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AIRCRAFT CONTROL BY USING LQG & LQR WITH OPTIMUM ESTIMATION-KALMAN FILTER DESIGN

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ABSTRACT

A major part of this paper defines LQG and LQR robust controller for the lateral and longitudinal flight dynamics of an aircraft control system. The controller is used to achieve robust stability and good dynamic performance against the variation of aircraft parameters. The LQG and LQR robust control scheme is implemented through the simulation. The proposed robust controller for aircraft stability is designed using MATLAB software/ Simulink. Simulation results confirm the performance of the proposed controller for aircraft control system. Now the, the Kalman filter has been the subject of wide-range research application, especially in the area of autonomous or assisted navigation. i.e, to determine the velocity of an aircraft or sideslip angle, one could use a radar Doppler, the velocity indications of an inertial navigation system, or the relative wind info in the air data system. Rather than ignore any of these outputs, a Kalman filter could be built to combine all of this data and knowledge of the various systems dynamics to generate an overall best estimate of pitch, roll and sideslip angle.

Keywords: Aircraft motion; LQG controller; LQR controller stability; State estimator- Kalman filter.

1. INTRODUCTION

A feedback loop is powerful and common to use when design a control system. This feedback loop control system is commonly used in manufacturing mining, and military hardware purpose. The Flight dynamics characterize the motion of a flight vehicle in the atmosphere. it can be very useful for a systems dynamic in which the system studies are a flight vehicle. The actual response a of the vehicle to aerodynamic, propulsive, and gravitational forces, and to control inputs from the pilot determine the attitude of the vehicle and the destination flight path. [1]

The field of flight dynamics can be divided into further

- **Performance**: In which the short time scales of response are ignored, and the forces are assumed to be in quasi-static equilibrium. Here the issues are maximum and minimum flight speeds, rate of climb, maximum range, and time aloft (endurance).
- **Stability and Control**: in which the short- and intermediate-time response of the attitude and velocity of the vehicle is considered. Stability considers the response ofthe vehicle to perturbations in flight conditions from some dynamic equilibrium, while control considers the response of the vehicle to control inputs.
- **Navigation and Guidance**: In which the control inputs required to achieve a particular trajectory are considered. These control systems are being required to deliver more accurate and better overall performance in the face of difficult and changing operating conditions. In order to design

control systems to meet the demands of improved performance and robustness when controlling complicated processes, control engineers will require new design tools and better underlying theory. A standard method of improving the performance of a control system is to add extra sensors and actuators. This necessarily leads to a multi-input multioutput control system. Thus, it is a requirement for any modern feedback control system design methodology that it be able to handle the case of multiple actuators and sensors. Linear Quadratic Gaussian optimal control theory (LQG) is one of the major achievements of the modern control area. This controller design methodology enables a controller to be synthesized which is optimal with respect to a specified quadratic performance index. Furthermore, this theory takes into account the presence of Gaussian white noise disturbances acting on the system. Indeed, in many practical control problems, it is straightforward to translate the required performance objective into a problem of minimizing a quadratic cost functional. Also, in many practical control problems, the system is subject to disturbances and measurement noise which are most naturally modeled as stochastic white noise processes.

The LQG controller design methodology based on the Kalman filter who in 1960 published his famous paper describing a recursive solution to the discrete-data linear filtering problem. A more complete introductory discussion can be found in [2] which also contains some interesting historical narrative. More extensive references include [3], [4] and [5]. It has also been used for motion prediction [6] and it is used for multi-sensor. In practice, although it is possible to obtain process models either from first principles or from experimental measurements, these models will always be subject to errors. Thus, the control system needs to be designed to be robust against these modeling errors.

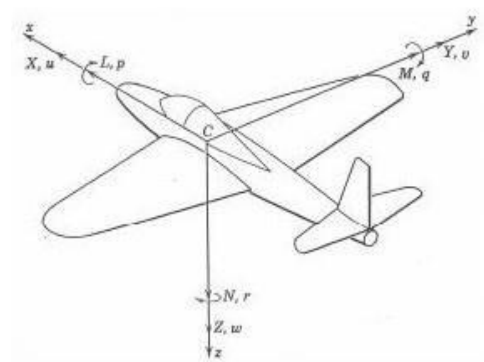
1.1 Aircraft control and movement

The following three basic sites for an aircraft to change its direction relative to the fleeting air.

