ANALYSIS OF PERISHABLE INVENTORY MANAGEMENT & CONTROL FUZZY MODEL WITH TRIANGULAR FUZZY NUMBER

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ABSTRACT

This paper introduces a model for managing perishable goods with a steady linear demand. In this model, the cost of holding these items rises as time progresses. Specifically, the items under consideration are perishable and deteriorate rate, denoted as θ. In most previous studies, the holding cost was assumed to remain constant, which doesn't accurately reflect real-world scenarios. In practical situations, costs like insurance, recordkeeping, and cold storage expenses tend to increase over time. This paper takes into account this time-dependent linear holding cost, where the cost of holding the items rises as time passes. The paper derives an approximate optimal solution for this scenario and provides numerical examples to illustrate the results.

KEY WORDS: Perishable inventory, Fuzzy EOQ model, Triangular fuzzy number.

INTRODUCTION

This paper aims to develop an inventory management strategy for perishable items such as food and fashion goods, which have a constant demand and a time-dependent holding cost. Previously, Muhlemann and Spanopoulos revised the traditional Economic Order Quantity (EOQ) model by incorporating time-dependent holding costs and permissible payment delays. In the classical EOQ model, only two costs, namely ordering and carrying costs, were considered, and the optimal solution was reached when the ordering cost equated the carrying cost. In this study, a more comprehensive model is presented. It takes into account a third cost, namely the deteriorating cost, and considers carrying costs that are not constant; they increase over time. Ghare and Schrader were among the first to introduce the concept of deterioration in inventory modelling. They developed a model that accounted for a constant rate of deterioration [1]. Subsequently, [2] extended this concept by formulating a model that considered variable rates of deterioration, employing a two-parameter Weibull distribution. This work was further expanded upon by [3]. Nahmais [4] extensively reviewed the existing literature pertaining to the establishment of suitable ordering strategies for two specific inventory types: fixed-life perishable inventory and inventory subject to continuous exponential decay. Cheng focused on an economic order quantity (EOQ) model that factored in demand-dependent unit production costs, solving it using geometric programming to derive the optimal cost [5].

Balancing stock-dependent demand and deteriorating items is a critical issue. Consequently, an inventory model was developed for deteriorating items with stock-dependent demand [6]. The research efforts described above paved the way for the development and successful resolution of inventory models designed to handle a range of demand scenarios. Among these models are two specifically tailored for managing deteriorating items with linearly time-dependent demand. In these models, the replenishment rate varies in response to fluctuations in demand and on-hand inventory levels, providing a practical approach to addressing such dynamic demand patterns [7]. In the same year, a study concentrated on extended EOQ-type deterministic inventory models for perishable products, where the demand rate depended on the on-hand inventory. This study also considered holding costs as nonlinear functions dependent on the length of time items were held in stock [8]. Building on the understanding of deteriorating items, a production model for such items was developed, offering optimal production stopping and restarting times within an EOQ model with deteriorating items [9].

Significant progress in the field of inventory management, particularly concerning perishable items, emerged from the work of [5]. This research unveiled the direct relationship between the inventory carrying cost and the cost of deteriorated items, resulting in the creation of an approximate solution. Building upon these findings, a

