Stochastic Modelling and Computational Sciences

STOCHASTIC MODEL FOR RELIABILITY ANALYSIS OF MULTI-COMPONENT SYSTEMS

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ABSTRACT

In this paper, we present an analysis of availability and maintenance for systems experiencing multiple failures, utilizing alternating renewal processes along with an M/E$_2$/1 queue model to represent the system's reliability and availability. The primary objective of this study is to demonstrate the findings obtained through the proposed model.

Keywords: Multi-failure systems, Availability analysis, Maintenance optimization, Alternating renewal processes, M/E$_2$/1queue model, System reliability and Failure rate.

INTRODUCTION

In this model, we analyze a Power Distribution System (PDS) to demonstrate the derived results. Globally, PDSs are among the most intricate infrastructures and are required to function with high standards of quality and reliability. Power systems aim to create an economical and dependable network to transport electric power from generation sites to customer locations. However, balancing economic constraints and reliability demands can be challenging, making power system operations complex. This analysis centers on the station and feeder design at the utilization end of the power grid (Sajeesh Babu, 2017). There are two primary failure modes in this setting: active and passive failure events. Active failure events, which are the more common in power systems, involve faults, such as a defect in a conductor component. In these cases, the defective conductor is disconnected from the rest of the system by a circuit breaker assigned to the protection zone, classifying this as an active failure. Passive failures, on the other hand, may arise from material defects, operator inexperience, and similar factors.

Consider a PDS where both active and passive failures are modeled with theoretical random failure times denoted by α_1 and α_2, respectively. Let $E_1(t)$ and $E_2(t)$ describe the distribution of sequential failure times, where $E_1(t) = E_2(t) = 1 - e^{-t}$. The successive service times, β_1 and β_2, follow an Erlang distribution with a shape parameter of 2 and a service rate of 6. Thus, setting $\eta_1 = \eta_2 = 1$ and $\zeta_1 = \zeta_2 = 6$, we get $\frac{2.1}{6} = \frac{1}{3} < 1$. Additionally, the values of σ and ω are -8 and -3, respectively. The results for this example are outlined below.

RELIABILITY ANALYSIS FOR POWER DISTRIBUTION SYSTEM (PDS)

An equation given by:

$$A(t) = R(t) + \sum_{l=0}^{\infty} \sum_{q=1}^{B} \int_0^{t-(l+1)\theta} A(t-(l+1)\theta - z) \frac{(\zeta_q - 2\eta_q)}{\omega - \sigma} \zeta_q e^{wz} - e^{\sigma z} \, dz \quad \text{…..(1)}$$

where, $A(t)$ represented the primary performance metric for complex systems

$R(t)$ represents the system's reliability.

ζ_q is the rate of repair of qth mode of failure.

η_q is the failure rate of qth failure mode.

θ is the length of inspection interval

B is the number of failure modes

With the help of equation (1), Using equation (1), the reliability of the PDS is assessed as shown in Table 1 and illustrated in Fig. 1

Time	**R(t) for PDS**	
	M/E$_2$/1 queue	**M/M/1 queue**
0.0	1.1	1.1
0.3	0.6704	0.6704
0.6	0.4495	0.4495
0.9	0.3015	0.3015
1.2	0.2022	0.2022
1.5	0.1356	0.1356
1.8	0.0910	0.0910
2.1	0.0611	0.0611
2.4	0.0411	0.0411
2.7	0.0276	0.0276
3.0	0.0186	0.0186

