{\lambda}{\mu}$$

…3

The average number of patients in queue (Lq) is calculated using:

$$L_q = L - \frac{\lambda}{\mu}$$

…4

The average time spent by patient in the hospital (W) is calculated using:

$$W = \frac{L}{\lambda}$$

…5

The average number of patients in the queue (W_q) of the system is calculated using:

$$W_q = \frac{L_q}{\lambda}$$

…6

A discrete model for ED process flow has been developed while incorporating the arrival, triage and service rate. The model has been simulated for triage processes to analyze the impact on patient flow and service delivery.

3. RESULTS AND DISCUSSION

3.1. Patient flow in the Emergency Department

The data collected from the hospital ED showed that an average number of patients approached hospitals in the time between 8 am to 5 pm, the data was collected at the interval of each hour, hence the first observation received at 9:00 am suggests the patients arrived from 8:00 am. The average number of patients who arrived at the hospital is shown in Figure 2. The number of patients arriving increases towards the middle of the day and the maximum number of patients approaches from 10:00 am to 11: 00 am and the least number of patients reach the hospital in the post-lunch hours i.e. 02:00 pm to 03:00 pm and again the patient arrival increases towards the evening hours. Figure 3, suggests the average number of patients who visited the hospital during the different days of the week, where on average 180 patients approached the hospital daily during the study period maximum patients reached during the weekends and the minimum patient load received on Tuesday or middle of the week.

