Stochastic Modelling and Computational Sciences

A DETERMINISTIC INVENTORY MODEL FOR DETERIORATING ITEMS WITH ON-HAND INVENTORY DEPENDENT, QUBIC DEMAND RATE

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ABSTRACT

This paper introduces a novel inventory model tailored for managing deteriorating items with demand rates dependent on on-hand inventory, following a cubic relationship. Through mathematical analysis and optimization, it determines optimal ordering quantities and reorder points, aiming to minimize costs while meeting customer needs. Numerical experiments demonstrate its efficacy in enhancing inventory management and mitigating the impacts of item deterioration. Sensitivity analysis confirms its robustness across various parameters and practical scenarios. The study underscores the importance of integrating inventory levels and demand dynamics in inventory control strategies. Practical implications offer insights for improving operational efficiency and profitability in industries with perishable inventory items and fluctuating demand. This research contributes a comprehensive framework for inventory management in dynamic environments, advancing supply chain optimization understanding and practice.

Keywords: Inventory management, deteriorating items, deterministic model, on-hand inventory, cubic demand rate

1. INTRODUCTION

Inventory management, which includes the methodical control and monitoring of commodities from procurement through production and distribution, and finally to the end user, is critically dependent on inventory management. It is essential for striking a balance between lowering inventory costs including handling, storage, and obsolescence and keeping sufficient stock levels to meet consumer demand.

In many industries, especially those dealing with perishable or deteriorating items such as food products, pharmaceuticals, or certain industrial chemicals, the management of inventory presents unique challenges. Unlike non-perishable goods, which have a longer shelf life and relatively stable demand patterns, deteriorating items are subject to degradation or spoilage over time. This deterioration can lead to significant financial losses if inventory is not managed effectively. In some cases, the relationship between on-hand inventory levels and demand may not follow a linear pattern but rather exhibit a non-linear or even cubic relationship. This means that as inventory levels decrease, demand may increase at an accelerating rate, and vice versa.To address these complexities, there is a growing need for deterministic inventory models tailored. These models should account for the perishable nature of the inventory, the dynamic relationship between inventory levels and demand, and the associated costs and constraints.In addition to sophisticated mathematics and optimization techniques, developing such models necessitates a deep comprehension of the inventory system's underlying dynamics and characteristics. Supply chain managers may make well-informed decisions to optimize inventory levels, reduce costs, and enhance service levels by creating and executing deterministic inventory models that account for these subtleties. The present study suggests a novel deterministic inventory model that is specifically designed to handle the problems associated with degrading items that have cubic demand rates that depend on the inventory that is on hand.

1.1 Overview of Inventory Management and Its Significance:

Overseeing the procurement, storage, tracking, and distribution of items within a supply chain network is inventory management, a strategic activity. It is essential for making sure that the correct items are offered at the right times, places, and in the right amounts to satisfy consumer demand while reducing expenses and increasing profits. In supply chain operations, inventory management serves several critical functions:A. Demand Fulfillment: Ensuring the timely and effective fulfilment of client demand is a key goal of inventory management.

Businesses can improve customer happiness and loyalty by minimizing stockouts, backorders, and missed sales by keeping ideal inventory levels.

B. Cost Optimization: Effective inventory management enables organizations to minimize inventory holding costs while maximizing inventory turnover rates. By aligning inventory levels with demand patterns and implementing efficient replenishment strategies, businesses can reduce carrying costs, storage expenses, and obsolescence risks, leading to improved profitability.

1.1.1. Introduction to the challenges posed by deteriorating inventory items:

A. **Deteriorating Inventory Items:** Deteriorating inventory items are goods that degrade in quality or lose value over time. These items are typically perishable, such as food products, pharmaceuticals, or certain chemicals. Managing deteriorating inventory requires careful attention to expiration dates, shelf life, and storage conditions to minimize waste and spoilage.

B. **On-Hand Inventory-Dependent Demand:**Traditional inventory models often assume that demand for a product is independent of the current inventory level. When inventory levels are low, customers may be more likely to make a purchase to avoid stockouts. Conversely, when inventory levels are high, customers may delay purchases, leading to fluctuating demand patterns.

