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# ON MAXIMUM DEGREE ENERGY OF GRAPHS

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**ABSTRACT**
*The sum of the absolute values of all Maximum degree eigenvalues $E_{md}(G)$ of a graph $G$ is called as the Maximum degree energy of $G$. A few upper and lower constraints on the Maximum degree energy are obtained in this study.*

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## 1. INTRODUCTION

$V(G) = \{v_1, v_2, \dots, v_n\}$ is the set of all the vertices in a simple graph G. The adjacency matrix $A(G)$ for graph $G$ is a square matrix of order n. The $(i;\ j) - \text{entry}$ is equal to 1 if the vertices $v_k$ and $v_j$ are next to each other, and to $0$ otherwise. If you assume that the eigenvalues of $A(G)$ don't go up in order, the eigenvalues of the graph $G$ are $\boldsymbol{\delta_1, \delta_2, \dots, \delta_n}$. In 1978, I. Gutman was the first person to say that $G's$ energy was equal to the sum of its eigenvalues, which are

$$\boldsymbol{E(G) = \sum_{k=1}^{n} |\delta_k|}$$

Since, I.Gutman[5] first talked about the graph energy $E(G)$ of a simple graph $G$, a lot of studies have been written on the subject. Numerous matrix types, including Incidence [10], Distance [9], Lapalcian [6], and others are established and researched for graphs, with inspiration drawn from the adjacency matrix (AM) of a graph.

Let $G$, be a simple graph with vertex set $V(G) = \{v_k,\ 1 \le k \le n\}$ and edge set $E(G) = \{v_e,\ 1 \le k \le n\}$. The following kind of matrix, known as the Maximum Degree Matrix (MDM) of $G$, was introduced by C.Adiga [1]. The $n \times n$ matrix, $M_D(G) = [m_{kj}]$ is the MDM of $G$, whose, $kj - \text{th}$ element is given by

$$m_{kj} = \begin{cases} \max(d_k, d_j), & \text{when } v_k \text{ and } v_j \text{ are adjacent} \\ 0 & \text{othewise,} \end{cases}$$

where, $d_k$ is the degree of the vertex $v_k$ in $G$. The expression $\chi(G : \eta) = det\big(\eta I - M_D(G)\big)$,

defines the characteristic polynomial of $M_D(G)$. The maximum degree eigenvalues(ME), of the graph G are nothing but the eigenvalues of $M_D(G)$. The matrix $M_D(G)$ is real as well as symmetric and its trace is zero. The real numbers that make up the eigenvalues of $M_D(G)$ are organized as follows: $\eta_1 \ge \eta_2 \ge \dots \ge \eta_n$. The formula

$$E_{md}(G) = \sum_{k=1}^{n} |\eta_k|,$$

Defines $G's$ Maximum Degree energy (MdE).

We see that $\eta_1 + \eta_2 + \ldots + \eta_n = 0$ and $\sum_{k=1}^{n} {\eta_k}^2 = 2|\mathfrak{C}_2|$, in which

$\mathfrak{C}_2 = -\sum_{k=1}^{n}(t_k + s_k){d_k}^2$, where $t_k$ the number of vertices in the neighbourhood of $v_k$, whose degrees are less than $d_k$ and $s_k$ is the number of vertices $v_k$, j > i, in the neighbourhood of $v_k$, whose degrees are equal to $d_k$. $\eta_1$ is the spectral radius, the highest MdE of a graph G. We derive some upper and lower bounds for the MdE, $E_{md}(G)$, in this study.

## 2. UPPER BOUNDS FOR MAXIMUM DEGREE ENERGY

Upper bounds for MdE's of graph $G$ are achieved in this section.

**Theorem 2.1. If G is a non-empty graph having order n and size m, then**

$$E_{md}(G) \leq \sqrt{2|\mathfrak{C}_2|^2 + \frac{n^2}{2}}.$$

**Proof:** Let $e_k$, $f_k$, $g_k$ $and$ $h_k$ are real sequence and $p_k$ , $q_k$ be non-negative numbers for $1 \leq k \leq n$. Then from [4], we have the inequality,

$$2\sum_{k=1}^{n} p_k e_k g_k \sum_{k=1}^{n} q_k f_k h_k \leq \left(\sum_{k=1}^{n} p_k e_k^2\right)\left(\sum_{k=1}^{n} q_k f_k^2\right) + \left(\sum_{k=1}^{n} p_k g_k^2\right)\left(\sum_{k=1}^{n} q_k h_k^2\right) \qquad (2.1)$$

