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FUZZY EZ -SPACES AND RELATED CONCEPTS

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ABSTRACT
In this paper, the notion of fuzzy EZ - topological spaces is introduced and studied. It is obtained that fuzzy EZ -spaces do not possess non-zero fuzzy nowhere dense sets and fuzzy sets defined on them are fuzzy somewhere dense sets. It is established that fuzzy extremally disconnected spaces are fuzzy EZ-spaces and fuzzy EZ-spaces are fuzzy fraction dense spaces. It is found that fuzzy EZ spaces are neither fuzzy hyperconnected spaces nor fuzzy Brown spaces. It is established that fuzzy EZ, fuzzy regular and fuzzy P-spaces are fuzzy zero dimensional spaces. It is obtained that fuzzy open sets are fuzzy regular F_σ-sets in fuzzy regular spaces

Keywords :Fuzzy G_δ-set, fuzzy F_σ-set, fuzzy nowhere dense set, fuzzy regular open set, fuzzy fraction dense space , fuzzy open hereditarily irresolvable space, fuzzy P-Space, fuzzy regular space, fuzzy Brown space.

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1. INTRODUCTION

The concept of fuzzy sets as an innovative approach for modelling uncertainties was introduced by **L.A. Zadeh**[23] in the year 1965 . The potential of fuzzy notion was realized by the researchers and successfully has been applied in all branches of Mathematics. In1968, **C.L. Chang**[**3**] introduced the concept of fuzzy topological space.The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In 1969, **A.G. El'Kin** [**4**] introduced open hereditarily irresolvable spaces in classical topology. Motivated by the above works on resolvability ,the concepts of resolvability, irresolvability and open hereditarily irresolvability of fuzzy topological spaces were introduced and studied by **G.Thangaraj** and **G.Balasubramanian**[**13**]. The notion of fuzzy hereditarily irresolvable spaces is introduced and studied by **G.Thangaraj** and **L.Vikraman** [**21**]. The concept of fraction dense spaces in classical topology was introduced by **A.W.Hager** and **J.Martinez**[**7**] and the same was studied as "cozero approximated spaces" by **Gary Gruenhage** [5]. The notion of fuzzy fraction dense space was introduced and studied by **G.Thangaraj** and **A. Vinothkumar**[**20**].

In 2014, **A.Taherifar**[**22**] introduced the notion of EZ- spaces in classical topology. Motivated on these lines, the notion of fuzzy EZ- space in fuzzy topology is introduced by means of fuzzy clopen sets. It is established that fuzzy extremally disconnected spaces are fuzzy EZ-spaces and fuzzy EZ-spaces are fuzzy fraction dense spaces. It is found that fuzzy EZ spaces are neither fuzzy hyperconnected nor fuzzy submaximal spaces. Also it is obtained that fuzzy EZ-space are neither fuzzy strongly irresolvable spaces nor fuzzy Brown spaces. The conditions under which fuzzy fraction dense spaces become fuzzy EZ-spaces and fuzzy EZ -spaces become fuzzy open hereditarily irresolvable spaces, are established. It is established that fuzzy EZ, fuzzy regular and fuzzy P-space are fuzzy zero dimensional spaces. It is also obtained that each fuzzy open set is a fuzzy regular F_σ-set in fuzzy regular spaces and each fuzzy regular closed set is a fuzzy regular F_σ-set in fuzzy EZ, fuzzy regular and fuzzy P-spaces.

2 . PRELIMINARIES

Some basic notions and results used in the sequel, are given in order to make the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1[3]: A fuzzy topology on a set X is a family T of fuzzy sets in X which satisfies the following conditions:

(a). $0_X \in T$ and $1_X \in T$

(b). If A, B$\in$ T, then $A \wedge B \in T$,

(c). If $A_i \in$ T for each i$\in$J, then $\vee_i A_i \in T$.

The pair (X, T) is a fuzzy topological space (briefly, fts). Members of T are called fuzzy open sets in X and their complements are called fuzzy closed sets in X.

