Stochastic Modelling and Computational Sciences

A STUDY ON FREE VIBRATION OF NON - HOMOGENEOUS ORTHOTROPIC ELLIPTICAL PLATE WITH INCONSISTENT THICKNESS VARIATION ALONG WITH THERMAL EFFECT

Ashish Kumar Sharma[1] and Pooja Sharma[2*]
[1]Associate Professor, Department of Mathematics, IEC University, Baddi, H.P
[*2]Research Scholar Department of Mathematics, IEC University, Baddi, H.P
[1]ashishk482@gmail.com and [2]psharma5991@gmail.com

ABSTRACT
To study temperature effect on vibration of non-homogeneous orthotropic elliptical plate in which temperature and thickness are taken as linear in both the directions. The variation in density is assumed as linear along a line through plate centre, which raise non-homogeneity of the plate materials and make problem interesting as introducing variation in non-homogeneity of the material mass density reduce the problem practical importance in comparison to homogeneous plate. For visco-elastic, basic elastic and viscous elements are combined in parallel. Here we have taken Kelvin model for visco-elasticity that is the combination of viscous and elastic elements. Here the elastic element means the spring and viscous element means the dashpot. The governing equation has been solved by the Rayleigh-ritz method. Frequency corresponding to the first two modes of vibration of a clamped non-homogeneous visco-elastic elliptic plate for various value of thermal gradient, taper constant, aspect ratio are obtained and shown graphically.

Keywords – Elliptic plate , Variable thickness , Thermal gradient , Non-homogeneous

INTRODUCTION
In modern technology an interest towards the effect of high temperature on non-homogeneous plates of variable thickness is developed due to applications in various engineering branches such as nuclear, power plants, aeronautical, chemicals etc. where metals and their alloys exhibits visco-elastic behavior. Therefore for these changes the structures are exposed to high intensity, heat fluxes and material properties undergo significant changes. Several authors [1-4] have studied the thermal effect on vibration of homogeneous plates of variable thickness but none of the authors have so far considered thermal effect on vibration of non-homogeneous elliptical plate of varying thickness. Therefore solution of vibration of non-homgeneous plate of variable thickness with thermal gradient is more important as non-homogeneity play an important role in the theory of vibration of plates. Khanna and Sharma [5] has studied the transverse vibration of square plate of variable thickness with thermal gradient. Khanna, Kumar & Bhatia [6] has studied A Computational Prediction on Two Dimensional Thermal Effect on Vibration of Visco-elastic Square Plate of Variable Thickness. Bhardwaj, Gupta, Choong, Wang & Ohmori [8] has studied the transverse Vibrations of clamped and simply-supported circular plates with two dimensional thickness variation. Khanna, Kaur & Sharma [9] has discussed the effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate. Khanna & Sharma [10-12] has studied a computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect and mechanical vibration of visco-Elastic Plate with thickness Variation and effect of thermal gradient on vibration of visco-elastic Plate with Thickness Variation. De and Debnath[13] has discussed the vibration of orthotropic circular plate with Thermal Effect in Exponential thickness and quadratic temperature distribution. Khanna & Sharma [14] has studied the natural vibration of visco-elastic elate of varying thickness with thermal effect. Kumar Sharma, A., & Sharma, S. K. [15] has discussed the free vibration analysis of visco-elastic orthotropic rectangular plate with bi- parabolic thermal effect and bi-linear thickness variation. S. K., & A. K. [16] has studied the mechanical vibration of orthotropic rectangular plate with 2D linearly varying thickness and thermal effect. Kumar Sharma, A., & Sharma, S. K. [17] has studied the vibration computational of visco-elastic plate with sinusoidal thickness variation and linearly thermal effect in 2D. Sharma, S. K., & Sharma, A. K. [18] effect of bi-parabolic thermal and thickness Variation on Vibration of Visco-Elastic Orthotropic Rectangular Plate.

The aim of the present study is to determine the thermal effect on vibration of a clamped non-homogeneous visco-elastic elliptic plate of variable thickness. Rayleigh-ritz has been applied to obtain corresponding natural frequencies. The poisson ratio is assumed to remain constant. Frequency for the first two modes of vibration is obtained for various numerical values of thermal gradient, tapering constant and non-homogenous constant. Results are presented in graphical and tabular form.