Pitch (when nose movements up or down),

Roll (when aircraft rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) .

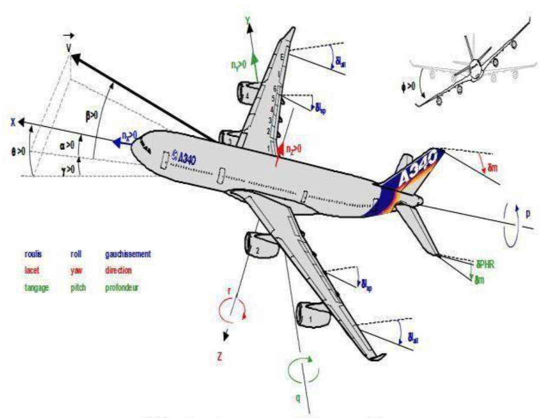
Yaw (the movement of the nose to left or right.) Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero. [7]



Aircraft control and rotation about axis.

Standard notation for aerodynamic forces and moments, and linear and rotational velocities in body- axis system [2]

1. AIRCRAFT LONGITUDINAL MOTION.



Aircraft control and rotation about axis.

- **General Equation of Motion**

$$m\frac{du}{dt} = \sum Fe \quad , \quad \frac{dc}{dt} = \sum Me$$

- **Equation of longitudinal notion**

$$Q = p = r = \Phi$$

Longitudinal equation can be written as:

$$u = \frac{Xu}{m}u + \frac{Xw}{m}w - \theta_0 + \triangle.$$

$$w = \frac{Zu}{Zw}u + \frac{Zw}{Zu}v + Mu + \frac{mgSin\theta}{mz^{u}+z^{c}}$$

$$q = -\frac{[M_u + Z_{U}{}_T]}{l_{yy}} + \frac{[M^{\cdot} U_u + z_T]}{l_{yy}}w + \mu_q$$

$$\theta^{\cdot} = q$$

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}-0.3149 & 235.8928 & 0\\ -0.0034 & -0.4282 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}-5.5079\\ 0.0021\\ 0\end{bmatrix}\delta_e$$

$$y = [0\ 0\ 1]\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + [0]$$

$$\triangle X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\delta_p$$

$$\triangle Z^c = \frac{Z_{\delta_e}}{m - Z_\omega}\delta_e + \frac{Z_{\delta_p}}{m - Z_w}\delta_p$$

$$\triangle M^c = \frac{M_{\delta_e} + Z_{\delta_e}}{l_{yy}}\delta_e + \frac{M_{\delta_p} + Z_{\delta_p}}{l_{yy}}\delta_p$$

Write it into a state space form.

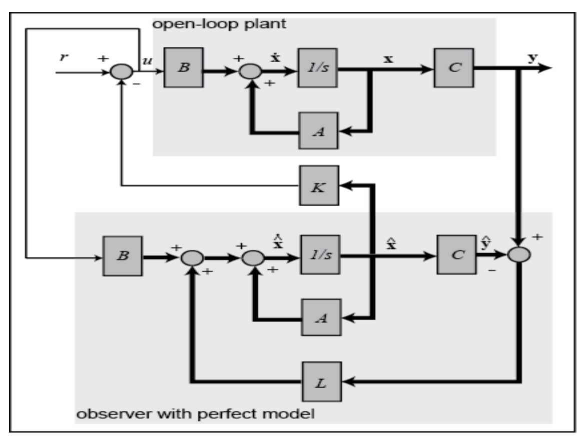
Since $u = 0$ in this mode, then $u' = 0$ and can be eliminate the X Force element.

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{Z_w}{m - Z_{\dot{\omega}}} & \frac{Z_q + mU_o}{m - Z_{\dot{\omega}}} & \frac{-mgsin\theta_o}{m - Z_{\dot{\omega}}}\\ \frac{\left[M_w + Z_w\frac{M_{\dot{w}}}{m}\right]}{l_{yy}} & \frac{[M_q + (mU_o)\acute{\Gamma}]}{l_{yy}} & \frac{-mgsin\theta_o}{l_{yy}}\theta\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\triangle Z^c\\ \triangle M^c\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{Z_w}{m} & U_o & gsin\theta_o\\ \frac{\left[M_w + Z_w\frac{M_{\dot{w}}}{m}\right]}{l_{yy}} & \frac{\left[M_q + (mU_o)\frac{M_{\dot{w}}}{m}\right]}{l_{yy}} & \frac{-mgsin\theta_o}{l_{yy}}\frac{M_{\dot{w}}}{m}\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\triangle Z^c\\ \triangle M^c\\ 0\end{bmatrix}$$

The transfer function from this state space form can be stated as following equations.