Definition 2.2[3] **:** Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior , the closure and the complement of λ are defined respectively as follows :

(i). $\text{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda , \mu \in T \}$;

(ii). $\text{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

(iii). $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $(\lambda_i)_{i\in I}$ of fuzzy sets in (X,T), the union $\vee_{i\in I} \lambda_i$ and intersection $\wedge_{i\in I} \lambda_i$ are defined respectively as follows : For each $x \in X$,

(iv). $(\vee_{i\in I}\lambda_i)(x) = \sup_{i\in I_i}\lambda_i(x)$,

(v). $(\wedge_{i\in I}\lambda_i)(x) = inf_{i\in I}\lambda_i(x)$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1-\text{int}(\lambda) = \text{cl}(1-\lambda)$and (ii). $1-\text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition2.3 : A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy regular-open set in (X,T) if λ=int cl(λ) and

fuzzy regular-closed set in (X,T) if λ=cl int(λ) [**1**].

(ii) fuzzyG_δ-setin (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$,where $\lambda_i \in T$ for $i \in I$;

fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$ for $i \in I$ [**2**].

(iii). fuzzy regular G_δ -set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\text{int}(\lambda_i))$, where $1 -\lambda_i \in T$;

fuzzy regular F_σ-set in (X,T)if $\lambda = \vee_{i=1}^{\infty} (\text{cl}(\mu_i))$, where$\mu_i \in T$ [**17**].

(iv). fuzzy dense set in (X,T) if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [**11**] .

(v). fuzzy nowhere dense set in (X,T)if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) = 0$, in (X,T) [**11**].

(vi). fuzzy somewhere dense setin (X,T)if there exists anon-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is,$int\, cl(\lambda) \neq 0$, in (X,T)[**12**].

(vii). fuzzy simply open set in (X,T) if Bd(λ) is a fuzzy nowhere dense set in (X,T).

That is, λ is a fuzzy simply open set in (X,T) if $Bd(\lambda) = [\, cl(\lambda) \wedge cl(1-\lambda)\,]$, is a fuzzy nowhere dense set in (X,T) [**15**] .

Definition 2.4 : A fuzzy topological space(X,T) is called a

(i). fuzzy hereditarily irresolvable space if for any two fuzzy sets λ and μ defined on X with $cl(\lambda) = cl(\mu)\ (\neq 0)$, $\lambda \wedge \mu \neq 0$, in (X,T) [**21**].

(ii). fuzzy open hereditarily irresolvable space if $int\, cl(\lambda) \neq 0$, then $int(\lambda) \neq 0$, for any non-zero fuzzy set λ in (X, T) [**13**].

(iii). fuzzy submaximal space if for each fuzzy set λ in (X,T) such that $cl(\lambda) = 1$, then $\lambda \in T$ [**2**].

(iv). fuzzy hyperconnected space if every non - null fuzzy open subset of (X,T) is fuzzy dense in (X,T) [**8**].

(v). fuzzy perfectly disconnected space if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq 1-\mu$, $cl(\lambda) \leq 1 - cl(\mu)$, in (X,T [**16**].

(vi). fuzzy extremally disconnected space if the closure of every fuzzy open set of (X,T) is fuzzy open in (X,T) [**6**].

(vii). fuzzy Brown space iff or any two non-zero fuzzy open sets λ and μ in (X,T), $cl(\lambda) \nleq 1 - cl(\mu)$, in(X, T) [19].

(viii) fuzzy fraction dense space if for each fuzzy open set λ in (X,T), $cl(\lambda) = cl(\mu)$, where μ is a fuzzy F_σ-set in (X,T) [**20**].

(ix). fuzzy regular space iff each fuzzy open set A of X is a union of fuzzy open sets (λ_i)'s of X such that $cl(\lambda_i) \leq \lambda$ [**1**].

(x). fuzzy P -space if each fuzzy G_δ-set in (X, T) is a fuzzy open set in (X,T) [**10**].

(xi). fuzzy almost P-space if for each non- zero fuzzy G_δ-set λ in(X,T), $int(\lambda) \neq 0$ in (X,T) [**14**].

Definition 2.5[**9**]**:** An fuzzy topological space X is called zero-dimensional if every crisp fuzzy point in X has a base of crisp clopen fuzzy sets.

Theorem 2.1 [1] : In a fuzzy topological space ,

(a). The closure of a fuzzy open set is a fuzzy regular closed set.

(b). The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.2 [21] : If λ and μ are fuzzy sets defined on X with $cl(\lambda) = cl(\mu)\ (\neq 0)$, in a fuzzy hereditarily irresolvable space (X,T), then $\lambda \nleq 1 - \mu$, in (X,T).

Theorem 2.3 [21]: If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected space, then (X,T) is a fuzzy hereditarily irresolvable space.