EQUATION OF MOTION

The governing differential equation of transverse motion of visco-elastic non-homogeneous plate of variable thickness is

$$D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\,\partial y}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\vartheta\frac{\partial^2 W}{\partial x^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\vartheta\frac{\partial^2 W}{\partial x^2}\right)+2(1-\vartheta)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2 W}{\partial x\,\partial y}-\rho h p^2 W=0$$
(1)

Assuming that the elliptical plate has a steady two dimensional temperature distribution which is represented by

$$\tau = \tau_0\left(1 - \frac{x}{a} - \frac{y}{b}\right) \qquad (2)$$

Where τ denotes the temperature excess above the reference temperature at any point on the diameter from the centre of the elliptic plate and τ_0 denotes the temperature at any point on the boundary of the elliptic plate i.e. $1-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$.

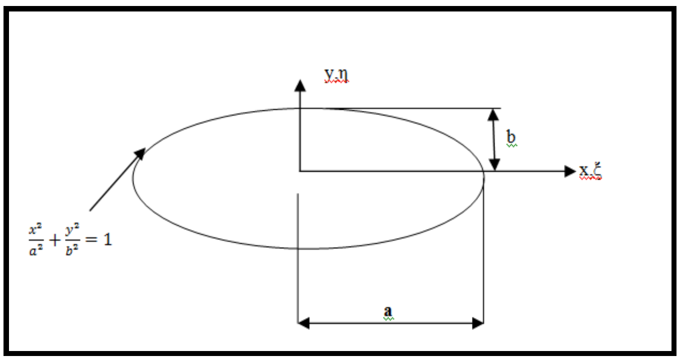

Fig. 1.1 Elliptical plate

The temperature dependent modulus of elasticity is taken as

$$E(\tau) = E_0\,(1 - \gamma\,\tau) \qquad (3)$$

Where E_0 is the Young's modulus and γ is taken as slope variation.

From equation (1) and (2) , we have

$$E = E_0\left[\,1 - \gamma\,\tau_0\left(1 - \frac{x}{a} - \frac{y}{b}\right)\right]$$

$$E = E_0\left[\,1 - \alpha\left(1 - \frac{x}{a} - \frac{y}{b}\right)\right] \qquad (4)$$

Where $\alpha = \gamma\,\tau_0$ ($0 \leq \alpha < 1$), a parameter.

It is assumed that thickness and non-homogeneity of the plate (which is non-homogeneity of the material mass density) varies along a diameter respectively as

$h = h_0 [1 - \beta \left(\frac{x}{a} + \frac{y}{b} \right)]$ (5)

and

$\rho = \rho_0 [1 - c_1 \left(\frac{x}{a} + \frac{y}{b} \right)]$ (6)

Where β is known as tapering constant and c_1 is non − homogeneity constant.

Since, the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$\left.\begin{array}{l} W = W_x = 0 \text{ at } x = 0, a \\ W = W_y = 0 \text{ at } y = 0, a \end{array}\right\} \quad (7)$$

Deflection function W(x,y) of plate is assumed to be

$W = A_1 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2 + A_2 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^3$ (8)

where A_1, A_2 are constants to satisfy boundary conditions.

Now, unit less variables having no dimension are using for us convince as

$X = \frac{x}{a}, Y = \frac{y}{a}$ (9)

Solution by Rayleigh-Ritz Method

Rayleigh – Ritz method is used to find an appropriate vibrational frequency. This method works on the phenomena that maximum strain energy (P_E) must equal to maximum kinetic energy (K_E). An equation in the following form is obtained as

$\delta(P_E - K_E) = 0$ (10)

The expression for kinetic and strain energy are

$$K_E = \frac{1}{2} p^2 \int_0^1 \int_0^{\sqrt{1-\xi^2}} \rho h\, W^2 dx\, dy \quad (11)$$

$$P_E = \frac{1}{2} \int_0^1 \int_0^{\sqrt{1-\xi^2}} D \left[\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2(1-\nu) \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dx\, dy \quad (12)$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$ is plate's flexural rigidity and ρ is density of the plate.