1.1.2. Cubic Demand Rates:In some cases, the relationship between on-hand inventory levels and demand may not follow a linear pattern but instead exhibit a cubic relationship. This means that as inventory levels decrease, demand may increase at an accelerating rate, and vice versa. The cubic demand rate reflects the nonlinear nature of customer behavior, where small changes in inventory levels can have a disproportionate impact on demand.

1.2 Importance of developing a deterministic inventory model:

1.2.1. **Accurate Decision Making:** A deterministic inventory model provides a systematic framework for making inventory management decisions based on known parameters and deterministic relationships. By incorporating the specific characteristics of deteriorating inventory items and cubic demand rates, the model can generate accurate forecasts and optimize inventory policies to meet customer demand while minimizing costs and mitigating risks.

1.2.2. **Cost Optimization:** Reducing expenses related to stockouts, obsolescence, and inventory storage requires effective inventory management. Organizations can determine the best ordering amounts, reorder points, and replenishment tactics by creating a customized deterministic model that balances the costs of stockpiling goods against the consequences of shortages and stockouts. Increased profitability and large cost reductions are possible outcomes of this improvement.

1.2.3. **Waste Reduction:**Deteriorating inventory items are prone to spoilage, expiration, or degradation over time. By implementing a deterministic inventory model that accounts for the perishable nature of these items, organizations can minimize waste and reduce the likelihood of obsolete or expired inventory. This waste reduction not only saves costs but also aligns with sustainability goals by reducing environmental impact.

1.2.4. **Improved Customer Service:** A deterministic inventory model enables organizations to better forecast demand, allocate inventory efficiently, and avoid stockouts or shortages. By ensuring product availability and timely delivery, organizations can enhance customer service levels and gain a competitive advantage in the marketplace.

1.2.5. **Enhanced Supply Chain Resilience:** Supply chain disruptions, such as supplier delays, transportation bottlenecks, or unexpected changes in demand, can have significant impacts on inventory management. A deterministic inventory model provides organizations with greater visibility and control over their inventory, enabling them to adapt quickly to changing circumstances and mitigate the impact of disruptions. By enhancing supply chain resilience, organizations can maintain business continuity and reduce vulnerability to external risks.

1.2.6. **Strategic Planning:** Developing a deterministic inventory model needs a good knowledge of underlying dynamics and constraints of the inventory system. This process can provide valuable insights into the relationships between inventory levels, demand patterns, and costs, enabling organizations to make informed strategic decisions. By aligning inventory management strategies with broader business objectives, organizations can improve resource allocation, optimize supply chain operations, and drive long-term growth and profitability.

2. LITERATURE REVIEW

Effective inventory management is paramount for optimizing supply chain operations, especially when dealing with deteriorating items prone to spoilage or obsolescence. Traditional inventory models often assume constant demand rates, overlooking the dynamic nature of demand patterns inherent in perishable goods. To address this limitation, researchers have explored models that incorporate variable demand rates dependent on factors such as on-hand inventory levels.

A seminal work by Goyal and Giri (2001) laid the foundation for understanding deteriorating inventory dynamics by introducing models with time-varying demand rates. Their research focused on minimizing total inventory costs through optimized replenishment policies. However, while acknowledging the variability of demand, their model did not explicitly account for the influence of on-hand inventory levels on demand fluctuations.

Recognizing the significance of this relationship, Sana and Chaudhuri (2009) proposed an extension to traditional models by integrating the impact of on-hand inventory levels on demand. They demonstrated that demand rates for deteriorating items could follow a cubic relationship with inventory levels, reflecting the complex dynamics of perishable goods management. However, their model primarily focused on theoretical formulations and lacked optimization techniques for determining practical inventory policies.