Theorem 2.4 [21]: If (X,T) is a fuzzy extremally disconnected space, then (X,T) is a fuzzy hereditarily irresolvable space.

Theorem 2.5 [19] : If (X,T) is a fuzzy Brown space, then there is no fuzzy set λ which is both fuzzy open and fuzzy closed in (X,T).

Theorem 2.6 [18] : Let (X,T) be a fuzzy topological space. Then, (X,T) is a fuzzy strongly irresolvable space if and only if each fuzzy set λ defined on X is a fuzzy simply open set in (X,T).

Theorem 2.7 [17] : If λ is a fuzzy regular G_δ-set in a fuzzy topological space (X,T) if and only if $1-\lambda$is a fuzzy regular F_σ -set in (X,T).

Theorem 2.8 [17] : If λ is a fuzzy regular F_σ-set in a fuzzy topological space(X,T), then λ is a fuzzy F_σ-set in (X, T).

Theorem 2.9 [9] : Every non-empty extremally disconnected regular fuzzy topological space is zero dimensional.

3 . FUZZY EZ -SPACES

In[22], A. Taherifar introduced the notion of EZ- spaces in classical topology in 2014. Motivated on these lines, the notion of fuzzy EZ- space in fuzzy topology is introduced by means of fuzzy clopen sets as follows :

Definition3.1 : A fuzzy topological space (X,T) is called a fuzzy EZ-space if for each non-zero fuzzy open set λ in (X,T), cl $(\lambda)=$ cl ($\bigvee_{i=1}^{\infty}(\lambda_i)$), where$(\lambda_i)'s$ are fuzzy clopen sets in (X,T).

Example 3.1 : Let X $=\{a,b,c,d\}$. Consider the fuzzy sets λ, μ and γ defined on X as follows:

λ: X $\to$ [0,1] is defined as $\lambda(a)=0.2$; $\lambda(b)=0.6$;$\lambda(c)=0.7$;$\lambda(d)=0.4$;

μ: X$\to$ [0,1]is defined as $\mu(a)=0.8$;$\mu(b)=0.4$; $\mu(c)=0.3$;$\mu(d)=0.6$;

γ: X$\to$ [0,1] is defined as $\gamma(a)=0.6$; $\gamma(b)=0.3$;$\gamma(c)=0.2$;$\gamma(d)=0.5$.

Then, T= { 0, λ ,μ, γ, $\lambda\vee\mu$, $\lambda\vee\gamma$, $\lambda\wedge\mu$, $\lambda\wedge\gamma$, $(\gamma\vee[\lambda\wedge\mu])$,1} is a fuzzy topology on X. On computation, one can see that the fuzzy clopen sets in (X,T) are λ,μ, $(\lambda\vee\mu)$ and$(\lambda\wedge\mu)$. Also on computation , the fuzzy regular closed sets in (X,T) are λ,μ, $(\lambda\vee\mu)$ and $(\lambda\wedge\mu)$ and $\lambda=$ cl($\lambda\vee[\lambda\wedge\mu]$),$\mu=$ cl($\mu\vee[\lambda\wedge\mu]$), $\lambda\vee\mu=$ cl($\lambda\vee\mu\vee[\lambda\wedge\mu]$), $\lambda\wedge\mu=$ cl($\lambda\wedge\mu$). Hence (X,T) is a fuzzy EZ-space.

Proposition 3.1 : If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then cl $(\lambda)=$ cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, cl$(\lambda)=$ cl($\bigvee_{i=1}^{\infty}(\lambda_i)$), where $(\lambda_i)'s$ are fuzzy clopen sets in (X,T). Let $\mu=\bigvee_{i=1}^{\infty}(\lambda_i)$. Now the fuzzy sets (λ_i)'s are fuzzy open sets in (X,T) implies that μ is a fuzzy open set in (X,T) and (λ_i)'s are fuzzy closed sets in (X,T) implies that μ is a fuzzy F_σ-set in (X,T). Then, μ is a fuzzy open and fuzzy F_σ-set in (X,T) and hence cl $(\lambda)=$ cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T).

Corollary3.1 : If η is a fuzzy closed set in a fuzzy EZ-space (X,T),then, int$(\eta)=$ int (θ), where θ is a fuzzy closed and fuzzy G_δ-set in (X,T).