The equations results show the pitch angle and pitch rate development of aircraft control system. Therefore, use the linear quadratic controller to control the pitch

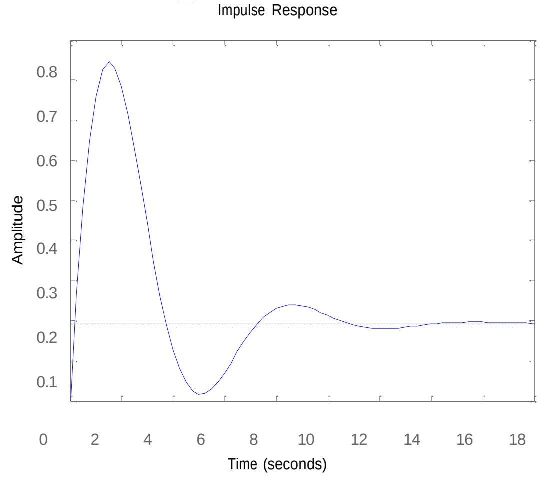


Open loop plant and observer model

Open loop Pitch Response

```
%open loop pitch response clear
all; close all;
```

```
A = [-0.313 56.7 0; -0.0139 -0.426
0; 0 56.7 0];
B = [0.232; 0.0203; 0];
C = [0 0 1];
D = [0];
pitches = ss(A,B,C,D)
impulse(pitch_ss)
```



Open-loop impulse response (without LQG)

Aerodynamic forces and moments are strongly dependent upon the ambient density standardize performance calculations, standard values of atmospheric properties have been developed, under the assumptions that the atmosphere is static (i.e., no winds), that atmospheric properties are a function only of altitude h, that the temperature is given by a specified piecewise linear function of altitude, and that the acceleration of gravity is constant (technically requiring that properties be defined as functions of geopotential altitude.

3.LATERAL DYNAMICS OF AIRCARFT

By the following similar procedure state space equation of literal dynamics state as:

$$x^{\cdot} = Ax + Bu$$

$$x = \begin{bmatrix} \beta \\ \rho \\ r \\ \Phi \end{bmatrix}$$

$xT = [\beta\ \rho\ r\ \phi]\ T$: State vector

$u^T = [\delta_A\ \delta_r\ ]$: *control vector*

δa , δr =all iron and rudder deflection

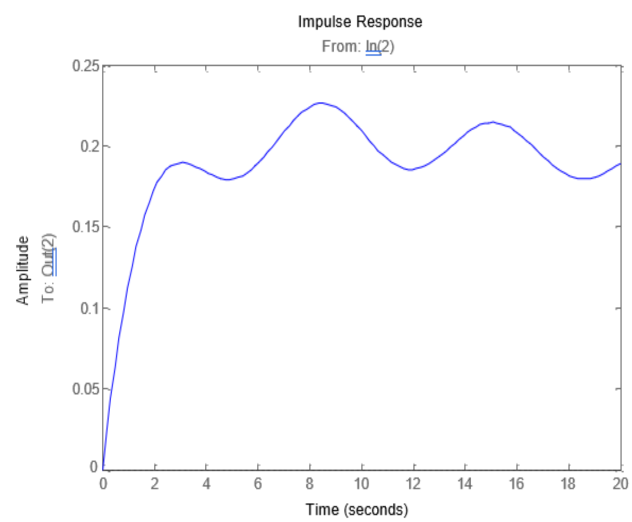
$B\theta$ = slide slip and roll angle P,r: roll and yaw angle