comprehensive inventory model was introduced for deteriorating items. This model accommodated both stockdependent demand and time-varying demand, with a specific focus on a finite planning horizon [10]. Additionally, an Economic Production Quantity (EPQ) model was developed to address perishable items characterized by stock-dependent demand. This model seamlessly integrated the concept of deteriorating items into the production facet of inventory management, further enhancing our understanding of how to effectively manage such inventory items [11].In addition to addressing crucial elements of inventory management like inflation and the time value of money, Kuo-Lung Hou [12] devised an inventory model tailored for items undergoing deterioration. This model took into account a consumption rate dependent on stock levels, as well as considerations for shortages, inflation impact, and time discounting. Moreover, by introducing the concept of permissible payment delays, a separate model was developed to guide retailers in making optimal replenishment decisions when dealing with deteriorating items. This model also considered two levels of trade credit policy, offering insights into supply chain management within the context of the Economic Production Quantity (EPQ) [13]. [14] discusses the intricate challenge known as the inventory routing problem, where the management of inventory and vehicle routing decisions becomes interrelated rather than being treated as separate problems. By integrating these decisions, it becomes possible to discover solutions that surpass the mere combination of optimal solutions obtained from smaller subproblems. [15] closely examines an integrated fuzzy model designed to reduce delays in the location-routing of perishable multi-product materials, all while taking into account environmental constraints. This approach aims to optimize the process of storing and transporting goods while considering environmental impact. [16] delves into the application of neural network control for the management of perishable inventory. This particularly addresses products with fixed shelf lives that experience fluctuating and uncertain demand over time. The use of neural networks helps in making dynamic decisions to manage such inventory efficiently. [17] presents innovative models that assist managers in making environmentally responsible decisions when it comes to inventory operations. These models offer a framework for optimizing inventory processes while considering factors like changing demand patterns and environmental costs. They support sustainable and responsible decision-making. [18] In the specific context of this research, a bi-level programming method is introduced to address the location-inventory-routing challenge within a two-tier supply chain. This supply chain consists of central warehouses in the first tier and retailers in the second tier. The key focus is on managing perishable products, which are vulnerable to unpredictable fluctuations in demand. The overarching objective is to minimize the overall operational costs at both levels while adhering to capacity limitations. This approach helps in efficiently managing perishable goods throughout the supply chain.

This research provides a framework that retailers can use to determine the ideal cycle period and economic order quantity for items with holding costs that escalate over time. It emphasizes that prolonged storage of such items can significantly increase costs, as demonstrated in the numerical examples. The study assumes that the holding of items increases linearly with time and assesses the sensitivity of decision variables, ultimately deriving optimal solutions as demonstrated in the numerical examples.

PRIMILINARIES

A fuzzy set X on the given universal set is a set of order pairs $\tilde{A} = \{ (x, \mu_A(x)) : x \in X \}$, $\mu_A: X \rightarrow [0,1]$, is **called** membership function.

The α – cut of \tilde{A} is defined by

 $A_{\alpha} = \{x: \mu_A(x) = \alpha, \alpha \ge 0\}$

If R is a real line, then a fuzzy number is a fuzzy set \tilde{A} with the membership function $\mu_{\tilde{A}}$: X \rightarrow [0,1].

Definition 1. A Triangular fuzzy number $\tilde{A} = (a, b, c)$ is represented with membership function $\mu_{\tilde{A}}$

Definition 2. Let $\tilde{A} = (a, b, c)$ be a fuzzy set defined on R. The Signed Distance Method of \tilde{A} is defined as

$$
P(\widetilde{A_1 \cdot 0}) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha
$$

 $a+2b+c$ 4

NOTATIONS

The notations are presented below are the result of thorough research conducted in this particular field. These detailed conditions are provided to precisely define the model.

- 1. Demand, $D(t) = a + b t$.
- 2. The deterioration rate, $\theta(t) = \theta_1 + \theta_2 t$.
- 3. C: Purchasing cost per unit time.
- 4. OC: Ordering Cost.
- 5. DC: Deteriorating Cost.
- 6. HC: Holding Cost.
- 7. C_1 : Holding cost per unit time.
- 8. T: Total time per cycle.
- 9. \tilde{D} : Fuzzy demand.
- 10. \tilde{h} : Fuzzy holding cost.
- 11. $\tilde{\theta}$: Fuzzy deterioration rate.
- 12. \tilde{C} : Fuzzy Purchase cost.
- 13. \widetilde{TC} : Fuzzy total cost per unit time.
- 14. $\widetilde{TC_{SD}}$: Signed distance Defuzzification method