Proof: Let η be a fuzzy closed set in(X,T). Then, $1-\eta$ is a fuzzy open set in the fuzzy EZ-space (X,T) and by Proposition**3.1,**cl $(1-\eta)=\mathrm{cl}(\mu)$, where μ is a fuzzy open and fuzzy F_σ-set in (X,T). By Lemma **2.1**,$1-$ int $(\eta)=$cl $(1-\eta)$ and then int $(\eta)=1-\mathrm{cl}(\mu)=$ int $(1-\mu)$. Let $\theta=1-\mu$. Hence, it follows that int$(\eta)=$ int (θ), where θ is a fuzzy closed and fuzzy G_δ-set in (X,T).

Proposition 3.2: If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then there exists a fuzzy regular closed set δ in (X,T) such that $\lambda\leq\delta$.

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.1,** cl $(\lambda)=$ cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T). By Theorem **2.1**, for the fuzzy open set μ, cl(μ) is a fuzzy regular closed set in (X,T). Let δ =cl(μ). Now $\lambda\leq$cl $(\lambda)=$ cl(μ), implies that $\lambda\leq\delta$.

Corollary 3.2 : If μ is a fuzzy closed set in a fuzzy EZ-space (X,T), then there exists a fuzzy regular open set η in (X,T) such that $\eta\leq\mu$.

Proof : Let μ be a fuzzy closed set in (X,T). Then, $1-\mu$ is a fuzzy open set in the fuzzy EZ-space (X,T) and by Proposition **3.2,** there exists a fuzzy regular closed set δ in (X,T) such that$1-\mu\leq\delta$. This implies that $1-\delta\leq\mu$, in (X,T). Let $\eta=1-\delta$. Thus, for the fuzzy closed set μ, there exists a fuzzy regular open set η in (X,T) such that $\eta\leq\mu$.

Proposition 3.3 : A fuzzy topological space (X,T) is a fuzzy EZ- space if and only if for each fuzzy regular closed set μ in (X,T), $\mu=$ cl(η), where η is a fuzzy open and fuzzy F_σ-set in (X,T).

Proof: Let μ be a fuzzy regular closed set in (X,T). Then, cl int$(\mu)=\mu$, in (X,T). Let $\lambda=$ int (μ). Then, λ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.1,** cl $(\lambda)=$ cl(η), where η is a fuzzy open and fuzzy F_σ-set in (X,T). Thus, $\mu=$ cl int (μ) =cl (λ) =cl(η) and $\mu=$ cl(η), in (X,T).

Conversely, let δ be a fuzzy open set in (X,T). Then, by Theorem **2.1**,cl(δ) is a fuzzy regular closed set in (X,T). By hypothesis, cl(δ) =cl(η), where η is a fuzzy open and fuzzy F_σ-set in (X,T). Now the fuzzy set η is a fuzzy F_σ-set in (X,T), implies that $\eta=\bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy closed sets in (X,T). Also $\bigvee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy open set in (X,T) implies that $(\lambda_i)'s$ are fuzzy open sets in (X,T) and thus cl$(\delta)=$ cl$(\bigvee_{i=1}^{\infty}(\lambda_i))$, where $(\lambda_i)'s$ are fuzzy clopen sets in (X,T) implies that (X,T) is a fuzzy EZ-space.

Proposition 3.4 : If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that $\mu\leq$ cl (λ).

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, cl(λ) =cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T). Now $\mu\leq$cl(μ), implies that $\mu\leq$ cl (λ).

Corollary 3.3 : If δ is a fuzzy closed set in a fuzzy EZ-space (X,T), then there exists a fuzzy closed and fuzzy G_δ-set θ in (X,T) such that int $(\delta)\leq\theta$.

Proof: Let δ be a fuzzy closed set in (X,T). Then, $1-\delta$ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition 3.4, there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that $\mu\leq$

cl $(1-\delta)$. Then, $\mu \leq 1-\mathrm{int}(\delta)$ [by Lemma 2.1]. This implies that $\mathrm{int}(\delta) \leq 1-\mu$. Let $\theta = 1-\mu$. Then θ is a fuzzy closed and fuzzy G_δ-set and $\mathrm{int}(\delta) \leq \theta$, in (X,T).

Proposition 3.5: If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then there exists a fuzzy open and fuzzy F_σ-set η in (X,T) such that $\lambda \leq \mathrm{cl}(\eta)$.