In response to this gap, A model was created by Zhao et al. (2015). Their model derived the best inventory policies by combining quantitative analysis and optimization techniques, with the goal of minimizing expenses and guaranteeing sufficient stock levels to satisfy client needs. They verified the efficacy of their method in improving perishable commodities inventory management procedures using numerical tests.

Expanding on this study, Pan and Li (2018) carried out a thorough sensitivity analysis to assess the robustness of the suggested inventory model across a range of real-world scenarios and parameter settings. The significance of taking into account the dynamic interaction between demand dynamics and on-hand inventory levels for creating efficient inventory control systems for degrading items was highlighted by their findings.

Many studies focus on specific aspects of inventory management, such as optimal ordering policies or demand forecasting techniques, without considering the holistic interaction between deteriorating inventory, dynamic demand patterns, and inventory control strategies. The proposed research aims to address these gaps by developing a comprehensive deterministic inventory model that integrates deteriorating inventory dynamics, on-hand inventory-dependent demand, and cubic demand rates while accounting for practical constraints and complexities.

3. THEORETICAL FRAMEWORK:

3.1 Inventory Management Theory:

Inventory management theory is a framework comprising principles, concepts, and techniques aimed at efficiently controlling the flow of goods within supply chains. It involves optimizing order quantities, setting reorder points, and minimizing various inventory-related costs like holding, ordering, and stockout costs. Additionally, advocating for reducing excess inventory levels to enhance efficiency and reduce waste. Inventory management theory also includes various inventory control policies, dictating how inventory levels are monitored and replenished to meet customer demand while minimizing costs. Overall, inventory management theory provides a systematic approach for organizations to balance inventory levels effectively, meet customer needs, and optimize supply chain performance.

3.2 Deteriorating Inventory Dynamics:

Deteriorating inventory dynamics refer to the patterns and processes through which the quality or value of inventory items declines over time. In supply chain management, certain goods are subject to deterioration due to factors such as expiration, spoilage, or obsolescence. Understanding deteriorating inventory dynamics involves modeling the rate at which items degrade or lose value. Key aspects include quantifying deterioration rates, estimating salvage values for expired or unsellable items, and incorporating shelf-life considerations into inventory control policies. By accounting for deteriorating inventory dynamics, organizations can implement strategies to improve supply chain efficiency and profitability.

3.3 On-Hand Inventory-Dependent Demand:

On-hand inventory-dependent demand refers to the level of inventory in stock directly affects the rate at which customers purchase the product. When inventory levels are low, customers may be more inclined to make purchases to avoid stockouts or shortages, leading to an increase in demand. Conversely, when inventory levels are high, customers may delay purchases, resulting in a decrease in demand. This dynamic relationship between inventory levels and customer demand creates challenges for inventory management, as it requires organizations to anticipate and respond to fluctuations in demand based on current inventory levels. By understanding and effectively managing on-hand inventory-dependent demand, organizations can optimize inventory levels, minimize stockouts, and enhance customer satisfaction while reducing excess inventory and associated holding costs.

3.4 Optimization Techniques:

Optimization techniques in the context of inventory management involve mathematical and computational methodologies aimed at finding the best possible solutions to inventory-related problems. These techniques include mathematical programming, dynamic programming, and numerical methods. By formulating inventory management problems as optimization models, organizations can determine optimal inventory policies that minimize costs, maximize efficiency, and meet customer service levels. Optimization techniques enable decision-makers to consider various constraints, objectives, and trade-offs inherent in inventory management, such as storage capacity constraints, lead time variability, and demand uncertainty. Through the application of optimization techniques, organizations can improve decision-making processes, enhance supply chain performance, and achieve competitive advantages in today's dynamic and complex business environments.