Table 1: Time vs Reliability Analysis

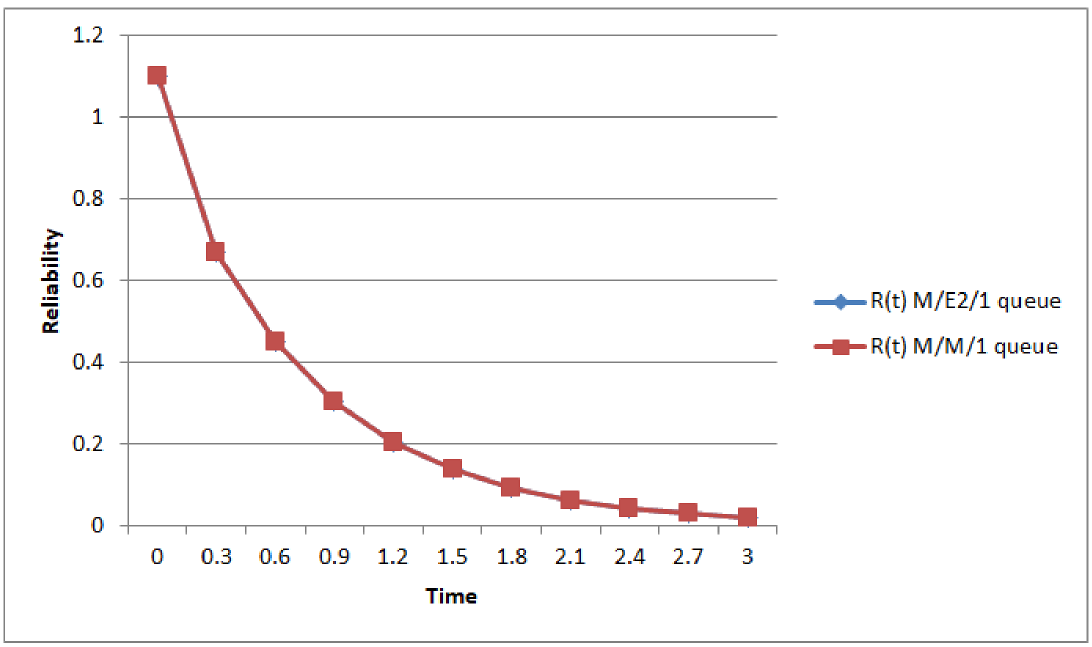

Fig 1: PDS Reliability function R (t) vs Time (t)

ANALYSIS OF AVAILABILITY FOR THE POWER DISTRIBUTION SYSTEM

$$A = \frac{\int_0^\infty R(t)\,dt}{\sum_{l=0}^{\infty}(l+1)\theta[R(l\theta)-R(l+1)\theta]+\sum_{q=1}^{B}E(Z_q)\int_0^\infty R(t)\eta_q(t)dt} \quad \text{....(2)}$$

The point availability, A(t), and the long-term availability, A(θ), of the PDS can be determined using equations (1) and (2). The values of A(t) and A(θ) are presented in Tables 2 and 3 and are shown graphically in Figures 2 and 3, respectively.

Time	$A(t)$	
	M/E$_2$/1 queue	M/M/1 queue
0.0	1.1	1.1
0.3	0.6704	0.6704
0.6	0.4495	0.4495
0.9	0.3015	0.3015
1.2	0.2022	0.2022
1.5	0.1358	0.1358
1.8	0.2715	0.5271
2.1	0.3936	0.5139
2.4	0.4029	0.4038
2.7	0.3511	0.2927
3.0	0.3800	0.2044
3.3	0.2494	0.3619
3.6	0.2760	0.4717
3.9	0.3210	0.4500
4.2	0.3453	0.3700
4.5	0.3376	0.3246

Table2: Point Probability vs Time

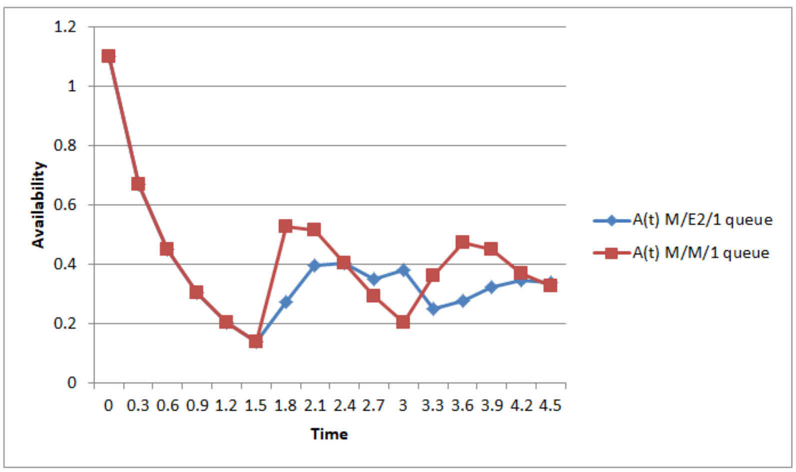

Fig 2: PDS Availability function A (t) vs Time (t)