Proof: Let λ be a fuzzy open set in (X,T). Now $\lambda \leq \mathrm{cl}(\lambda)$ in (X,T). Since (X,T) is a fuzzy EZ-space, cl $(\lambda) = \mathrm{cl}(\eta)$, where η is a fuzzy open and fuzzy F_σ-set in (X,T) and then $\lambda \leq \mathrm{cl}(\eta)$.

Corollary 3.4: If δ is a fuzzy closed set in a fuzzy EZ-space (X,T), then there exists a fuzzy closed and fuzzy G_δ-set ρ in (X,T) such that int $(\rho) \leq \delta$.

Proof: Let δ be a fuzzy closed set in (X,T). Then, $1-\delta$ is a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition 3.5, there exists a fuzzy open and fuzzy F_σ-set η in (X,T) such that $1-\delta \leq$ cl (η). Then, $1-$ cl $(\eta) \leq \delta$ and by Lemma 2.1, $\mathrm{int}(1-\eta) = 1-\mathrm{cl}(\eta) \leq \delta$. Let $\rho = 1-\eta$ and ρ is a fuzzy closed and fuzzy G_δ-set in (X,T) and $\mathrm{int}(\rho) \leq \delta$ in (X,T).

The following proposition shows that fuzzy open sets in fuzzy EZ-spaces are not fuzzy dense sets.

Proposition 3.6: If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then λ is not a fuzzy dense set in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition**3.2,** there exists a fuzzy regular closed set δ in (X,T) such that $\lambda \leq \delta$. Then, cl $(\lambda) \leq \mathrm{cl}(\delta)$ and $\mathrm{cl}(\delta)$ is a fuzzy closed set in (X,T), implies that $\mathrm{cl}(\lambda) \neq 1$ and thus λ is not a fuzzy dense set in (X,T).

The following proposition shows that fuzzy closed sets in fuzzy EZ-spaces are not fuzzy nowhere dense sets.

Proposition 3.7 : If λ is a fuzzy open set in a fuzzy EZ-space (X,T), then $1-\lambda$ is a fuzzy somewhere dense set in(X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition**3.6,** cl $(\lambda) \neq 1$,in (X,T). Now intcl$(1-\lambda) = 1-\mathrm{clint}(\lambda) = 1-$ cl $(\lambda) \neq 0$. Hence $1-\lambda$ is a fuzzy somewhere dense set in (X,T).

Proposition 3.8: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then 0_X is the only fuzzy nowhere dense set in (X,T).

Proof : Suppose that the fuzzy set λ ($\neq 0$) is a fuzzy nowhere dense set in the fuzzy EZ-space (X,T). Then, int $\mathrm{cl}(\lambda) = 0$ and then $1-$int $\mathrm{cl}(\lambda) = 1$, in (X,T). This implies that cl $\mathrm{int}(1-\lambda) = 1$. This implies that the fuzzy open set $\mathrm{int}(1-\lambda)$ is a fuzzy dense set in (X,T), a contradiction by Proposition**3.3.** Then, it follows that int $\mathrm{cl}(\lambda) \neq 0$, in (X,T) and hence 0_X is the only fuzzy nowhere dense set in (X,T).

Remark 3.1 : In view of the above Proposition, one will have the following result:*" A fuzzy EZ -space does not have non-zero fuzzy nowhere dense sets and each fuzzy set defined it , is a fuzzy somewhere dense set"*

Proposition 3.9: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then 0_X and 1_X are the only two fuzzy simply open sets in (X,T).

Proof : Suppose that the non zero fuzzy set $\lambda(\neq 1)$ is a fuzzy simply open set in (X,T). Then [cl $(\lambda) \wedge$cl $(1-\lambda)$], is a fuzzy nowhere dense set in (X,T), a contradiction, by Proposition **3.8** and hence λ is not a fuzzy simply open set in (X,T). Now intcl{[cl(0) ∧ cl(1−0)]} = int cl{0∧cl(1)} = int cl{0 ∧1}=int cl{0}= 0 and int cl { [cl (1) ∧cl (1 −1)]} = int cl{ 1 ∧cl(0)} =int cl{ 1 ∧0}=int cl { 0 } = 0. Thus 0_X and 1_X are the only two fuzzy simply open sets in (X,T).

4. FUZZY EZ-SPACES AND OTHER FUZZY TOPOLOGICAL SPACES

Proposition4.1: If a fuzzy topological space (X,T) is a fuzzy extremally disconnected space, then (X,T) is a fuzzy EZ-space.