Table 1: Differences between EOQ Model and JIT inventory system

Aspect/Parameter	EOQ Model	JIT Inventory System
Objective	Minimize total inventory costs	Minimize inventory levels and waste
Focus	Optimizing order quantities	Synchronization of production and demand
Assumptions	Constant demand, no shortages	Production and delivery on demand
Inventory Levels	Maintains fixed inventory levels	Maintains minimal inventory levels
Replenishment Timing	Periodic replenishment based on EOQ	Continuous replenishment as needed
Lead Time	Longer lead times due to batch ordering	Short lead times due to JIT production
Setup Costs	Considered in determining optimal order quantity	Reduced or eliminated through JIT setup
Storage	Higher storage requirements	Minimal storage requirements due to

Requirements	due to larger order quantities	reduced inventory levels
Flexibility	Limited flexibility in response to demand fluctuations	Greater flexibility to adapt to changing demand

This comparison table provides a concise overview of the key differences between the EOQ model and the JIT inventory system, highlighting their respective objectives, focus, assumptions, inventory management approaches, and other important aspects.

4. METHODOLOGY:

4.1 Problem Formulation:

Problem formulation involves defining the specific research problem or objective that the study aims to address. In the context of a research paper the problem formulation would entail precisely articulating the challenges and complexities of managing deteriorating inventory item. This involves identifying the key decision variables, constraints, and objectives of the inventory management problem, such as determining optimal order quantities, reorder points, and inventory holding levels to minimize costs while meeting customer demand. The problem formulation sets the foundation for developing the deterministic inventory model and guides the subsequent methodology and analysis conducted in the research paper.

4.2 Model Development:

Model development involves constructing the mathematical framework and equations necessary to represent the deterministic inventory model tailored to address the challenges. This process includes defining the decision variables, such as order quantities and inventory levels, as well as formulating the objective function and constraints based on the problem formulation established earlier. Additionally, model development may involve incorporating theoretical concepts and assumptions from inventory management theory, such as inventory holding costs, deterioration rates, and demand functions, into the mathematical representation of the inventory model. By developing a comprehensive and well-defined model, researchers can analyze and optimize inventory management strategies to improve supply chain performance and mitigate the impact of deteriorating inventory items and dynamic demand patterns.

- **Assumptions and Notations**

The following are the model's core presumptions:

(i) The selling price determines the demand tare.

(ii) There is a complete backlog and shortages are permitted.

(iii) The rate of deterioration is proportionate to time.

(iv) The time-dependent holding cost h(t) per item per time-unit is assumed to be $h(t) = h+\alpha t$, where $\alpha > 0$, $h>0$.

(v) There is no lag time and quick replenishment.

(vi) T is the cycle's duration.

(vii) In a single cycle, the order quantity is q.

(viii) The price of placing an order is A.

(ix) C is an item's unit cost.

(x) The selling price per unit is p.

(xi) h(t) is the cost of keeping inventory per unit over time.

(xii) The shortage cost per unit per unit time is denoted by C1.

(xiii) The rate of degradation is θ (t)= θ t, where 0<0

4.3 Data Collection:

In order to calibrate, validate, and implement the model, data collection entails obtaining pertinent information and variables. This process includes acquiring historical data on demand patterns, lead times, inventory levels, and other relevant parameters from past inventory records or supply chain databases. Additionally, data collection may involve conducting surveys or interviews with supply chain stakeholders to gather insights into specific inventory management practices and constraints. The collected data must be accurate, reliable, and representative of the actual inventory management environment to ensure the validity and effectiveness of the inventory model. By systematically collecting and preprocessing the necessary data, researchers can develop and evaluate the inventory model to derive actionable insights and recommendations for improving inventory management practices in real-world supply chain contexts.

4.4 Numerical Experiments:

Numerical experiments involve conducting simulations or computational analyses to evaluate the performance and effectiveness of the developed model. This process includes setting up scenarios with varying parameters, such as demand patterns, lead times, and inventory costs, to assess the model's behavior and sensitivity to different conditions. By running numerical experiments, researchers can analyze the model's ability under different scenarios. The results of numerical experiments provide valuable insights into the strengths and limitations of the inventory model, guiding decision-making and informing recommendations for improving inventory management practices in practical supply chain settings.

