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DEGREE OF AN EDGE IN UNION AND JOIN OF NEUTROSOPHIC FUZZY GRAPHS

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ABSTRACT
In this paper, edge degree in union and join of two neutrosophic fuzzy graphs is studied. Also we see the conditions for finding the degree of edge in these graphs.

Keywords: neutrosophic fuzzy graph, degree of a vertex in neutrosophic fuzzy graph, edge degree in fuzzy graph,union of neutrosophic fuzzy graph, join of neutrosophic fuzzy graph.

AMS Subject Classification: 05C12, 03E72, 05C72.

1 INTRODUCTION
Presently, science and technology is featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle various types of systems containing elements of uncertainty. A large number of these models is based on an extension of the ordinary set theory, namely, fuzzy sets. Graph theory has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, and transportation. In 1965, Zadeh[12] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, computer networks and automata theory. In 1994, Zhang[13, 14] initiated the concept of neutrosophic fuzzy sets as a generalization of fuzzy sets. Neutrosophic fuzzy sets are an extension of fuzzy sets whose membership degree range is [−1, 1]. In a neutrosophic fuzzy set, the membership degree of an element means that the element is irrelevant to the corresponding property, the membership degree (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree [−1, 0) of an element indicates that the element somewhat satisfies the implicit counter-property.

2 PRELIMINARIES
We present some known definitions related to fuzzy graphs and neutrosophic fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1. A fuzzy graph G : (σ, μ) is a pair of functions (σ, μ), where σ : V →[0,1] is a fuzzy subset of a non empty set V and μ : V XV →[0, 1] is a symmetric fuzzy relation on σ such that for all u, v in V, the relation μ(u, v) ≤ σ(u) ∧ σ(V) is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation μ(u, v) = σ(u) ∧ σ(V) is satisfied.

Definition 2.2. Let G : (N, M) be a single - valued neutrosophic graph where N and M are represented by two neutrosophic sets on V and E, respectively, which satisfy the following.

$T_M(a, b) \leq \min(T_N(a), T_N(b))$, $I_M(a, b) \geq \max(I_N(a), I_N(b))$, $F_M(a, b) \geq \max(F_N(a), F_N(b))$

Here a and b are two vertices of G and (a, b) ∈ E is an edge of G. Where T_M denote the truth value, I_M denoted the indeterminacy value and F_M denoted the false value.

Definition 2.3. Let G : (N, M) be an single - valued neutrosophic graph and a ∈ V be vertex of G. The degree of vertex a is the sum of the truth membership values, the sum of the indeterminacy membership values, and the sum of the membership values of falsity of all the arcs that are adjacent to vertex a .The degree of vertex a is denoted by $d(a) = (d_T(a), d_I(a), d_F(a))$ where $d_T(a) = \sum_{a \in V} T_M(a, b)$, $d_I(a) = \sum_{a \in V} I_M(a, b)$, $d_F(a) = \sum_{a \in V} F_M(a, b)$.

Here d_T (a),d_I (a), and d_F (a) are the degree of the truth membership value, the degree of the indeterminacy membership value, and the degree of falsity membership value, respectively, of vertex a.

Definition 2.4. Let G_N : (N, M) be a neutrosophic fuzzy graph on G_N^* : (V, E). The truth membership degree of an edge is defined as $d_T(sy) = d_T(s) + d_T(y) - 2T_M(sy)$, The indeterminacy membership degree of of an edge is defined as $d_I(sy) = d_I(s) + d_I(y) - 2I_M(sy)$.The falsity membership degree of of an edge is defined as $d_F(sy) = d_F(s) + d_F(y) - 2F_M(sy)$. Then the degree of Edge is is $d(sy) = (d_T(sy), d_I(sy), d_F(sy))$.

Definition 2.5. Let G_N : (N, M) be a neutrosophic fuzzy graph on G_N^* : (V, E). The order of G is denoted by $O(G)=(O^T(G),O^I(G),O^F(G)))$ where $O_T(G)=\sum T_N(v)$, $O_I(G)=\sum I_N(v)$, $O_F(G)=\sum F_N(v)$.