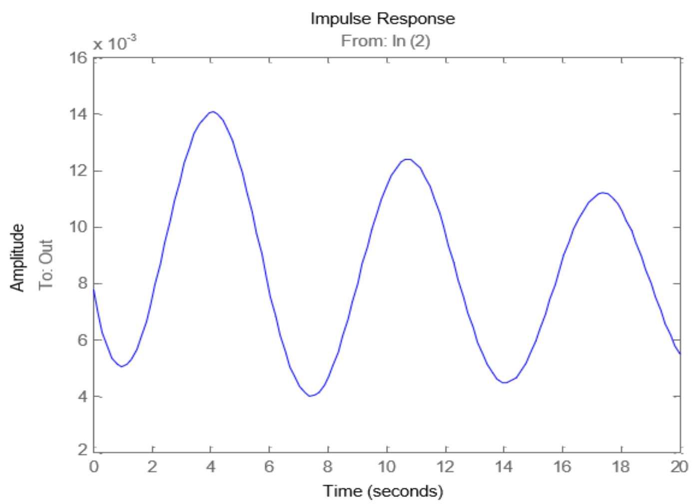
Here if we assume here our outputs are sideslip angle β and roll angle φ , so hereby given parameters we can covert our matric A,B,C are :

A=[-0.0558 -0.9968 0.0802 0.0415;
0.5980 -0.1150 -0.0318 0;
-3.0500 0.3880 -0.4650 0.
0 0.0805 1.000 0]

B= [0.0729 0.000.
-4.7500 0.00775
0.15300 0.1430
0 0 0 0].
C= [1 0 0 0.
0 0 0 1]

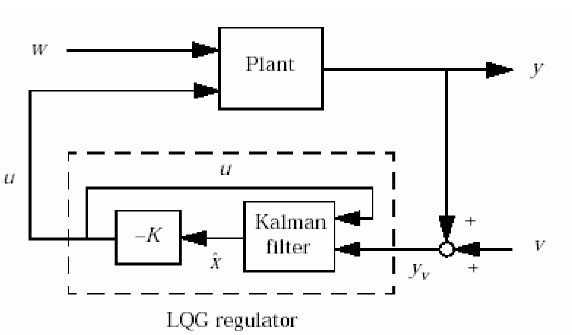
Now open loop response of slide slip and roll angle.





4 LINEAR QUADRATIC GUSSIAN CONTROLLER (LQG) DESIGN

The inference of LQG controller design delivers a reliable controller design procedure that assures the stability for the linearized closed-loop system model. In Figure shows the block diagram for the system that is implied by the state feedback controller.



LQG controller

The state space equations of above figure written as:

$$\dot{x} = Ax + Bu + Gw$$
$$y(t)=Cx+Du+Hw+v$$

here w and v modeled as white noise.

The combined form of linear quadratic regulator (LQR) and Kalman estimator is also called linear quadratic gaussian controller (LQG). Linear Quadratic Gaussian (LQG) controller provide a good performance and robustness in the design applied to aircraft Naturally, in LQG controller, it is necessary to select weighting matrices to solve the Algebraic-Riccati Equations and to get the possible best solution by connecting Kalman estimation and optimal state-feedback. Hence the LQG design is the combination of linear optimal gain and state estimators. LQG is generally not robust, but its t always gives a good result when the model of the system is reasonably accurate.

The regulation performance measured by the quadratic performance criteria

$$J(u) = \int \{x^T Qx + 2x^T Nx + u^T Ru\}$$

Here Q, N, R are the weighting matrices to check the control effect and how fast it goes to zero(regulation performance).

5 KALMAN ESTIMATION

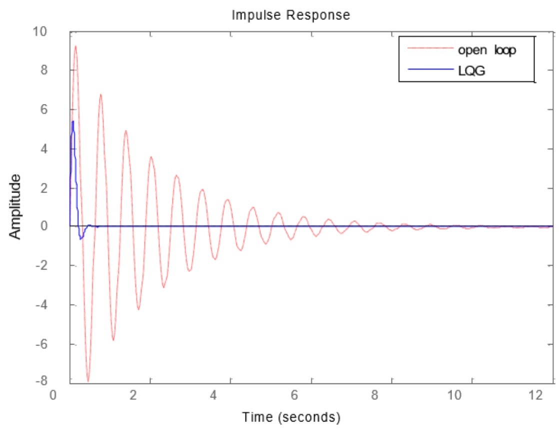
The state estimation is done by Kalman filter. Our aim is to regulate the plant output y around zero. The input disturbance d is low frequency with power spectral density (PSD) concentrated below 10 rad/sec. For LQG design purposes, it is modeled as white noise driving a low-pass filter with a cutoff at 10 rad/sec, the noise intensity is given as