Now using the values of E and h from equations (3.3) and (3.4) in 'D' , We get

$$D = \frac{E_0 h_0^3 [1 - \alpha \left(1 - \frac{x}{a} - \frac{y}{b}\right)][1 + \beta \left(\frac{x}{a} + \frac{y}{b}\right)]^3}{12(1-\nu^2)} \quad (13)$$

Substitute the values from equations (5) , (6) , (7) , (8) and (9) in equation (10) , we get

$\delta(P_E^* - \lambda^2 K_E^*) = 0$ (14)

Where,

$$K_E^* = \frac{1}{2} p^2 \rho_0 h_0 \int_0^1 \int_0^{\sqrt{1-\xi^2}} \left(1 - \alpha_2 \left(X + Y\frac{a}{b}\right)\right)\left(1 - \beta \left(X + Y\frac{a}{b}\right)\right) W^2 dX\, dY \quad (15)$$

$$P_E^* = \frac{1}{2}\int_0^1 \int_0^{\sqrt{1-\xi^2}} \left[\left[1-\alpha\left(1-X-Y\frac{a}{b}\right)\right]\left[1+\beta\left(X+Y\frac{a}{b}\right)\right]^3\left[\left(\frac{\partial^2 w}{\partial X^2}\right)^2+\left(\frac{\partial^2 w}{\partial Y^2}\right)^2+ 2\nu\frac{\partial^2 w}{\partial X^2}\frac{\partial^2 w}{\partial y^2}+2(1-\nu)\left(\frac{\partial^2 w}{\partial X\,\partial Y}\right)^2\right]dX\,dY$$

(16)

and frequency $\lambda^2 = \frac{12p^2\rho_0 a^4(1-\nu^2)}{E_0 h_0^2}$

Now, on substituting the value of W, equation consist of two unknown constants i.e. A_1 & A_2 which is evaluate as follow:

$\frac{\partial(P_E^* - \lambda^2 K_E^*)}{\partial A_n} = 0$, for n= 1,2 (17)

On simplifying (3.16), we get

$mn_1A_1 + mn_2A_2 = 0$, for n= 1, 2 (18)

Where mn_1, mn_2 (n = 1,2) comprises parametric constant and the frequency parameter.

For non-trivial solution, the determinant of the co-efficient of equation (3.17) must be zero.

So, we get the frequency equation as

$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = 0$ (19)

With the help of equation (3.18), we get quadratic equation in λ^2 from which the two values of λ^2 can be found. These two values of λ^2 represent the frequency vibration of two modes i.e. λ_1 (first mode) & λ_2 (second mode) for different values of taper constant and thermal gradient for a clamped plate.

RESULT AND DISCUSSION

Frequency equation (3.18) is quadratic in λ^2, so it will give two roots. The frequency is derived for the first two modes of vibration for non – homogeneous elliptical plate having linearly varying thickness in both the directions, for the various values of taper constant (β), and thermal gradient (α), non – homogeneity constant (c_1). The value of passion ratio ν has been taken 0.345. All the results are calculated with the help of MAPLE software.

The results are shown in figures (1-4) for the first two modes of vibration for the elliptic plate.

Fig. 1, represents thermal gradient versus frequency with fixed value of Poisson ratio ($\nu = 0.345$). It is clearly seen that as thermal gradient (α) increases from 0 to 1 results frequency increases. Figure 1 has shown the results for the following three cases:

i) $\beta = \xi = C_1 = 0.0$, a/b = 1.5, ii) $\beta = \xi = C_1 = 0.2$, a/b = 1.5 iii) $\beta = \xi = C_1 = 0.4$, a/b = 1.5

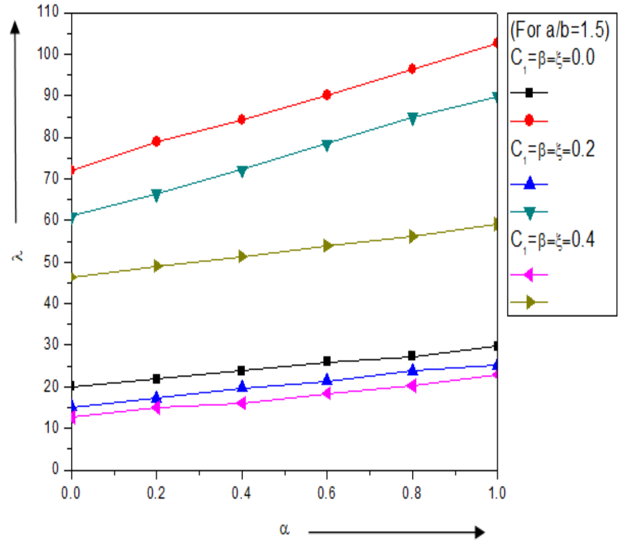

Fig. 1 Thermal gradient v/s Frequency

Fig. 2represents taper constant versus frequency with fixed value of Poisson ratio (ν = 0.345). It is clearly seen that as taper constant (β) increases from 0 to 1 results frequency decreases. Figure 2 has shown the results for the following three cases:

i) $\alpha = \xi = C_1 = 0.0$, a/b = 1.5, ii) $\alpha = \xi = C_1 = 0.2$, a/b = 1.5 iii) $\alpha = \xi = C_1 = 0.4$, a/b = 1.5