MATHEMATICAL (CRISP) MODEL

Let $O(t)$ represent the on-hand inventory at any moment in time, where time is constrained within the range $0 \le t$ \leq T. The reduction in unit inventory is influenced by two primary factors: demand and deterioration. The differential equation serves as the governing equation that describes the instantaneous state of $Q(t)$ at any given point in time. It accounts for how the inventory changes over time due to these two contributing factors.

$$
\frac{dI_1(t)}{dt} = -D(t) = -(a+bt)
$$

Boundary condition, I₁(0) = M, I₁(t) = M - a T -
$$
\frac{bT^2}{2}
$$
 (1)

$$
\frac{dI_z(t)}{dt} + \theta(t)I_2(t) = -D(t), \mu \leq t \leq T,
$$

Ordering Cost (OC): A

Deterioration Cost (DC)- Cost due to Deterioration

$$
[M - \int_{\mu}^{T} (a + bt) dt] C
$$

\n
$$
DC = C \left\{ M - \left[a(T - \mu) + \frac{b}{2} (T^{2} - \mu^{2}) \right] \right\}
$$

\nHolding Cost (HC):
\n
$$
\int_{0}^{\mu} (h_{1} + h_{2}t) I_{1}(t) dt + \int_{\mu}^{T} (h_{1} + h_{2}t) I_{2}(t) dt
$$

\nTotal Cost (TC) = (OC+DC+HC)/T
\n
$$
\frac{A}{T} \left\{ M - \left[a(T - \mu) + \frac{b}{2} (T^{2} - \mu^{2}) \right] (C - \beta C) \right\} \frac{1}{T} +
$$

$$
\begin{pmatrix}\nh_{1}(T-\mu)+\frac{h_{2}}{2}(T^{2}-\mu^{2})+\theta_{1}(T-\mu)(h_{1}+h_{2}) \\
-\left(h_{1}\theta_{1}\left(\frac{T^{2}-\mu^{2}}{2}\right)+h_{2}\theta_{1}\left(\frac{T^{2}-\mu^{2}}{3}\right)\right) \\
+\frac{\theta_{2}\mu^{2}}{2}\left(h_{1}(T-\mu)+\frac{h_{2}}{2}(T^{2}-\mu^{2})\right) \\
+\alpha\mu\left(h_{1}(T-\mu)+\frac{h_{2}}{2}(T^{2}-\mu^{2})\right) \\
+\alpha\mu\left(h_{1}(T-\mu)+\frac{h_{2}}{2}(T^{2}-\mu^{2})\right) \\
-h_{1}\left(\frac{\pi^{2}-\mu^{2}}{3}\right)-h_{2}\left(\frac{\pi^{4}-\mu^{4}}{4}\right)) \\
+\frac{1}{3}\left(\frac{\alpha\theta_{2}+2b\theta_{1}}{2}\right)\left\{\mu^{3}\left(h_{1}(T-\mu)\right)\right\} \\
+\frac{h_{2}}{2}(T^{2}-\mu^{2})-h_{1}\left(\frac{\pi^{4}-\mu^{4}}{4}\right)-h_{2}\left(\frac{\pi^{4}-\mu^{4}}{5}\right)\right) \\
+\frac{h_{2}}{2}\left(\mu^{2}\left(T^{2}-\mu^{2}\right)-\left(\frac{\pi^{4}-\mu^{4}}{5}\right)\right) \\
-\theta_{1}(\alpha\mu\left(h_{1}\left(\frac{\pi^{2}-\mu^{2}}{2}\right)+h_{2}\left(\frac{\pi^{2}-\mu^{2}}{3}\right)\right) \\
-\theta_{1}(\alpha\mu\left(h_{1}\left(\frac{\pi^{2}-\mu^{2}}{2}\right)+h_{2}\left(\frac{\pi^{2}-\mu^{2}}{3}\right)\right)) \\
+\frac{h_{3}(\alpha_{2}+\alpha_{2})}{2}\left(\mu^{2}\left(h_{1}\left(\frac{\pi^{2}-\mu^{2}}{2}\right)+h_{2}\left(\frac{\pi^{2}-\mu^{2}}{3}\right)\right)\right) \\
+\frac{1}{3}\left(\frac{\alpha\theta_{2}+\alpha_{2}\theta_{1}}{2}\right)\left\{\mu^{3}\left(h_{1}\left(\frac{\pi^{2}-\mu^{2}}{2}\right)+h_{2}\left(\frac{\pi^{2}-\mu^{2}}{3}\right)\right)\right\} \\
-\theta_{2}\left\{\frac{\alpha\mu}{2}\left(h_{1}\left(\frac{\pi^{2}-\mu^{2}}{2}\right
$$