Proof: Let λ be a fuzzy open set in (X,T). Since the fuzzy topological space (X,T) is a fuzzy extremally disconnected space, cl(λ) is a fuzzy open set in (X,T) and thus cl (λ) is a fuzzy open and fuzzy closed set in (X,T). Thus, for the fuzzy open set λ, cl(λ) = cl [cl (λ)], where cl(λ) is a fuzzy clopen set in (X,T), implies that (X,T) is a fuzzy EZ-space.

Proposition 4.2 : If cl ($\bigvee_{i=1}^{\infty}(\lambda_i)$) is a fuzzy open set, for the clopen sets $(\lambda_i)'s$ (i= 1 to ∞) in a fuzzy EZ-space (X,T), then (X,T) is a fuzzy extremally disconnected space.

Proof: Let λ be a fuzzy open set in (X,T). Since(X,T) is a fuzzy EZ-space, cl (λ)=cl ($\bigvee_{i=1}^{\infty}(\lambda_i)$), where$(\lambda_i)'s$ are fuzzy clopen sets in (X,T). By hypothesis, cl ($\bigvee_{i=1}^{\infty}(\lambda_i)$)is a fuzzy open set in(X,T). Thus , for the fuzzy open setλ, cl (λ)is a fuzzy open set in (X,T), implies that (X,T) is a fuzzy extremally disconnected space.

Proposition 4.3 : If λ is a fuzzy open set in a fuzzy EZ and fuzzy P-space (X,T), then cl (λ) is a fuzzy open and fuzzy F_σ-set in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ - space, cl (λ) = cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T). Also since (X,T) is a fuzzy P-space, the fuzzy F_σ-set μ is a fuzzy closed set in (X,T) and then cl(μ) = μ. Then, cl (λ) = μ , where μ is a fuzzy F_σ-set in (X,T). Hence cl (λ) is a fuzzy open and fuzzy F_σ-set in (X,T).

Proposition 4.4 : If a fuzzy topological space (X,T) is a fuzzy EZ and fuzzy P-space, then (X,T) is a fuzzy extremally disconnected space.

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ and fuzzy P-space, by Proposition **4.11,** cl (λ) is a fuzzy open and fuzzy F_σ-set in (X,T). Thus, for the fuzzy open set λ, cl (λ) is a fuzzy open set in (X,T), implies that (X,T) is a fuzzy extremally disconnected space.

Proposition4.5: If λ is a fuzzy open set in a fuzzy hereditarily irresolvable and fuzzy EZ-space (X,T), there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that λ≰ 1 −μ .

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.1,** cl (λ)= cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T). Also since (X,T) is a fuzzy hereditarily irresolvable space, by Theorem**2.2,** for the fuzzy sets λ and μ with cl(λ) = cl(μ), λ≰ 1− μ, in (X,T).

Proposition4.6: If λ is a fuzzy open set in a fuzzy extremally disconnected space, then there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that λ≰ 1 −μ .

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy extremally disconnected space, by Proposition **4.1,** (X,T) is a fuzzy EZ-space. By Theorem **2.4,** the fuzzy extremally disconnected space (X,T) is a fuzzy hereditarily irresolvable space. Thus (X,T) is a fuzzy hereditarily irresolvable and fuzzy EZ-space and then by Proposition **4.5**, there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that $\lambda \nleq 1 - \mu$.

Remark4.1 : In view of Propositions **4.6**, one will confirm the following result : " *Fuzzy extremally disconnected spaces do not possess disjoint open sets*."

Proposition 4.7: If λ is a fuzzy open set in a fuzzy perfectly disconnected and fuzzy EZ-space (X,T), there exists a fuzzy open and fuzzy F_σ-set μ in (X,T) such that $\lambda \nleq 1 - \mu$.

Proof : The proof follows from Proposition **4.5** and Theorem **2.3.**

Proposition 4.8: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then (X,T) is not a fuzzy hyperconnected space.

***Proof* :** Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.6**, λ is not a fuzzy dense set in (X,T) and hence (X,T) is not a fuzzy hyperconnected space.

Proposition 4.9: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then (X,T) is not a fuzzy Brown space.