Definition 2.6. Let G_1: (N_1,M_1) and G_2:(N_2,M_2) where $N_1 = (T_{N_1}, I_{N_1}, F_{N_1})$, $M_1 = (T_{M_1}, I_{M_1}, F_{M_1})$,

$N_2 = (T_{N_2}, I_{N_2}, F_{N_2})$, $M_2 = (T_{M_2}, I_{M_2}, F_{M_2})$ be two fuzzy graphs. The union $G_1 \cup G_2$ is defined as

$$(T_{N_1} \cup T_{N_2})(x) = \begin{cases} T_{N_1}(x) & x \in V_1 - V_2 \\ T_{N_2}(x) & x \in V_2 - V_1 \\ T_{N_1}(x) \vee T_{N_2}(x) & x \in V_1 \cap V_2 \end{cases}$$

$$(I_{N_1} \cup I_{N_2})(x) = \begin{cases} I_{N_1}(x) & x \in V_1 - V_2 \\ I_{N_2}(x) & x \in V_2 - V_1 \\ I_{N_1}(x) \wedge I_{N_2}(x) & x \in V_1 \cap V_2 \end{cases}$$

$$(F_{N_1} \cup F_{N_2})(x) = \begin{cases} F_{N_1}(x) & x \in V_1 - V_2 \\ F_{N_2}(x) & x \in V_2 - V_1 \\ F_{N_1}(x) \wedge F_{N_2}(x) & x \in V_1 \cap V_2 \end{cases}$$

$$(T_{M_1} \cup T_{M_2})(xy) = \begin{cases} T_{M_1}(x) & xy \in E_1 - E_2 \\ T_{M_2}(x) & xy \in E_2 - E_1 \\ T_{M_1}(x) \vee T_{M_2}(x) & xy \in E_1 \cap E_2 \end{cases}$$

$$(I_{M_1} \cup I_{M_2})(xy) = \begin{cases} I_{M_1}(x) & xy \in E_1 - E_2 \\ I_{M_2}(x) & xy \in E_2 - E_1 \\ I_{M_1}(x) \wedge I_{M_2}(x) & xy \in E_1 \cap E_2 \end{cases}$$

$$(F_{M_1} \cup F_{M_2})(xy) = \begin{cases} F_{M_1}(x) & xy \in E_1 - E_2 \\ F_{M_2}(x) & xy \in E_2 - E_1 \\ F_{M_1}(x) \wedge F_{M_2}(x) & xy \in E_1 \cap E_2 \end{cases}$$

Definition 2.7. Let G_1: (N_1,M_1) and G_2:(N_2,M_2) where $N_1 = (T_{N_1}, I_{N_1}, F_{N_1})$, $M_1 = (T_{M_1}, I_{M_1}, F_{M_1})$,

$N_2 = (T_{N_2}, I_{N_2}, F_{N_2})$, $M_2 = (T_{M_2}, I_{M_2}, F_{M_2})$ be two neutrosophic fuzzy graphs. The join $G_1 + G_2$ is defined as

Let $x \in V_1 \cup V_2$

$$(T_{N_1} + T_{N_2})(x) = (T_{N_1} \cup T_{N_2})(x), (I_{N_1} + I_{N_2})(x) = (I_{N_1} \cup I_{N_2})(x), (F_{N_1} + F_{N_2})(x) = (F_{N_1} \cup F_{N_2})(x)$$

If xy$\in E_1 \cup E_2$

$$(T_{N_1} + T_{N_2})(xy) = (T_{N_1} \cup T_{N_2})(xy), (I_{N_1} + I_{N_2})(xy) = (I_{N_1} \cup I_{N_2})(xy), (F_{N_1} + F_{N_2})(xy) = (F_{N_1} \cup F_{N_2})(xy)$$

If xy$\in E'$ where E' is the set of all edges joining vertices of V_1 and V_2

$$(T_{N_1} + T_{N_2})(xy) = (T_{N_1} \vee T_{N_2})(xy), (I_{N_1} + I_{N_2})(xy) = (I_{N_1} \wedge I_{N_2})(xy), (F_{N_1} + F_{N_2})(xy) = (F_{N_1} \wedge F_{N_2})(xy).$$

3 DEGREE OF AN EDGE IN UNION OF TWO NEUTROSOPHIC FUZZY GRAPHS

In this section, degree of an edge in union of two neutrosophic fuzzy graph is defined.

For any uv $\in E_1 \cup E_2$, let v$\in V_1 \cap V_2$

Case 1: $V_1 \cap V_2 = \emptyset$. Here $E_1 \cap E_2 = \emptyset$. Therefore uv$\in E_1$ or uv $\in E_2$ but not both. Then

$$(T_{M_1} \cup T_{M_2})(uv) = \begin{cases} T_{M_1}(uv) & uv \in E_1 - E_2 \\ T_{M_2}(uv) & uv \in E_2 - E_1 \end{cases}$$

$$(I_{M_1} \cup I_{M_2})(uv) = \begin{cases} I_{M_1}(uv) & uv \in E_1 - E_2 \\ I_{M_2}(uv) & uv \in E_2 - E_1 \end{cases}$$

$$(F_{M_1} \cup F_{M_2})(uv) = \begin{cases} F_{M_1}(uv) & uv \in E_1 - E_2 \\ F_{M_2}(uv) & uv \in E_2 - E_1 \end{cases}$$