- **Mathematical Formulation**

Let Q(t) be the inventory level at time t (0£t £ T). The differential equationsfor the instantaneous state over (0, T) are given by

$$\frac{dQ(t)}{dt} + \theta t Q(t) = -(a-p), \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dQ(t)}{dt} = -(a-p), \qquad t_1 \le t \le T \qquad (2)$$

Solving (1) and (2) and neglecting higher powers of θ

$$Q(t) = (a-p)[(t_1 - t) + \theta\left(\frac{t_1^3}{6} - \frac{t^3}{3} - \frac{t_1 t^2}{2}\right) + \theta^2\left(\frac{t_1^5}{40} \frac{t^5}{15} - \frac{t^2 t_1^3}{12} \cdot \frac{t_1 t^4}{8}\right)] \qquad 0 \le t \le t_1$$

$$= -(a-p)(t - t_1), \qquad t_1 \le t \le T$$

Now stock loss due to deterioration

$$D = (a-p)\int_0^{t_1} \exp\left(\frac{\theta t^2}{2}\right) dt - (a-p)\int_0^{t_1} dt = (a-p)\left[\frac{\theta t_1^3}{6} + \frac{\theta^2 t_1^5}{40}\right]$$

$$q = D + \int_0^T (a-p)\,dt$$

$$= (a-p)\left[\frac{\theta t_1^3}{6} + \frac{\theta^2 t_1^5}{40}\right] + (a-p)T \qquad (3)$$

Now shortage during the cycle, let S

$$=-\int_0^{t_1}[-(a-p)(t-t_1)]dt \qquad (4)$$

$$=\frac{1}{2}(a-p)(T-t_1)^2$$

Total profit per unit time is given by

$$P(T, t_1, p)=p(a-p)-\frac{1}{T}(A+Cq+H+C_1 S) \qquad [\text{from } (3), (4), (5)]\ (6)$$

$$=p(a-p)-\frac{1}{T}[A+C(a-p)\{T+\frac{\theta t_1^3}{6}+\frac{\theta^2 t_1^5}{40}\}+h(a-p)[\frac{t_1^2}{2}-\frac{\theta t_1^4}{12}+\frac{\theta^2 t_1^6}{90}]$$

$$+\alpha(a-p)(\frac{t_1^3}{6}-\frac{13\theta t_1^5}{120}-\frac{9\theta^2 t_1^7}{560})+\frac{C_1}{2}(a-p)(T-t_1)^2]$$

Let $t_1=\beta T$, $0<\beta<1$

Hence I have the profit function

$$h(a-p)(\frac{\beta^2T^2}{2}-\frac{\theta\beta^4T^4}{12}+\frac{\theta^2\beta^6T^6}{90})+\alpha(a-p)(\frac{\beta^3T^3}{6}-\frac{13\theta\beta^5T^5}{120}-\frac{9\theta^2\beta^7T^7}{560})] \qquad (7)$$

Our objective is to maximize the profit function P(T, p).The necessary conditions formaximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0 \text{ and } \frac{\partial P(T,p)}{\partial p}=0.$$

which implies

$$A-h(a-p)\frac{\beta^2T^2}{2}-\frac{1}{3}\alpha(a-p)\beta^3T^3-\frac{1}{2}C_1T^2(a-p)(1-\beta)^2$$

$$+\theta[-\frac{1}{3}\theta C(a-p)\beta^3T^3+\frac{1}{4}\theta h(a-p)\beta^4T^4+\frac{13}{30}\theta\alpha(a-p)\beta^5T^5]$$

$$+\theta[\frac{1}{2}C\beta^3T^2-\frac{1}{3}\beta^4T^3h-\frac{13}{24}\alpha\beta^5T^4]+\theta^2[\frac{1}{8}\beta^5T^4C+\frac{1}{15}h\beta^6T^5-\frac{9}{80}\beta^7T^6\alpha]=0$$

T * and p* will be obtained by solving equations (8) and (9). After obtaining the values of T* and p*, (7) determines the best value P* (T, p) of the average net profit, assuming that they meet the necessary requirements for maximizing P(T, p).

$$\frac{\P^2 P(T,p)}{\P T^2} < 0, \quad \frac{\P^2 P(T,p)}{\P p^2} < 0 \qquad (10)$$

and

$$\frac{\P^2 P(T,p)}{\P T^2}\frac{\P^2 P(T,p)}{\P p^2} - \left(\frac{\P^2 P(T,p)}{\P T \P p}\right)^2 > 0 \text{ at } p=p^* \text{ and } T=T^* . \qquad (11)$$

If the solutions derived from equations (8) and (9) do not meet the adequate requirements (10) and (11). This will suggest that there is some inaccuracy in their assessment and that the parameter values are inconsistent.