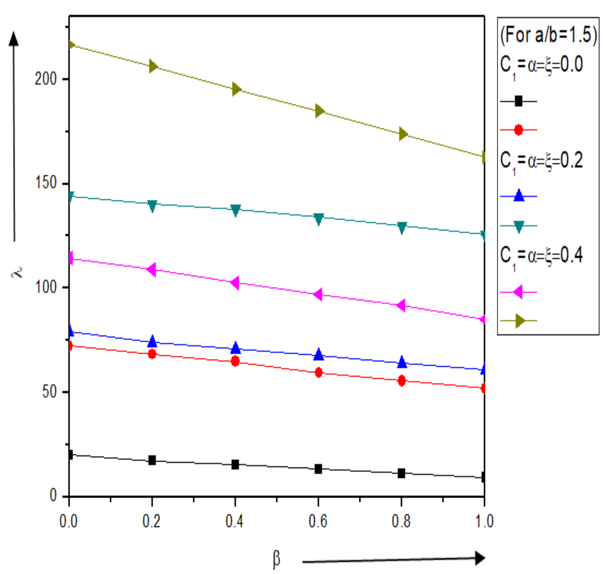

Fig. 2 Taper constant (β) vs frequency

Fig. 3, It is observed that for both modes of vibration, frequency parameter increases with the increase in aspect ratio a/b 0.5 to 3. Figure 3 has shown the results for the following three cases:

i) $\alpha = \beta = \xi = C_1 = 0.0$ ii) $\alpha = \beta = \xi = C_1 = 0.2$ iii) $\alpha = \beta = \xi = C_1 = 0.4$

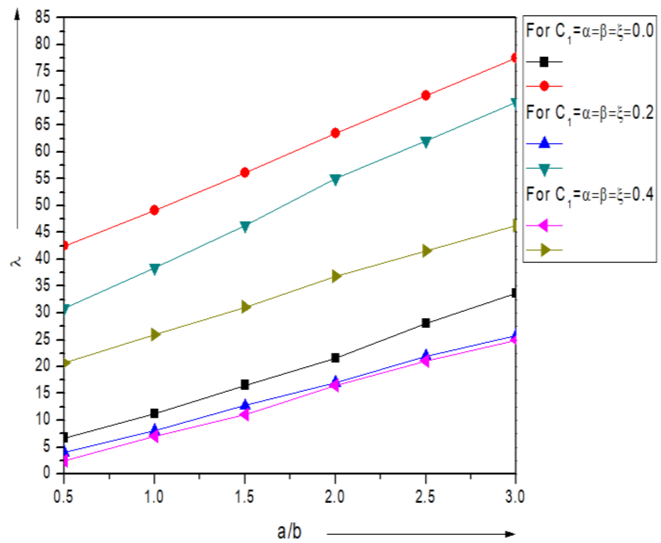

Fig. 3 Aspect ratio (a/b) vs frequency

Fig. 4, It is observed that for both modes of vibration, frequency parameter increases with the increase in ξ 0 to 1. Figure 4 has shown the results for the following three cases:

i) $\alpha = \beta = C_1 = 0.0$, a/b = 1.5 ii) $\alpha = \beta = C_1 = 0.2$, a/b = 1.5 iii) $\alpha = \beta = C_1 = 0.4$, a/b = 1.5