By definition,

$$d^T_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v} (T_{M_1} \cup T_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u} (T_{M_1} \cup T_{M_2})(wv)$$

If uv$\in E_1$, $d^T_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1, w \neq v} (T_{M_1})(uw) + \sum_{wv \in E_1, w \neq u} (T_{M_1})(wv)$

$d^T_{G_1 \cup G_2}(uv) = d^T_{G_1}(uv)$

$$d^I_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v} (I_{M_1} \cup I_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u} (I_{M_1} \cup I_{M_2})(wv)$$

If uv$\in E_1$, $d^T_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1, w \neq v} (I_{M_1})(uw) + \sum_{wv \in E_1, w \neq u} (I_{M_1})(wv)$

$d^I_{G_1 \cup G_2}(uv) = d^I_{G_1}(uv)$

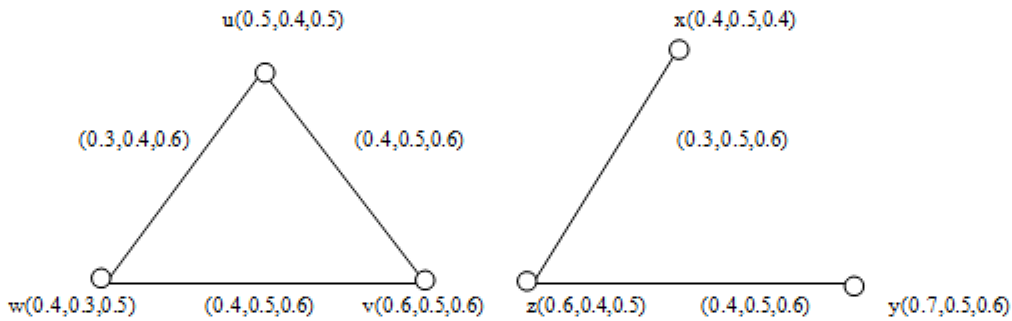
$$d^F_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v} (F_{M_1} \cup F_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u} (F_{M_1} \cup F_{M_2})(wv)$$

If uv$\in E_1$, $d^F_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1, w \neq v} (F_{M_1})(uw) + \sum_{wv \in E_1, w \neq u} (F_{M_1})(wv)$

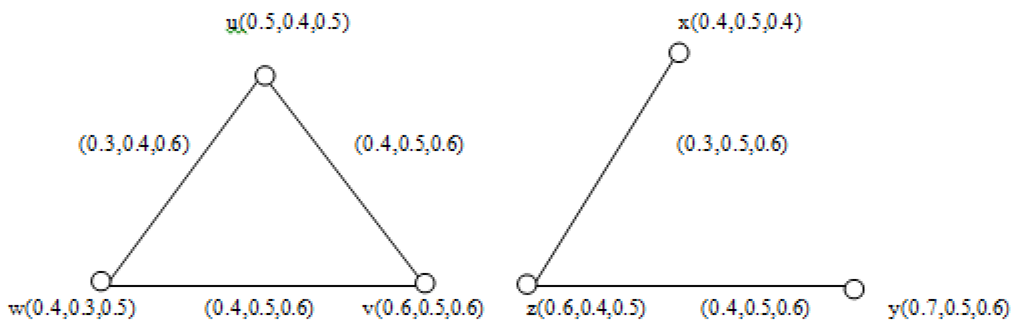
$d^F_{G_1 \cup G_2}(uv) = d^F_{G_1}(uv)$

Similarly if uv$\in E_2$, $d^T_{G_1 \cup G_2}(uv) = d^T_{G_2}(uv)$, $d^I_{G_1 \cup G_2}(uv) = d^I_{G_2}(uv)$, $d^F_{G_1 \cup G_2}(uv) = d^F_{G_2}(uv)$

Example 3.1 Consider a neutrosophic fuzzy graph on $G^*(V,E)$.