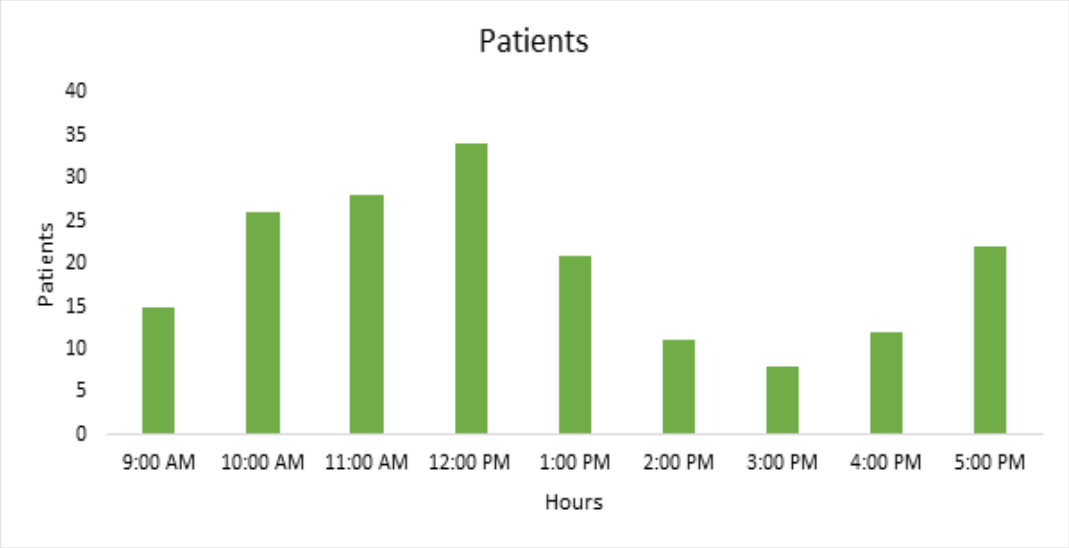

Figure 2: Arrivals of patients during a day at the Emergency Department

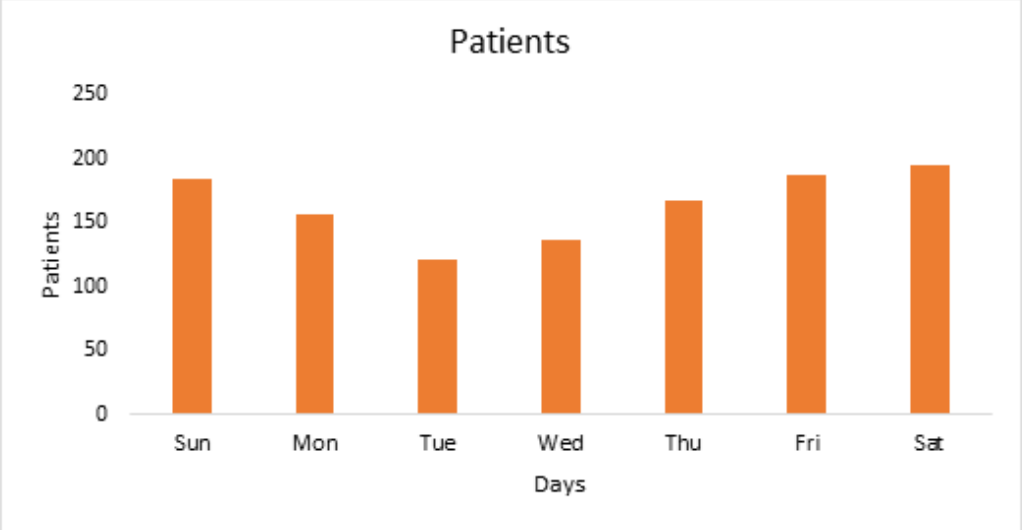

Figure 3: Arrival of patients on different days of the week

3.2. Patient flow in Triage

The arrival time at the ED and the service rate of the EMTs are the main factors directing the study. The data collection for the arrival rate is done when the patients register themselves at the ED arrival zone with the EMTs and observational data for the service time of EMTs is calculated for the time it took for the triage of the patients on arrival. After the data collection, the analysis was done with the help of Microsoft Excel to clean and model the data. This data is used to determine the waiting time of patients at ED and optimize the patient flow. The Input analyzer of the ARENA Simulation software is used to determine the distribution of the data. The queuing analysis of patient arrival and departure in the Emergency Department is shown in Table 1. The maximum number of patients arrived on Saturday followed by Sunday is 196 and 185 with operation efficiency of 83.8% and 81.5 %, respectively. The average number of patients in the queue was found highest on Saturday at 4.32 and the lowest was found on Wednesday at 1.54, as shown in Figure 4. The residence time of patients in the queue was found low throughout the week.

Table 1: Queuing analysis of patients who arrived Emergency department

Day	Arrival (λ)	Service (μ)	Avg operation (ρ)	Avg patient in queue (L_q)	Avg patient in system (L)	Avg wait in queue (W_q)	Avg time in system (W)
Sunday	185	227	0.815	3.590	4.405	0.019	0.024
Monday	156	211	0.739	2.097	2.836	0.013	0.018
Tuesday	121	156	0.776	2.682	3.457	0.022	0.029
Wednesday	136	197	0.690	1.539	2.230	0.011	0.016
Thursday	168	211	0.796	3.111	3.907	0.019	0.023
Friday	188	245	0.767	2.531	3.298	0.013	0.018
Saturday	196	234	0.838	4.320	5.158	0.022	0.026

The AIIMS Triage Protocol (ATP) was used to triage patients into critical (red) and moderate (yellow) categories before allocating the respective services. After being triaged patient entered into the regular process of an emergency department. The scheduling of patients and their respective data is monitored solely by the operational desk in the emergency department. The average number of patients who arrived in triage is 137, however, the maximum number of patients arrived on Saturday followed by Friday i.e. 172 and 161, respectively. All the patients were treated without any waiting time in the triage room and worked with an efficiency of 100%.

Table 2: Queueing analysis of patients who arrived Triage room

Day	Arrival (λ)	Service (μ)	Avg operation (ρ)	Avg patient in queue (L_q)	Avg patient in system (L)	Avg wait in queue (W_q)	Avg time in system (W)
Sunday	148	148	1	0	0	0	0
Monday	126	126	1	0	0	0	0
Tuesday	98	98	1	0	0	0	0
Wednesday	111	111	1	0	0	0	0
Thursday	143	143	1	0	0	0	0
Friday	161	161	1	0	0	0	0
Saturday	172	172	1	0	0	0	0

The arrival follows the exponential distribution and service time follows the exponential distribution with an arrival rate is 0.1/min and service rate are 0.12/min it gives us a utilization rate of 41.67% with an average service time of 8.33 mins and an average waiting time of 5.38 mins with all the servers are busy (P_0) at 78.65%.

The average daily patient arrival in the emergency bed area is 104 with operational efficiency of 72.2%. The maximum arrival was observed on Saturday however maximum operational efficiency was found on Thursday i.e. 79.6% followed by Friday i.e. 76.7%. Approximately, 3 patients were found in the queue and 4 were found in the system on Thursday. The minimum number of patients were observed on Sunday nearly 1.5 in the system and 1 in the queue, as shown in Figure 5.