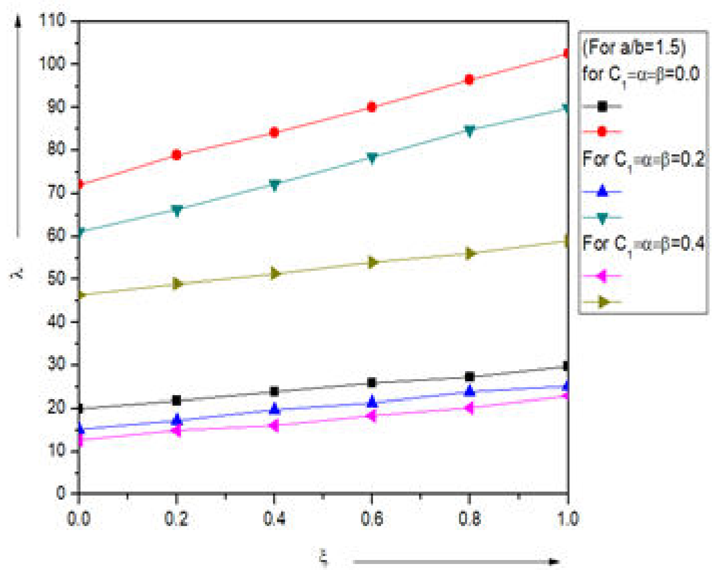

Fig.4 Frequency vs ξ

CONCLUSION

It can be clearly seen from the figures that frequency parameter decreases with an increase in taper constant and increases with the increase in thermal gradient. Also, frequency increases with increase in the value of aspect ratio. Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers. Therefore, mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

REFERENCES

[1]. Khanna A., Sharma A.K., 2011, Vibration Analysis of Visco-Elastic Square Plate of Variable Thickness with Thermal Gradient, International Journal of Engineering and Applied Sciences, Turkey, **3(4)**,1-6.

[2]. Khanna, A., & Sharma, A. K. "Analysis of free vibrations of Visco elastic square plate of variable thickness with temperature effect" International Journal of Applied Engineering Research, 2(2), 2011, 312-317.

[3]. Khanna, A., Kumar, A., & Bhatia, M. "A Computational Prediction on Two Dimensional Thermal Effect on Vibration of Visco-elastic Square Plate of Variable Thickness" Presented and Published in Proceeding of CONIAPS XIII, 2011, held in UPES, Deharadun.

[4]. N. Bhardwaj, A.P Gupta, K.K Choong, C.M Wang & Hiroshi Ohmori, "Transverse Vibrations of clamped and simply-supported circular plates with two dimensional thickness variation", Shock and Vibration 19 (2012), 273-285.

[5]. Khanna, A., Kaur, N., & Sharma, A. K. "Effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate" Indian Journal of Science and Technology, 5(9), 2012, 3263-3267.
and Technology, 5(9), 2012, 3263-3267.

[6]. Khanna, A., & Sharma, A. K. "A computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect" International journal of emerging in engineering and development, 2(3), 2012, 191-196.

[7]. Khanna, A., & Sharma, A. K. "Mechanical Vibration of Visco-Elastic Plate with Thickness Variation" International Journal of Applied Mathematical Research, 1(2), 2012,150-158.

[8]. Khanna, A., & Sharma, A. K. "Effect of Thermal Gradient on Vibration of Visco-Elastic Plate with Thickness Variation" American Journal of Computational and Applied Mathematics, 2(1), 2012, 34-36.

[9]. Anukul De and D. Debnath, "Vibration of Orthotropic Circular Plate with Thermal Effect in Exponential Thickness and Quadratic Temperature Distribution", J. Mech. Cont. & Math. Sci., Vol.-8, No.-1 (2013), 1121-1129.

[10]. Khanna A., Sharma A.K., 2013, Natural Vibration of Visco-Elastic Plate of Varying Thickness with Thermal Effect, Journal of Applied Science and Engineering, **16(2)**, 135-140.

[11]. Kumar Sharma, A., & Sharma, S. K. "Free Vibration Analysis of Visco-elastic Orthotropic Rectangular plate with Bi- Parabolic Thermal Effect and Bi-Linear Thickness Variation" Journal of Advanced Research in Applied Mechanics & Computational Fluid Dynamics, 1(1), 2014, 10-23.

[12]. Sharma, S. K., & Sharma, A. K. "Mechanical Vibration of Orthotropic Rectangular Plate with 2D Linearly Varying Thickness and Thermal Effect" International Journal of Research in Advent Technology, 2(6), 2014, 184-190.

[13]. Kumar Sharma, A., & Sharma, S. K. "Vibration Computational of Visco-Elastic Plate with Sinusoidal Thickness Variation and Linearly Thermal effect in 2D" Journal of Advanced Research in Applied Mechanics & Computational Fluid Dynamics, 1(1), 2014, 46-54.

[14]. Sharma, S. K., & Sharma, A. K. "Effect of Bi-Parabolic Thermal and Thickness Variation on Vibration of Visco-Elastic Orthotropic Rectangular Plate" Journal of Advanced Research in Manufacturing, Material Science & Metallurgical Engineering, 1(2), 2014, 26-38.