$$E(n^2) = 0.01$$

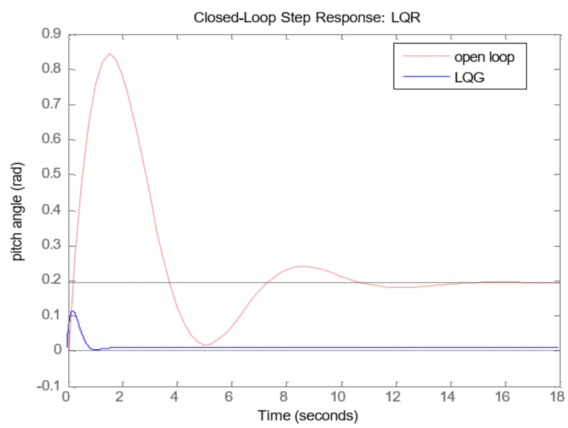
$J(u)=\int^{\infty} (10y^2 + u^2)\, dt$: cost function

Open loop state space model written as

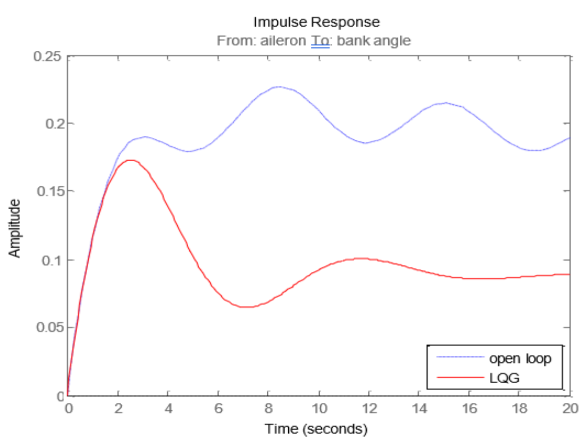
$$\dot{x} = Ax + Bu + Bd \quad \text{(state space equation)}$$
$$y = Cx + n \quad \text{(measurements)}$$



(a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Sideslip angle).



(b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Pitch angle)



(c)) Comparison of Open- and Closed-Loop Impulse Response for the LQG Example (Roll angle)

6 KALMAN FILTERING

The Kalman filter is the optimal filter (in the least mean squared error sense) for track prediction The Kalman filter is used heavily by control theorists. Kalman filter can be use as predictor, estimator, and observer, but in our case use the filter as estimator .

For aircraft tracking consider a discrete plant

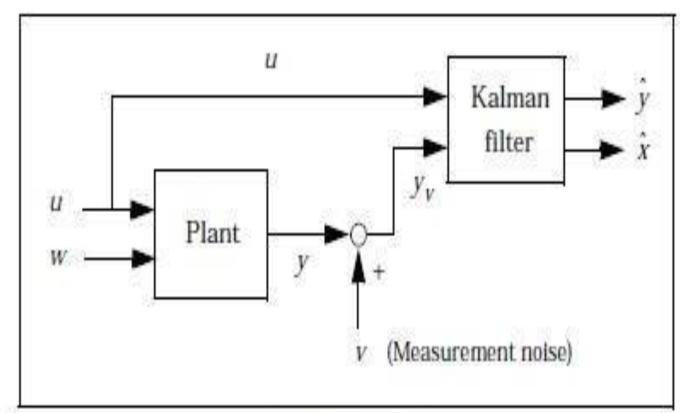
$$x(n+1) = A\dot{x}(n)+B(u(n)+w(n))$$

$$y(n)=Cx(n)$$

here we add the white noise w(n) at input u(n). our goal is to design a Kalman filter that provide the estimated value of output y(n) where the input u(n) and noisy measurement.

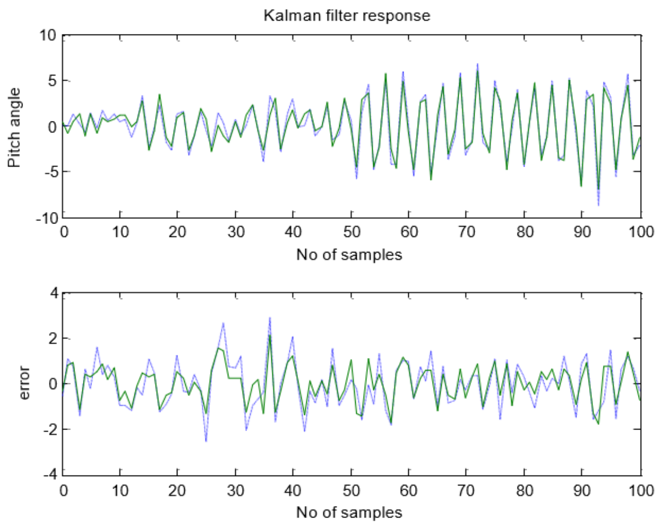
$$y_r(n) = Cx(n) + v(n)$$

V(n) is gaussian white noise (white Gaussian noise (WGN) is a basic noise model used in Information theory to mimic the effect of many random processes that occur in nature.