G_1 G_2



$G_1 \cup G_2$

Here, $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv)$ = (0.7,0.9,1.2) + (1.0,1.0,1.2) – 2(0.4,0.5,0.6)

=(1.7,1.9,2.4) - (0.8,1.0,1.2) = (0.9,0.9,1.2)

Case 2: $V_1 \cap V_2 = \emptyset$. Here $E_1 \cap E_2 = \emptyset$. Therefore uv $\in E_1$ or uv $\in E_2$. If u,v $\notin V_1 \cap V_2$ then it is of case 1. So, let u $\in V_1 \cap V_2$ or v$\in V_1 \cap V_2$ or u,v $\in V_1 \cap V_2$

Sub case 1: $u \in V_1 \cap V_2$ or v $\in V_1 \cap V_2$. Let uv$\in E_1$, if u $\in V_1 \cap V_2$

$d^T_{G_1 \cup G_2}(uv) = d^T_{G_1 \cup G_2}(uv) + d^T_{G_1 \cup G_2}(uv) - 2(T_{M_1} \cup T_{M_2})(uv)$

$= d^T_{G_1}(u) + d^T_{G_2}(u) + d^T_{G_1}(v) - 2T_{M_1}(uv)$

$= d^T_{G_1}(u) + d^T_{G_1}(v) - 2T_{M_1}(uv) + d^T_{G_2}(u)$

$= d^T_{G_1}(uv) + d^T_{G_2}(u)$

$d^I_{G_1 \cup G_2}(uv) = d^I_{G_1 \cup G_2}(uv) + d^I_{G_1 \cup G_2}(uv) - 2(I_{M_1} \cup I_{M_2})(uv)$

$= d^I_{G_1}(u) + d^I_{G_2}(u) + d^I_{G_1}(v) - 2I_{M_1}(uv)$

$= d^I_{G_1}(u) + d^I_{G_1}(v) - 2I_{M_1}(uv) + d^I_{G_2}(u)$

$=d^{I}_{G_1}(uv) + d^{I}_{G_2}(u)$

$d^{F}_{G_1\cup G_2}(uv) = d^{F}_{G_1\cup G_2}(uv) + d^{F}_{G_1\cup G_2}(uv) - 2(F_{M_1} \cup F_{M_2})(uv)$

$= d^{F}_{G_1}(u) + d^{F}_{G_2}(u) + d^{F}_{G_1}(v) - 2F_{M_1}(uv)$

$= d^{F}_{G_1}(u) + d^{F}_{G_1}(v) - 2F_{M_1}(uv) + d^{F}_{G_2}(u)$

$=d^{F}_{G_1}(uv) + d^{F}_{G_2}(u)$

Similarly if v $\in V_1 \cap V_2$

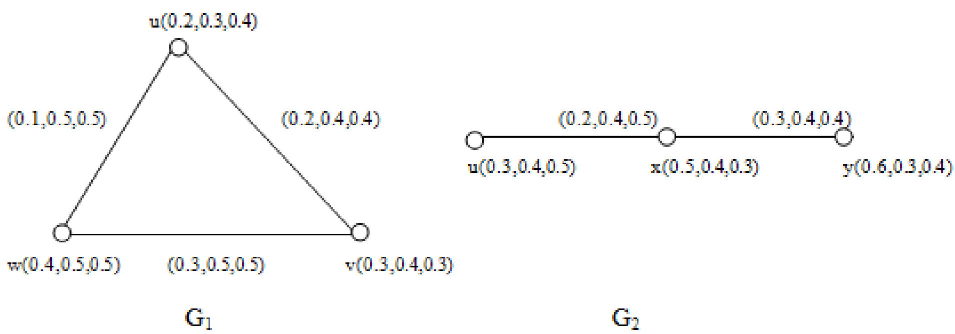
$d^{T}_{G_1\cup G_2}(uv)=d^{T}_{G_1}(uv) + d^{T}_{G_2}(v), d^{I}_{G_1\cup G_2}(uv)=d^{I}_{G_1}(uv) + d^{I}_{G_2}(v), d^{F}_{G_1\cup G_2}(uv)=d^{F}_{G_1}(uv) + d^{F}_{G_2}(v)$

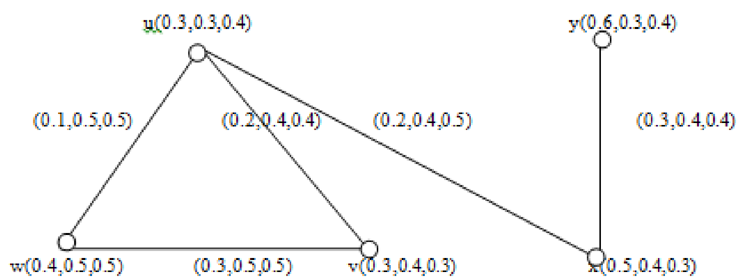
If uv $\in E_2$, $d^{T}_{G_1\cup G_2}(uv) = \begin{cases} d^{T}_{G_2}(uv) + d^{T}_{G_1}(u) & u \in V_1 \cap V_2 \\ d^{T}_{G_2}(uv) + d^{T}_{G_1}(v) & v \in V_1 \cap V_2 \end{cases}$

$$d^{I}_{G_1\cup G_2}(uv) = \begin{cases} d^{I}_{G_2}(uv) + d^{I}_{G_1}(u) & u \in V_1 \cap V_2 \\ d^{I}_{G_2}(uv) + d^{I}_{G_1}(v) & v \in V_1 \cap V_2 \end{cases}$$

$$d^{F}_{G_1\cup G_2}(uv) = \begin{cases} d^{F}_{G_2}(uv) + d^{F}_{G_1}(u) & u \in V_1 \cap V_2 \\ d^{F}_{G_2}(uv) + d^{F}_{G_1}(v) & v \in V_1 \cap V_2 \end{cases}$$

Example 3.2 Consider a neutrosophic fuzzy graph on $G^*(V,E)$.