θ	$A(\theta)$	
	$M/E_2/1$ queue	$M/M/1$queue
0.1	0.5218	0.71429
1.1	0.3097	0.36860
2.1	0.2004	0.22349
3.1	0.1444	0.15589
4.1	0.1122	0.11901
5.1	0.0917	0.09615
6.1	0.0775	0.08065

7.1	0.0671	0.06945
8.1	0.0592	0.06098
9.1	0.0530	0.05435
10.1	0.04782	0.04902

Table 3: Steady-State Availability vs Time

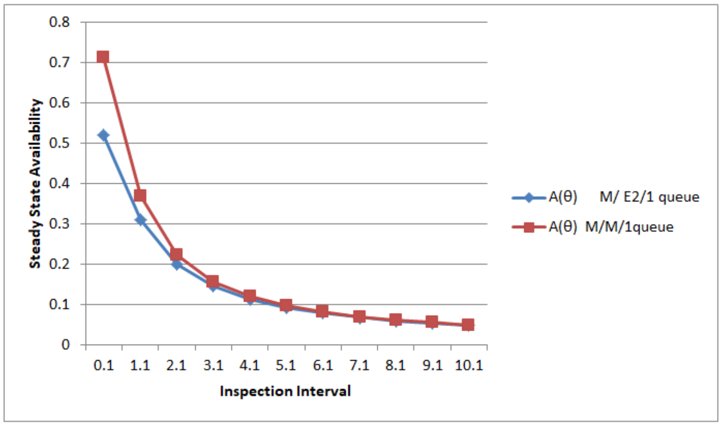

Fig. 3: Steady-state availability $A(\theta)$ versus θ for PDS

Time	***A(t)***		
	When $\theta = 1$	When $\theta = 1.5$	When $\theta = 2$
0.1	1.1	1.1	1.1
0.7	0.3013	0.3013	0.3013
1.1	0.1354	0.1354	0.1354
1.7	0.4022	0.1154	0.0408
2.1	0.2800	0.4128	0.0184
2.7	0.3198	0.2586	0.4158
3.1	0.3362	0.1411	0.2996
3.7	0.3296	0.2513	0.1221
4.1	0.3351	0.3194	0.0606

Table 4: Sensitivity Analysis for PDS

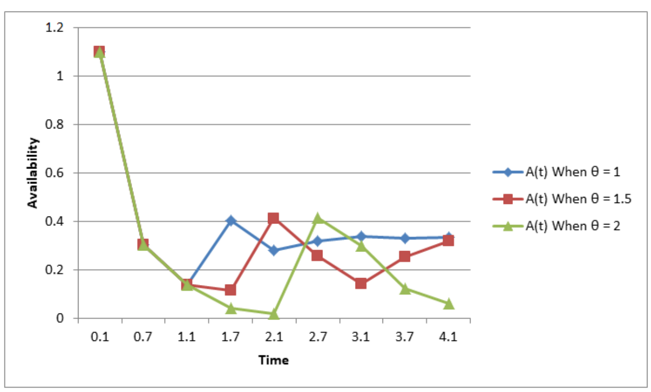

Fig. 4: Sensitivity analysis for PDS

MAINTENANCE COST ANALYSIS

Generally, conducting inspections more frequently raises inspection costs, while a prolonged interval between inspections results in higher penalties when the system fails. Although the system experiences similar lifetime and service time distributions, varying inspection periods can cause fluctuations in availability at different stages. Therefore, it is essential to select an appropriate inspection interval that enhances system availability while simultaneously lowering maintenance costs. This is clearly demonstrated in Figure 5, which shows that as the value of θ increases, the average long-run cost rate initially decreases in the time range [1, 1.5] before subsequently rising. Consequently, the analysis indicates that the minimum average long-run cost rate of 5.3782occurs at θ* = 0.5, which is identified as the optimal inspection interval.