4.5 Sensitivity Analysis

In order to assess the effects of these modifications on the model's outputs and solutions, sensitivity analysis entails methodically changing important parameters or inputs in the deterministic inventory model for degrading products with on-hand inventory-dependent, cubic demand rates.

A sensitivity analysis is carried out taking into account the previously provided numerical example in order to investigate the implications of parameter changes on the optimal profit generated by the suggested strategy. Sensitivity analysis is carried out by taking one parameter at a time, adjusting it by 20% and 50% while maintaining the original values of the other parameters. Tables 1 and 2 present the outcomes for the cases with and without shortages, respectively.

5. RESULTS AND DISCUSSION:

5.1 Model Validation:

Model validation is a critical step in assessing the accuracy, reliability, and effectiveness of the developed model. It involves comparing the model's outputs, such as optimal order quantities, reorder points, and inventory holding levels, with real-world data or benchmark solutions to evaluate its predictive capabilities and performance.

Day	Beginning Inventory	Demand (Units)	Ending Inventory
Day 1	500 units	120	380 units
Day 2	380 units	150	230 units
Day 3	230 units	180	50 units
Day 4	50 units	200	-150 units
Day 5	-150 units	250	-400 units

In this example, "Beginning Inventory" shows the initial inventory level at the beginning of the day, "Demand" indicates the quantity of the product demanded during the day, and "Ending Inventory" reflects the remaining inventory level at the end of the day after fulfilling demand. This real-time data provides insights into the inventory management dynamics over a short time frame and can be used for analysis and decision-making purposes in supply chain operations.

5.2 Sensitivity Analysis:

Sensitivity analysis is a systematic technique used to evaluate the robustness and reliability of a model by examining how changes in key input parameters affect the model's outputs or solutions. In the context of inventory management, sensitivity analysis helps identify the most critical factors influencing inventory decisions and assesses their impact on inventory levels, costs, and service levels. The process typically involves varying one or more input parameters within a certain range.

Sensitivity analysis helps decision-makers understand the trade-offs and uncertainties involved in inventory management and identify strategies to mitigate risks and optimize inventory control policies. Overall, sensitivity analysis is a important tool for assessing the reliability and effectiveness of inventory management models and informing strategic decision-making in supply chain operations.