$G_1 \cup G_2$

Here, $d_{G_1\cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv)$= (0.4,1.0,1.0)+(0.2,0.4,0.5)=(0.6,1.4,1.5)

Sub case 2: $u, v \in V_1 \cap V_2$ Let uv$\in E_1$

Since the edges at u and v in G_1 and also in G_2 with same membership value in $G_1 \cup G_2$

$d^{T}_{G_1\cup G_2}(uv) = d^{T}_{G_1\cup G_2}(uv) + d^{T}_{G_1\cup G_2}(uv) - 2(T_{M_1} \cup T_{M_2})(uv)$

$= d^T_{G_1}(u) + d^T_{G_2}(u) + d^T_{G_1}(v) + d^T_{G_2}(v) - 2T_{M_1}(uv)$

$= d^T_{G_1}(u) + d^T_{G_1}(v) - 2T_{M_1}(uv) + d^T_{G_2}(u) + d^T_{G_2}(v)$

$= d^T_{G_1}(uv) + d^T_{G_2}(u) + d^T_{G_2}(v)$

$d^I_{G_1 \cup G_2}(uv) = d^I_{G_1 \cup G_2}(uv) + d^I_{G_1 \cup G_2}(uv) - 2(I_{M_1} \cup I_{M_2})(uv)$

$= d^I_{G_1}(u) + d^I_{G_2}(u) + d^I_{G_1}(v) + d^I_{G_2}(v) - 2I_{M_1}(uv)$

$= d^I_{G_1}(u) + d^I_{G_1}(v) - 2I_{M_1}(uv) + d^I_{G_2}(u) + d^I_{G_2}(v)$

$= d^I_{G_1}(uv) + d^I_{G_2}(u) + d^I_{G_2}(v)$

$d^F_{G_1 \cup G_2}(uv) = d^F_{G_1 \cup G_2}(uv) + d^F_{G_1 \cup G_2}(uv) - 2(F_{M_1} \cup F_{M_2})(uv)$

$= d^F_{G_1}(u) + d^F_{G_2}(u) + d^F_{G_1}(v) + d^F_{G_2}(v) - 2F_{M_1}(uv)$

$= d^F_{G_1}(u) + d^F_{G_1}(v) - 2F_{M_1}(uv) + d^F_{G_2}(u) + d^F_{G_2}(v)$

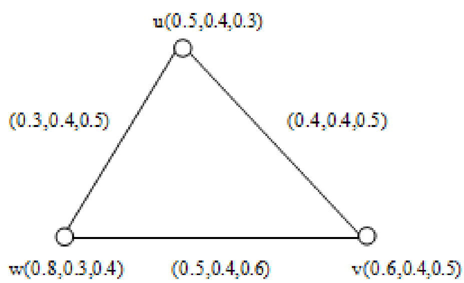
$= d^F_{G_1}(uv) + d^F_{G_2}(u) + d^F_{G_2}(v)$

Similarly if uv$\in E_2$,

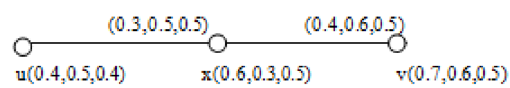
$d^T_{G_1 \cup G_2}(uv) = d^T_{G_2}(uv) + d^T_{G_1}(u) + d^T_{G_1}(v)$, $d^I_{G_1 \cup G_2}(uv) = d^I_{G_2}(uv) + d^I_{G_1}(u) + d^I_{G_1}(v)$,

$d^F_{G_1 \cup G_2}(uv) = d^F_{G_2}(uv) + d^F_{G_1}(u) + d^F_{G_1}(v)$.