FUZZY MODEL

A fuzzy number is defined by a membership function that assigns varying degrees of membership to different values within a specified range. Various types of fuzzy numbers exist, and in this paper, our focus is on triangular fuzzy numbers. Each type exhibits a unique shape of membership function, capturing the uncertainty associated with the value it represents. Fuzzy numbers provide a means to represent uncertainty in numerical data more realistically, enabling a nuanced approach when dealing with uncertain information.

Fuzzy total cost is defuzzied by the following defuzzification method

Signed Distance Method = $\frac{1}{4}$ ($\widetilde{TC}1 + 2\widetilde{TC}2 + \widetilde{TC}3$)

Case 1: Crisp Model

Example 1

For the defined model, the values of the following parameters are to be taken in appropriate units will be A= 125; a=1; b=1; C=10; C₁ =5; β =1.2; h₁ = 2.1; h₂= 2.5; θ ₁ = 0.5; θ ₂ = 0.5; μ = 1; T = 3.7618;

TC= (OC+DC+HC)/T

OC=33.2288; DC=2,10,890; HC= 98,448;

The average total cost per cycle TC=1,86,950;

Case 2: Fuzzy Model

Example 2

For the defined model, the values of the following fuzzy parameters are to be taken in appropriate units will be α $=1, b=1, \mu = 1, \beta = 1.2,$

 $\tilde{A} = (120, 125, 130,);$ $\widetilde{C1} = (2.5,5,7.5);$ $\overline{h1}$ = (1.9, 2.1, 2.3); $\overline{h2} = (2.3, 2.5, 2.7);$ $\tilde{C} = (5,10,15)$; $\widetilde{\theta_1}$ = (0.45,0.50,0.55); $\widetilde{\theta_{2}} = (0.45, 0.50, 0.55);$ \widetilde{T}_1 = 4.0105, $\widetilde{OC1}$ = 29.9215, \widetilde{DC} 1 = 1,42,620, $\widetilde{HC1}$ = 1,19,920, \widetilde{TC} 1 = 1,10,340 $\widetilde{T}_2 = 3.7618, \widetilde{OC2} = 33.288, \widetilde{DC} = 2,10,890, \widetilde{HC2} = 98,448, \widetilde{TC2} = 1,86,950$ $\widetilde{T}_3 = 3.5571, \widetilde{OC3} = 36.5466, \widetilde{DC}3 = 2,45,610, \widetilde{HC3} = 84,770, \widetilde{TC3} = 2,47,820$ Defuzzification method: **Signed Distance Method =** $\frac{1}{7}(\widetilde{TC1} + 2\widetilde{TC2} + \widetilde{TC3})$

For Signed Distance Method, by using Mat lab software we obtain $\tilde{T} = 3.7728$,

 $\widetilde{OC} = 33.2314$; $\widetilde{DC} = 2.02.500$; $\widetilde{HC} = 1.00.400$;

The average fuzzy total cost per cycle $\widetilde{TC_{SD}}$ = 1,83,015.

NUMERICAL OBSERVATIONS

On conducting a numerical comparison between the crisp model and the fuzzy model, we observed that incorporating uncertainties in a suitable manner led to a reduction in time. To determine the optimal total cycle length T that minimizes the overall cost, we utilized the Defuzzification Signed Distance Methods. Upon comparing the defuzzification methods, it was evident that the Signed Distance Method yielded a more favourable optimum time. & total cost, $\tilde{T} = 3.7728$, $\tilde{T}\tilde{C_{SD}} = 1,83,015$ respectively.