Kalman estimator

Here we choose a discrete Kalman filter (one step ahead predictor). The filter produces an optimal estimated solution. the filter states simulation shows:



Kalman filter repose for pitch angle

The first plot shows the actual response (dashed line) for the pitch angle and the filtered output (solid line). The other two response of sideslip and roll angle by Kalman filter and error signal simulate in a similar way.

7 CONCLUSIONS

The simple models of pitch, roll and sideslip angle is very useful to understand the control system stragty.to control the pitch slip and role angle of aircraft system requires controllers to maintain the the angle values the desire location. this can be easily done by reduce the error signal between the u(n) input and y(n) output which is the difference between the actual angle and desired angle. The LQG is very good to the outputs of plant with a steady shift error limited and the Kalman filter is an optimal estimator when dealing with Gaussian white noise. Optimal estimation provides an alternative rationale for the choice of observer gains in the current estimator which is based on observer performance in the presence of process noise and measurement errors, the Kalman filter estimate the process by following a feedback control law: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. the actual equations for the Kalman filter divide into two major groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

References

[1] Robert C. Nelson.1998. Flight Stability and Automatic Control. University of Notre Dame.

[2] Anonymous. 2006: uPSD3400 PWM API with R/C Servomotors Control Example (PCM), AN2283 Application note, June, http://www.st.com.

[3] Wong, K.C .2006. UAV Design Activities in a University Environment. University of Sydney.

[4] Aly El-Osery.2004. MATLAB Tutorial, October.

[5] Michael, T. Rusnell. 2004. Morphing UAV Pareto Curve Shift for Enhanced Performance. University of Notre Dame.Reed Siefert Christiansen (2004). Design ofan Autopilot for Small Unmanned Aerial Vehicle. Brigham Young University.

[6] Tom Bateman, Barry Blakeley, Adam Dean, Jake Hanft and Mike Sheek (2004). UAV Stability Augmentation System (USAS), University of Colorado.

[7] Bahadir Karuv (2003). Building Blocks, Bergen County Academies Invention Lab.

[8] A. Maddi, “Modélisation et contrôle du vol latéral d’un avion”, Magister Thesis, Electronics Department, University of Blida, Algeia, 1997.

[9] M. Grewal and A. Andrews, “Kalman Filtering Theory and Practice Using MATLAB”, (Second ed.), New York, NY USA: John Wiley & Sons, Inc., 2001.

[10] P.C. Murphy and V. Klein, “Estimation of Aircraft Unsteady Aerodynamic Parameters From Dynamic Wind Tunnel Testing”, AIAA

2001- 4016, August 2001

[11] G. Bishop, G. Welch, "An Introduction to the Kalman Filter", University of North Carolina at Chapel Hill, Department of Computer Science, SIGGRAPH 2001.

[12] M. Grewal and A. Andrews, "Kalman Filtering Theory and Practice Using MATLAB", (Second ed.), New York, NY USA: John Wiley & Sons, Inc., 2001.

[13] P.C. Murphy and V. Klein, "Estimation of Aircraft Unsteady Aerodynamic Parameters From Dynamic Wind Tunnel Testing" , AIAA 2001- 4016, August 2001

[14] G. Bishop, G. Welch, "An Introduction to the Kalman Filter", University of North Carolina at Chapel Hill, Department of Computer Science, SIGGRAPH 2001.

[15] D. Simon, "Optimal state estimation: Kalman, H-infinity, and nonlinear approaches", John Wiley & Sons, 2006.

[16] P. S. Maybeck, "Applied Optimal Estimation—Kalman Filter Design and Implementation", notes for a continuing education course offered by the Air Force Institute of Technology, Wright-Patterson AFB, Ohio, semiannually since December 1974.

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