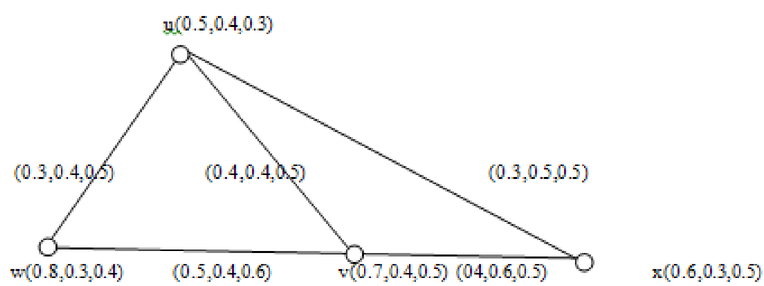
Example 3.3. Consider a neutrosophic fuzzy graph on $G^*(V,E)$.



G1



G2



G1∪G2

Here, uv $\in E_1$ an u, v $\in V_1 \cap V_2$.

$d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_1}(u) + d_{G_2}(v)$ = (0.8,0.8,1.1)+(0.3,0.5,0.5)+(0.4,0.6,0.5)

= (1.5,1.9,2.1).

Case 3: $V_1 \cap V_2 \neq \emptyset$. Here $E_1 \cap E_2 \neq \emptyset$. Then uv $\in E_1 \cap E_2$.

Sub case 1: No edge incident at u or v is either in E_1 or in E_2.

$d^T_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v}(T_{M_1} \cup T_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u}(T_{M_1} \cup T_{M_2})(wv)$

$= \sum_{uw \in E_1 - E_2, w \neq v}(T_{M_1})(uw) + \sum_{uw \in E_2 - E_1, w \neq v}(T_{M_2})(uw) + \sum_{wv \in E_1 - E_2, w \neq u}(T_{M_1})(wv) + \sum_{wv \in E_2 - E_1, w \neq u}(T_{M_2})(wv)$

$= \sum_{uw \in E_1 - E_2, w \neq v}(T_{M_1})(uw) + \sum_{wv \in E_1 - E_2, w \neq u}(T_{M_1})(wv) + \sum_{uw \in E_2 - E_1, w \neq v}(T_{M_2})(uw) + \sum_{wv \in E_2 - E_1, w \neq u}(T_{M_2})(wv)$

$d^T_{G_1 \cup G_2}(uv) = d^T_{G_1}(uv) + d^T_{G_2}(uv)$

$d^I_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v}(I_{M_1} \cup I_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u}(I_{M_1} \cup I_{M_2})(wv)$

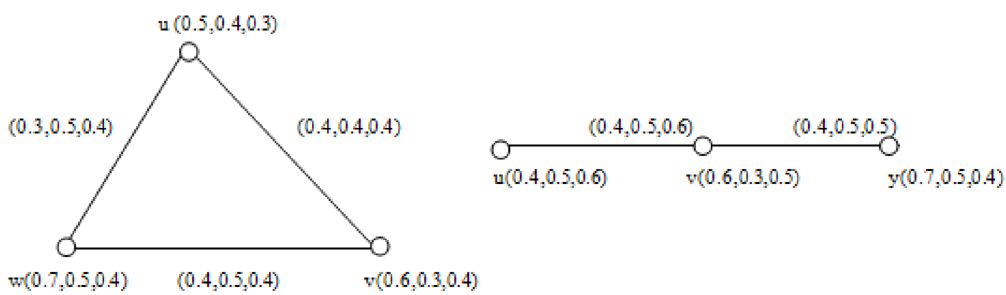
$= \sum_{uw \in E_1 - E_2, w \neq v}(I_{M_1})(uw) + \sum_{uw \in E_2 - E_1, w \neq v}(I_{M_2})(uw) + \sum_{wv \in E_1 - E_2, w \neq u}(I_{M_1})(wv) + \sum_{wv \in E_2 - E_1, w \neq u}(I_{M_2})(wv)$

$= \sum_{uw \in E_1 - E_2, w \neq v}(I_{M_1})(uw) + \sum_{wv \in E_1 - E_2, w \neq u}(I_{M_1})(wv) + \sum_{uw \in E_2 - E_1, w \neq v}(I_{M_2})(uw) + \sum_{wv \in E_2 - E_1, w \neq u}(I_{M_2})(wv)$

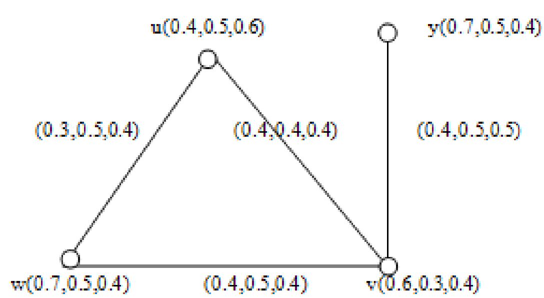
$d^I_{G_1 \cup G_2}(uv) = d^I_{G_1}(uv) + d^I_{G_2}(uv)$

Similarly, $d^F_{G_1 \cup G_2}(uv) = d^F_{G_1}(uv) + d^F_{G_2}(uv)$

Example 3.4 Consider a neutrosophic fuzzy graph on $G^*(V,E)$.