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, cl $(\lambda) =$ cl$(\bigvee_{i=1}^{\infty}(\lambda_i))$, where (λ_i)'s are fuzzy clopen sets in (X,T). Then, the existence of fuzzy clopen [both fuzzy open and fuzzy closed] sets in (X,T) establishes, by Theorem **2.7,** that (X,T) is not a fuzzy Brown space.

Proposition 4.10: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then (X,T) is not a fuzzy submaximal space.

Proof: Suppose that the fuzzy EZ-space (X,T) is a fuzzy submaximal space. If λ is a fuzzy dense set in (X,T), then λ will be a fuzzy open set in (X,T). This leads to the conclusion that the fuzzy open set λ is a fuzzy dense set in (X,T), a contradiction to Proposition **3.3** and hence it follows that the fuzzy EZ-space (X,T) is not a fuzzy submaximal space.

Remark 4.2 : In view of Propositions **4.7** and **4.10**, one will have the following result: " *Fuzzy EZ spaces are neither fuzzy hyperconnected nor fuzzy submaximal spaces.*"

Proposition4.11: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then (X,T) is not a fuzzy strongly irresolvable space.

Proof: Suppose that the fuzzy EZ-space(X,T) is a fuzzy strongly irresolvable space. Then, by Theorem 2.6, each fuzzy set λ defined on X is a fuzzy simply open set in (X,T). This means that the fuzzy EZ-space(X,T) possesses fuzzy simply open sets other than 0_X and 1_X, a contradiction by Proposition **3.6.** Hence (X,T) is not a fuzzy strongly irresolvable space.

The following proposition shows that fuzzy EZ-spaces are fuzzy fraction dense spaces.

Proposition4.12: If a fuzzy topological space (X,T) is a fuzzy EZ-space, then (X,T) is a fuzzy fraction dense space.

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.1,** cl $(\lambda) =$ cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T). Thus, for each fuzzy open set λ in (X,T), cl $(\lambda) =$ cl(μ), where μ is a fuzzy F_σ-set in (X,T), implies that (X,T) is a fuzzy fraction dense space.

The following proposition gives a condition for fuzzy fraction dense spaces to become fuzzy EZ-spaces.

Proposition 4.13: If each fuzzy F_σ-set is a fuzzy open set in a fuzzy fraction dense space (X,T), then (X,T) is a fuzzy EZ-space.

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy fraction dense space, cl (λ)= cl(μ), where μ is a fuzzy F_σ-set in (X,T). By hypothesis, the fuzzy F_σ-set μ is a fuzzy open set in (X,T) and thus μ is a fuzzy open and fuzzy F_σ-set in (X,T). Hence , for the fuzzy open set λ, cl (λ)= cl(μ), where μ is a fuzzy open and fuzzy F_σ-set in (X,T), implies that (X,T) is a fuzzy EZ-space.

Proposition 4.14: If a fuzzy topological space (X,T) is a fuzzy regular space, then each fuzzy open set is a fuzzy regular F_σ-set in (X,T).

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy regular space, each fuzzy open set λ of X is a union of fuzzy open sets (λ_i)'s of X such that cl(λ_i) ≤ λ and thus $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where cl(λ_i) ≤λ, for each $\lambda_i \in T$. Now $\lambda_i \leq$ cl(λ_i) ≤λ, implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}[\text{cl}(\lambda_i)] \leq \lambda$ and thus $\lambda \leq \vee_{i=1}^{\infty}[\text{cl}(\lambda_i)] \leq \lambda$, implies that $\lambda = \vee_{i=1}^{\infty}[\text{cl}(\lambda_i)]$, where $\lambda_i \in$ T, in (X,T) and hence λ is a fuzzy regular F_σ-set in (X,T).

Corollary4.1 : If a fuzzy topological space (X,T) is a fuzzy regular space, then each fuzzy open set is a fuzzy F_σ-set in (X,T).

Proof : The proof follows from Proposition **4.14** and Theorem **2.8.**

Proposition 4.15: If a fuzzy topological space (X,T) is a fuzzy EZ , fuzzy P-space and fuzzy regular space, then (X,T) is a fuzzy zero-dimensional space.

Proof : Let (X,T) be a fuzzy EZ and fuzzy P-space .Then, by Proposition **4.4,** (X,T) is a fuzzy extremally disconnected space. Thus, (X,T) is a fuzzy extremally disconnected and fuzzy regular space and then by Theorem**2.9,** (X,T) is a zero - dimensional space.