SENSITIVE ANALYSIS

Table – CRISP MODEL

OBSERVATION

Considering the sensitivity of each individual parameter as shown in Table, several noteworthy trends emerge:

- 1. **Parameter A:** When A increases, the setup cost rises, which results in an increase of total cost. However, the costs associated with deterioration, salvage value, and holding remain the same. This means that the overall cost increases due to the higher setup cost.
- 2. **Parameter C:** When C increases, the ordering cost and holding cost remain unchanged, but the deterioration and salvage value costs fluctuate. As a result, the total cost increases due to the impact of changes in C.
- 3. **Parameter C1:** Considering C1 along with the holding cost factors h1 and h2, the costs of deterioration and salvage value remain constant, while the cost of holding the item increases, leading to an increase in the total cost.
- 4. **Parameters** θ_1 **and** θ_2 **:** For θ_1 , the cost of ordering remains the same, while the costs of other expenses decrease. Conversely, for θ_2 , it is observed that the cost of ordering remains constant, but the costs of other expenses increase. These changes in θ_1 , θ_2 , have implications for the various components of the total cost.

In summary, the sensitivity of each parameter leads to specific changes in the different cost components, which collectively affect the total cost in the context of the analysis presented in the Table.

RESULTS & DISCUSSION

Figure 1: Analysis of Crisp and Fuzzy model

According to the data presented in Figure 1, our analysis involved a quantitative evaluation of the crisp model and the fuzzy model. During this analysis, we found that effectively incorporating uncertainties resulted in a noticeable reduction in the time required for the process. In our quest to determine the most efficient total cycle length (T) that minimizes the overall cost, we employed the Defuzzification Signed Distance Methods as a key tool. Upon comparing various defuzzification methods, our findings clearly demonstrated that the Signed Distance Method outperformed the alternatives by delivering a more favourable optimal time and overall cost reduction. This suggests that the careful integration of uncertainties through the fuzzy model, followed by the application of the Signed Distance Method, has the potential to significantly enhance both time efficiency and cost-effectiveness in the analysed process.

All these calculations are executed using MATLAB software, it is a powerful and versatile programming and numerical computing environment that finds applications in a wide range of fields for numerical calculations. Here are some common uses of MATLAB in numerical calculations in Linear Algebra, Numerical Analysis and

Optimization. In Linear Algebra it is widely used for solving systems of linear equations, matrix manipulation, and eigenvalue problems. In Numerical Analysis it is equipped with a variety of numerical methods, making it useful for solving problems related to interpolation, integration, differentiation, and solving differential equations. MATLAB can also be used for optimizing functions, making it suitable for problems related to resource allocation, and etc.,

CONCLUSION

This paper presents a novel model for effectively managing perishable items, assisting retailers in determining the optimal economic order quantity and cycle time for products with fluctuating holding costs. This model provides a user-friendly framework that enables retailers to make well-informed decisions about their ideal inventory levels without the need for intricate mathematical models. The holding cost, a pivotal component of the total inventory cost, plays a substantial role in influencing the optimal order quantity. Prior studies often assume that the holding cost remains constant, regardless of the cycle time, which can be an unrealistic assumption in practical scenarios. In contrast, our model permits the holding cost to increase linearly with time. Consequently, unless the ordering cost is exceptionally high, the optimal economic order quantity tends to be smaller, leading to a higher frequency of inventory replenishment cycles, as exemplified in numerical illustrations.

This study is grounded in the assumption of negligible lead time, but its adaptability extends to scenarios with constant lead times, allowing retailers to time their orders effectively. The proposed model offers room for expansion to accommodate additional factors, including addressing shortages, considering permissible payment delays, accounting for inflation, and incorporating time value for money. Furthermore, it can be extended to handle items with demand that depends on existing stock levels, making it a versatile tool for a wide array of inventory management challenges.

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