G_1 G_2



$G_1 \cup G_2$

Here, uv ∈ $E_1 \cap E_2$, $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv)$ = (0.7,1.0,0.8)+(0.4,0.5,0.5)=(1.1,1.5,1.3)

Sub case 2: some edge incident at u and v other than uv are in $E_1 \cap E_2$.

$$d^T_{G_1 \cup G_2}(uv) = \sum_{uw \in E_1 \cap E_2, w \neq v} (T_{M_1} \cup T_{M_2})(uw) + \sum_{wv \in E_1 \cap E_2, w \neq u} (T_{M_1} \cup T_{M_2})(wv)$$

$$= \sum_{w \neq v} (T_{M_1})(uw) + \sum_{w \neq v} (T_{M_2})(uw) + \sum_{w \neq u} (T_{M_1})(wv) + \sum_{w \neq u} (T_{M_2})(wv) + \sum_{w \neq v} (T_{M_1})(uw) \vee (T_{M_2})(wv) + \sum_{w \neq u} (T_{M_1})(uw) \vee (T_{M_2})(wv)$$

$$= \sum_{w \neq v} (T_{M_1})(uw) + \sum_{w \neq v} (T_{M_2})(uw) + \sum_{w \neq u} (T_{M_1})(wv) + \sum_{w \neq u} (T_{M_2})(wv) + \sum_{w \neq v} (T_{M_1})(uw) \vee (T_{M_2})(wv) + \sum_{w \neq u} (T_{M_1})(uw) \vee (T_{M_2})(wv) + \sum_{w \neq v} (T_{M_1})(uw) \wedge (T_{M_2})(wv) + \sum_{w \neq u} (T_{M_1})(uw) \wedge (T_{M_2})(wv) - \sum_{w \neq v} (T_{M_1})(uw) \wedge (T_{M_2})(wv) - \sum_{w \neq u} (T_{M_1})(uw) \wedge (T_{M_2})(wv)$$

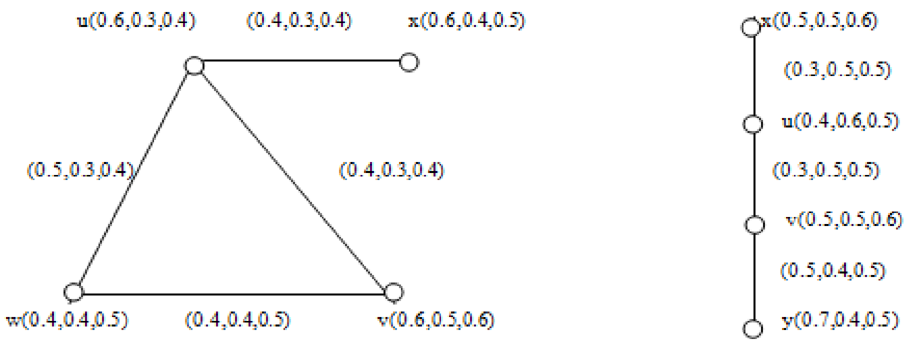
$$= \sum_{w \neq u} (T_{M_1})(uw) + \sum_{w \neq u} (T_{M_1})(wv) + \sum_{w \neq v} (T_{M_2})(uw) + \sum_{w \neq u} (T_{M_2})(wv) - \sum_{w \neq v} (T_{M_1})(uw) \wedge (T_{M_2})(uw) - \sum_{w \neq u} (T_{M_1})(wv) \wedge (T_{M_2})(wv)$$

$$d^T_{G_1 \cup G_2}(uv) = d^T_{G_1}(uv) + d^T_{G_2}(uv) - \sum_{w \neq v} (T_{M_1})(uw) \wedge (T_{M_2})(uw) - \sum_{w \neq u} (T_{M_1})(wv) \wedge (T_{M_2})(wv)$$