Table 3: Queuing analysis of patients who arrived in the emergency bedroom

Day	Arrival (λ)	Service (μ)	Avg operation (ρ)	Avg patient in queue (L_q)	Avg patient in system (L)	Avg wait in queue (W_q)	Avg time in system (W)
Sunday	112	186	0.602	0.911	1.514	0.008	0.014
Monday	95	132	0.720	1.848	2.568	0.019	0.027
Tuesday	74	105	0.705	1.682	2.387	0.023	0.032
Wednesday	85	111	0.766	2.503	3.269	0.029	0.038

Thursday	109	137	0.796	3.097	3.893	0.028	0.036
Friday	122	159	0.767	2.530	3.297	0.021	0.027
Saturday	131	187	0.701	1.639	2.339	0.013	0.018

Figure 4: Average no. of patient and waiting time in the emergency department

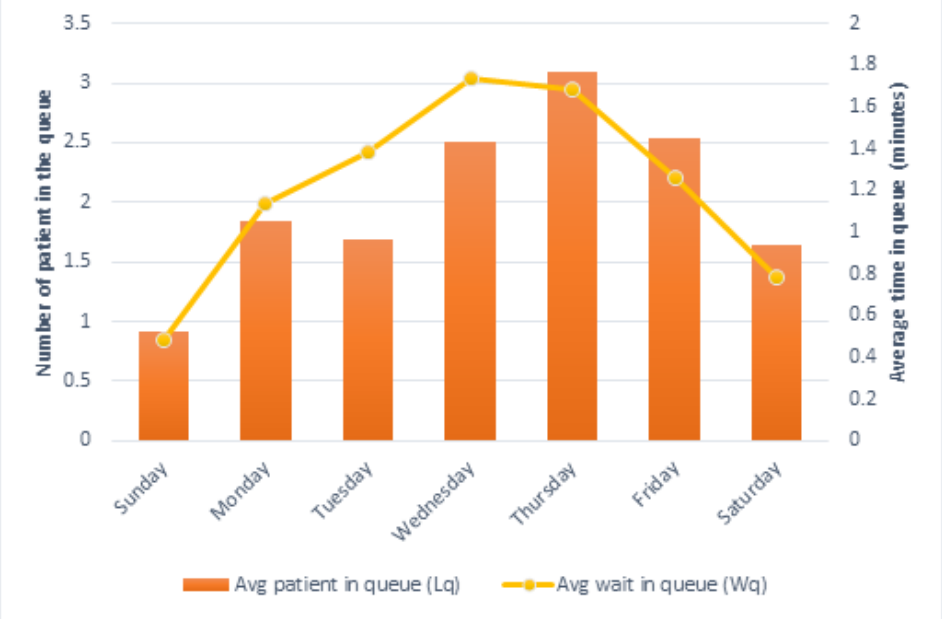

Figure 5: Average no. of patient and waiting time at room allocation

Based on the analysis and queueing theory principles, the service time in the emergency department of hospital management follows the exponential distribution (Williams, 1983), which means that the service times do not depend on external factors or the order of service provided and the completion of their treatment and being discharged within a certain time frame which generates the simple analytical framework for studying queuing

systems and analyzing their performance (Liu & Liu, 2012). The larger number of patients approach the hospital during the weekend and during the day most of the patients arrive near the noon time and further patient flow increases in the evening hours. The staff available in the emergency department varied based on the peak hour and peak day. The two doctors and 4 nurses are stationed every time in their spare to tackle the peak load and doctors are also positioned from the inpatient department to cater the load during the peak hours.

ED follows the triage protocol for the categorization of patients into Red, Yellow, and Green zones to ensure the delivery of adequate and quick services. The prescriptions or cards issued to patients are also colour-coded for easy classification of patients and coloured footsteps on the floor allow the patients to reach the specific department. ED also allows telephonic consultations for emergency patients to provide specialized consultation on laboratory tests and sometimes also work on the recategorization of patients based on the findings. As the load in the ED varies based on the day of the week and hour of the day, the average waiting time also varies and is expected to be near about 1 to 2 minutes at any time of the day.

4. CONCLUSION

In this study, the patient in queue has been studied to generate an efficient method for the working of the arrivals at the emergency department. The result states that we need more EMTs and a good priority system in place for the efficient working of the system so that it won't have bottlenecks and patients won't wait in line for the treatments they need. As with increasing queueing waiting time, the system needs to be working efficiently and it needs more servers and a better priority system to operate for the patients who need immediate attention and care. The data analysis and collection should be more efficient to obtain more insights into the functioning of the department. The data obtained is through observational statistics. The result defines the work needed in the department and how it can be done to make that happen. However, ED requires a formulated strategy to capture the required data for the triage performed by the EMTs on arrival, the arrivals log time was also not exact as the EMTs registering the entry of the patients are the ones performing triage which results in a bottleneck queue. This presents the limitations and several challenges in data collection. This problem has to be resolved if we are looking for a process of high volume of data collection.

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