θ	$W(\theta)$	
	***M/ E*₂/1 queue**	***M/M/*1 queue**
0.2	8.2424	9.3570
0.4	5.4816	5.6322
0.5	5.3782	5.4836
0.9	5.8660	6.0520
1.1	6.2008	6.4295
1.6	6.9369	7.2183
3.1	8.1853	8.4420
4.1	8.5893	8.8100
5.1	8.8474	9.0386
6.1	9.0259	9.1936
7.1	9.1565	9.3056
8.1	9.2562	9.3903

Table 5: Maintenance Cost vs. Time

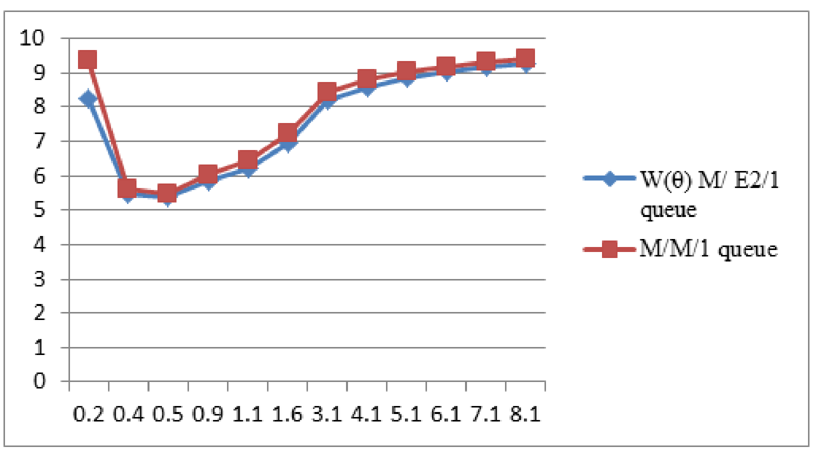

Fig. 5: Average long run cost rate for PDS

COMPARISON BETWEEN M/E$_2$/1 AND M/M/1 QUEUE MODELS IN A PDS SYSTEM

1. Analysis shows that the M/M/1 model demonstrates significantly better performance than the M/E$_2$/1 model.

2. As both $M/E_2/1$ and M/M/1 queue models are assumed to have identical failure rates, their reliability curves align, as illustrated in Figure 1.
3. For the $M/E_2/1$ model, the average waiting time to repair each failed component—whether from active or passive failures in the PDS—is considerably longer than in the M/M/1 model. This is supported by the fact that the long-run availability in the M/M/1 model is notably higher than in the $M/E_2/1$ model for the PDS system, as depicted in Figure 3.

Queuing Model	$A(\theta)$		
	$\theta = 1.1$	$\theta = 1.6$	$\theta = 2.1$
$M/E_2/1$	0.3097	0.2455	0.2004
$M/M/1$	0.36860	0.2812	0.22349

Table 6 : Steady state system availability with respect to $M/E_2/1$ and $M/M/1$ queuing models

Table 6 shows that the M/M/1 queuing model achieved a notable improvement in availability over the $M/E_2/1$ model, as illustrated in Figure 3. From Fig.5, it is evident that the minimum long-run cost rate for the M/M/1 model is slightly higher than that of the $M/E_2/1$ model, which may be an acceptable trade-off for a more reliable PDS system. Nevertheless, the optimal inspection interval is the same for both queuing models, at 0.4.

Queuing Model	$W(\theta)$
$M/E_2/1$ queue	5.3781
$M/M/1$ queue	5.4835

Table7: Minimum long run average cost for $M/E_2/1$ queue and M/M/1 queuing model

CONCLUSION

The parameters used to assess reliability and its characteristics are outlined in Table 8. Fig.1 illustrates that system reliability declines as time, t, increases. In Fig. 2, the relationship between instantaneous availability and time for a PDS following the $M/E_2/1$ queuing model is depicted. Since active and passive failure events occurring initially are only addressed during the first inspection, they are repaired afterward. As a result, A(t) initially rises in the interval [1.1,1.6] as t increases, followed by a decrease between [1.6, 2.1], and then gradually stabilizes at a steady value of 0.3097.

We also aim to evaluate the sensitivity of A(t) concerning the parameter θ. Fig. 4 shows that the station and feeder architecture at the power grid's utilization end in the Power Distribution System is more sensitive when the inspection interval θ is set to 2.1. This sensitivity is demonstrated across inspection intervals of θ = 1.1, 1.6, and 2.1. Notably, as θ increases, the system's availability declines, raising the likelihood of a failure event; thus, a higher θ results in lower availability. The data in Table 6 and Fig. 4 support this finding.

Additionally, it can be noted that when θ equals 1, the long-run availability is 0.3097, aligning with the point availability.

Parameters	Values
C_I	1
$C_{ME21} = C_{ME22}$	5
θ	1
C_d	10

Table 8: Parameter values for reliability, availability of PDS

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