For $p_k = q_k = e_k = f_k = 1$ and $g_k = h_k = |\eta_k|$, $1 \leq k \leq n$, the inequality (2.1) will be

$$2\left(\sum_{k=1}^{n}|\eta_k|\right)\left(\sum_{k=1}^{n}|\eta_k|\right) \leq \left(\sum_{k=1}^{n} 1\right)\left(\sum_{k=1}^{n} 1\right) + \left(\sum_{k=1}^{n}|\eta_k|^2\right)\left(\sum_{k=1}^{n}|\eta_k|^2\right).$$

Applying the known fact, $\sum_{k=1}^{n}|\eta_k|^2 = \sum_{k=1}^{n}{\eta_k}^2 = 2|\mathfrak{C}_2|$ in the above equation, we get,

$$2\, E_{md}(G) \cdot E_{md}(G) \leq n.n + 2|\mathfrak{C}_2| \cdot 2|\mathfrak{C}_2|$$

which gives,

$$2.\, E_{md}(G)^2 \leq n^2 + 4|\mathfrak{C}_2|^2.$$

Hence,

$$E_{md}(G) \leq \sqrt{2|\mathfrak{C}_2|^2 + \frac{n^2}{2}} \ .$$

**Theorem 2.2. If G is a non-empty graph having order $n$ and size $m$, then**

$$E_{md}(G) \leq \frac{1}{2}n + |\mathfrak{C}_2|.$$

**Proof:** Let $e_k,\ f_k,\ g_k\ and\ h_k$ be real sequence and $p_k\ and\ q_k$ be non-negative numbers for $1 \leq k \leq n$. Then from [4], we have the inequality,

$$2\sum_{k=1}^{n} p_k e_k g_k \quad \sum_{k=1}^{n} q_k f_k h_k \leq \left(\sum_{k=1}^{n} p_k e_k^2\right)\left(\sum_{k=1}^{n} q_k f_k^2\right) + \left(\sum_{k=1}^{n} p_k g_k^2\right)\left(\sum_{k=1}^{n} q_k h_k^2\right) \qquad (2.2)$$

For $p_k = q_k = e_k = f_k = h_k = 1$ and $g_k = |\eta_k|, 1 \leq k \leq n$, the inequality (2.2) gives

$$2\left(\sum_{k=1}^{n} |\eta_k|\right)\left(\sum_{k=1}^{n} 1\right) \leq \left(\sum_{k=1}^{n} 1\right)\left(\sum_{k=1}^{n} 1\right) + \left(\sum_{k=1}^{n} |\eta_k|^2\right)\left(\sum_{k=1}^{n} 1\right),$$

That is,

$$2n \cdot \sum_{k=1}^{n} |\eta_k| \leq n^2 + n \cdot \sum_{k=1}^{n} |\eta_k|^2 ,$$

Which gives,

$$2\, E_{md}(G) \leq n + 2|\mathfrak{C}_2|$$

and consequently,

$$E_{md}(G) \leq \frac{1}{2} n + |\mathfrak{C}_2|.$$

## 3. LOWER BOUNDS FOR MAXIMUM DEGREE ENERGY

Lower bound for MdE's of graph G are achieved in this section.

**Theorem 3.1. If G is a non-empty bipartite graph of order $n \geq 2$ and size $m$ with $\eta_1$ as spectral radius, then**

$$E_{md}(G) \geq 2\frac{|\mathfrak{C}_2|}{\eta_1}$$

**Proof**. Let $e_k\ and\ f_k$ be decreasing non-negative real sequences with $e_k,\ f_k \neq 0\ and\ j_k$ be non-negative for $1 \leq k \leq n$. Then we have the following inequality from [4]:

$$max\left\{f_1 \sum_{k=1}^{n} {e_k}^2 j_k , e_1 \sum_{k=1}^{n} {f_k}^2 j_k\right\} \sum_{k=1}^{n} e_k f_k j_k \geq \sum_{k=1}^{n} {e_k}^2 j_k \sum_{k=1}^{n} {f_k}^2 j_k \qquad 3.1$$

for $e_k = f_k = |\eta_k|\ and\ j_k = 1 , 1 \leq k \leq n,$ the inequality (3.1) becomes,

$$\eta_1 \sum_{k=1}^{n} |\eta_k| \geq \sum_{k=1}^{n} |\eta_k|^2.$$

That is,

$$\eta_1 E_{md}(G) \geq \sum_{k=1}^{n} |\eta_k|^2$$

Hence,

$$E_{md}(G) \geq 2\frac{|\mathfrak{C}_2|}{\eta_1}.$$

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