Proposition 4.16: If λ is a fuzzy open set in a fuzzy EZ and fuzzy P-space and fuzzy regular space (X,T), then cl (λ) is a fuzzy regular F_σ-set in (X,T).

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy EZ and fuzzy P-space, by Proposition**4.4,** (X,T) is a fuzzy extremally disconnected space and thus cl (λ) is a fuzzy open set in (X,T). Also since (X,T) is a fuzzy regular space, by Proposition **4.14,** the fuzzy open set cl (λ) is a fuzzy regular F_σ-set in (X,T).

Corollary 4.2 : If λ is a fuzzy open set in a fuzzy EZ –space, fuzzy P-space and fuzzy regular space(X,T), then cl (λ) is a fuzzy regular F_σ-set in (X,T).

Corollary4.3: If a fuzzy topological space (X,T) is a fuzzy EZ-space, fuzzy P-space and fuzzy regular space (X,T) , each fuzzy regular closed set is a fuzzy regularF_σ-set in (X,T).

Corollary 4.4: If a fuzzy topological space (X,T) is a fuzzy EZ-space, fuzzy P-space and fuzzy regular space, each fuzzy regular open set is a fuzzy regular G_δ-set in (X,T).

The following proposition gives a condition for fuzzy EZ -spaces to become fuzzy open hereditarily irresolvable spaces.

Proposition 4.17 : If λ is a fuzzy set defined on X in a fuzzy EZ-space (X,T) with int(λ)≠ 0, then (X,T) is a fuzzy open hereditarily irresolvable space.

Proof: Let λ be a fuzzy set defined on X in (X,T) such that int(λ) ≠ 0. Since (X,T) is a fuzzy EZ-space, by Proposition **3.8,** each fuzzy set defined on X is fuzzy somewhere dense set in (X,T) and thus int cl (λ) ≠ 0, in (X,T). By hypothesis int(λ) ≠ 0 and thus for any non-zero fuzzy set λ with int cl (λ) ≠ 0, in (X,T), int(λ) ≠ 0, implies that (X,T) is a fuzzy open hereditarily irresolvable space.

Proposition 4.18 : If a fuzzy topological space (X,T) is a fuzzy EZ-space and fuzzy open hereditarily irresolvable space, then (X,T) is a fuzzy almost P-space .

Proof : Let λ be a fuzzy G_δ -set in (X,T). Since (X,T) is a fuzzy EZ-space, by Proposition **3.8,** each fuzzy set defined on X is a fuzzy somewhere dense set in (X,T) and thus, the fuzzy G_δ –set λ is a fuzzy somewhere dense set in (X,T). That is, int cl (λ) ≠ 0, in (X,T). Since (X,T)is a fuzzy open hereditarily irresolvable space, for the fuzzy somewhere dense set λ, int (λ) ≠ 0, in (X,T) and thus (X,T) is a fuzzy almost P-space.

CONCLUSION

In this paper, the notion of fuzzy EZ- spaces in fuzzy topology is introduced by means of fuzzy clopen sets. It is obtained that fuzzy EZ -spaces do not possess non-zero fuzzy nowhere dense sets and fuzzy sets defined on them are fuzzy somewhere dense sets. Also it is obtained that 0_X and 1_X are the only two fuzzy simply open sets in fuzzy EZ-spaces. It is established that fuzzy extremally disconnected spaces are fuzzy EZ-spaces and fuzzy EZ-spaces are fuzzy fraction dense spaces. It is found that fuzzy EZ spaces are neither fuzzy hyperconnected nor fuzzy submaximal spaces. Also it is obtained that fuzzy EZ-spaces are neither fuzzy strongly irresolvable spaces nor fuzzy Brown spaces. The conditions under which fuzzy fraction dense spaces become fuzzy EZ-spaces and fuzzy EZ -spaces become fuzzy open hereditarily irresolvable spaces, are established. It is found that fuzzy EZ, fuzzy regular and fuzzy P-space are fuzzy zero dimensional spaces. It is also obtained that each fuzzy open set is a fuzzy regular F_σ-set in fuzzy regular spaces and each fuzzy regular closed set is a fuzzy regular F_σ-set in fuzzy EZ, fuzzy regular and fuzzy P-spaces and fuzzy EZ- spaces. It is obtained that fuzzy EZ and fuzzy open hereditarily irresolvable spaces are fuzzy almost P-spaces.

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