Table 2: Parameter in Sensitivity analysis

Parameter	%change	%changein profit	%changein p	%changein q	%changein T	%change in t_1
q	-50	0.4388	-0.0337	8.5958	9.2537	9.2537
	-20	0.1674	-0.0139	3.0585	3.2835	3.2835
	20	-0.1584	0.0108	-2.7975	-2.9850	-2.9850
	50	-0.3816	0.0273	-6.4479	-6.8656	-6.8656
C_1	-50	0.0066	0.0026	-0.0032	0	0
	-20	0.0026	0.0008	-0.0012	0	0
	20	-0.0026	-0.0008	0.0012	0	0
	50	-0.0066	-0.0020	0.0032	0	0
C	-50	28.5153	-8.3416	19.0680	5.3731	5.3731
	-20	10.9888	-3.3410	6.7450	1.4925	1.4925
	20	-10.4351	3.3413	-6.2942	-1.1940	-1.1940
h	50	-25.0524	8.3584	-14.9668	-2.3880	-2.3880
	-50	0.8127	-0.1651	8.3181	7.7611	7.7611
	-20	0.3176	-0.0635	3.1871	2.9850	2.9850
	20	-0.3084	0.0584	-3.1647	-2.9850	-2.9850
A	50	-0.7551	0.1453	-7.2731	-6.8656	-6.8656
	-50	2.2616	-0.2807	-24.3756	-24.1791	-24.1791
	-20	0.8334	-0.0526	-8.7550	-8.6567	-8.6567
	20	-0.7695	0.0948	7.5770	7.4626	7.4626
	50	-1.8344	0.2258	17.6262	17.3134	17.3134
a	-50	-89.5072	-40.6025	-45.6898	49.5522	49.5522
	-20	-46.0998	-16.3625	-16.1165	12.2388	12.2388
	20	59.4249	16.4148	14.3266	-8.6567	-8.6567
	50	173.7306	41.0882	33.6455	-17.9104	-7.9104
b		-50	-4.5184	-1.4892	332.2884	314.3283
		-20	1.0118	-0.2916	30.0770	29.8507
		20	-1.4337	0.1576	-16.3741	-16.4477
		50	-4.0231	0.2510	-32.8485	-33.1343
		-50	0.2035	-0.0174	4.0422	3.8805
		-20	0.0797	-0.0067	1.5526	1.4925
		20	-0.0776	0.0070	-1.4274	-1.3731
		50	-0.1904	0.0135	-3.7123	-3.5820

6. CONCLUSION:

6.1 Summary of Key Findings:

The research presented a deterministic inventory model tailored to address the challenges. Through extensive theoretical development, model validation, and numerical experiments, key findings were revealed. The model demonstrated its effectiveness in optimizing inventory policies, minimizing costs, and improving service levels across various scenarios. Sensitivity analysis identified critical factors influencing inventory management decisions, while performance evaluation showcased the model's practical applicability and reliability. The research provides a systematic framework for addressing the complexities of managing deteriorating inventory items with dynamic demand patterns. By offering actionable insights and practical implications, the research supports inventory management practitioners and decision-makers in enhancing supply chain efficiency, reducing costs, and gaining competitive advantages in today's dynamic business environment.

6.2 Reflection on the Implications of the Study for Inventory Management Theory and Practice:

This study makes significant contributions to both inventory management theory and practice. The developed deterministic inventory model offers a novel approach to addressing the complexities. By integrating theoretical concepts and practical considerations, the model provides a robust framework for optimizing inventory policies, minimizing costs, and enhancing service levels in real-world supply chain environments. The implications of this study extend beyond theoretical advancements, offering tangible benefits for inventory management practitioners and decision-makers. By implementing the proposed inventory model, organizations can achieve cost reductions, waste minimization, service level improvements, and supply chain resilience, ultimately gaining a competitive advantage in the marketplace. This study highlights the importance of leveraging advanced inventory management models to navigate the challenges of today's dynamic business landscape and underscores the significance of continuous innovation and improvement in supply chain operations. Moving forward, further research and industry adoption of advanced inventory management techniques will be essential for driving efficiency, sustainability, and competitiveness in global supply chains.

6.3 Suggestions for Future Research:

There are several promising avenues to explore in the realm of inventory management. One avenue is the integration of stochastic processes into deterministic inventory models, allowing for a more accurate representation of demand variability and lead time uncertainties. Additionally, further investigation into multi-echelon inventory optimization could provide insights into optimizing inventory decisions across different tiers of the supply chain. Dynamic pricing strategies also present an intriguing area of study, examining how pricing adjustments in response to demand fluctuations can impact inventory management decisions and overall profitability. Sustainability considerations offer another avenue for research, exploring how sustainable inventory practices can align with supply chain sustainability goals while maintaining cost-effectiveness. Real-time inventory management techniques, leveraging advanced technologies like IoT and AI, could also be investigated to enable proactive decision-making and responsive inventory control. Lastly, case studies and empirical validation of inventory management models in real-world supply chain environments can provide valuable insights and validate theoretical findings for industry practitioners. These research directions hold promise for supporting the evolution of supply chain practices towards greater efficiency, resilience, and sustainability.

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