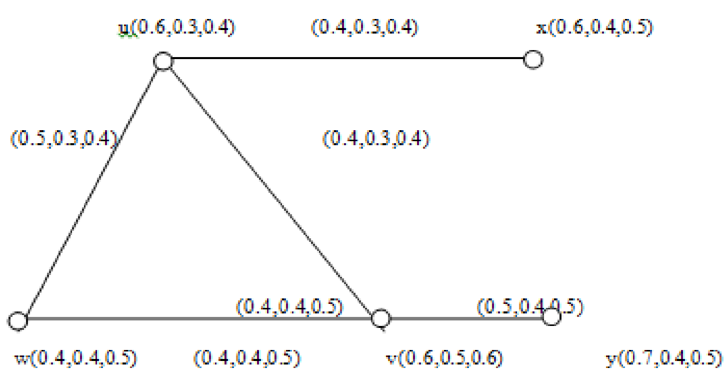
$$d^I_{G_1 \cup G_2}(uv) = d^I_{G_1}(uv) + d^I_{G_2}(uv) - \sum_{w \neq v} (I_{M_1})(uw) \wedge (I_{M_2})(uw) - \sum_{w \neq u} (I_{M_1})(wv) \wedge (I_{M_2})(wv)$$

$$d^F_{G_1 \cup G_2}(uv) = d^F_{G_1}(uv) + d^F_{G_2}(uv) - \sum_{w \neq v} (F_{M_1})(uw) \wedge (F_{M_2})(uw) - \sum_{w \neq u} (F_{M_1})(wv) \wedge (F_{M_2})(wv)$$

Example 3.5 Consider a neutrosophic fuzzy graph on $G^*(V,E)$.



G_1 G_2



$G_1 \cup G_2$

Here, uv $\in E_1 \cap E_2$, $d_{G_1 \cup G_2}(uv) = d_{G_1}(uv) + d_{G_2}(uv) - \sum_{w \neq v} M_1(ux) + M_2(ux)$

= (1.3,1.0,1.3)+(0.8,0.9,1.0) – (0.3,0.5,0.5)=(1.8,1.4,1.8)

4 Degree of an Edge in Join of two Neutrosophic Fuzzy Graphs

In this section, degree of an edge in join of two neutrosophic fuzzy graph is defined.

Here, $V_1 \cap V_2 = \emptyset$ and $E_1 \cap E_2 = \emptyset$.

$$(T_{M_1} + T_{M_2})(uv) = \begin{cases} T_{M_1}(uv) & uv \in E_1 \\ T_{M_2}(uv) & uv \in E_2 \\ T_{N_1}(u) \vee T_{N_2}(u) & uv \in E' \end{cases}$$

$$(I_{M_1} + I_{M_2})(uv) = \begin{cases} I_{M_1}(uv) & uv \in E_1 \\ I_{M_2}(uv) & uv \in E_2 \\ I_{N_1}(u) \vee I_{N_2}(u) & uv \in E' \end{cases}$$

$$(F_{M_1} + F_{M_2})(uv) = \begin{cases} F_{M_1}(uv) & uv \in E_1 \\ F_{M_2}(uv) & uv \in E_2 \\ F_{N_1(}u) \vee F_{N_2}(u) & uv \in E' \end{cases}$$

By definition,

$d^T_{G_1+G_2}(uv) = \sum_{uw \in E_1 \cup E_2 \cup E', w \neq v} T_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} T_{M_2}(wv)$

$= \sum_{uw \in E_1 \cup E_2, w \neq v} T_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} T_{M_2}(wv) +$

$\sum_{uw \in E', w \neq v} T_{M_1}(uw) + \sum_{uw \in E', w \neq v} T_{M_2}(wv)$

$d^I_{G_1+G_2}(uv) = \sum_{uw \in E_1 \cup E_2 \cup E', w \neq v} I_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} I_{M_2}(wv)$

$= \sum_{uw \in E_1 \cup E_2, w \neq v} I_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} I_{M_2}(wv) +$

$\sum_{uw \in E', w \neq v} I_{M_1}(uw) + \sum_{uw \in E', w \neq v} I_{M_2}(wv)$

$d^F_{G_1+G_2}(uv) = \sum_{uw \in E_1 \cup E_2 \cup E', w \neq v} F_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} F_{M_2}(wv)$

$= \sum_{uw \in E_1 \cup E_2, w \neq v} F_{M_1}(uw) + \sum_{uw \in E_1 \cup E_2 \cup E', w \neq u} F_{M_2}(wv) +$

$\sum_{uw \in E', w \neq v} F_{M_1}(uw) + \sum_{uw \in E', w \neq v} F_{M_2}(wv)$

For any $uv \in E_1$,

$d^T_{G_1+G_2}(uv) = \sum_{w \neq v} T_{M_1}(uw) + \sum_{w \neq u} T_{M_1}(wv) + \sum_{uw \in E'} T_{N_1}(u) \wedge T_{N_1}(w) + \sum_{wv \in E'} T_{N_1}(v) \wedge T_{N_1}(w)$

$= d^T_{G_1}(uv) + \sum_{uw \in E'} T_{N_1}(u) \wedge T_{N_1}(w) + \sum_{wv \in E'} T_{N_1}(v) \wedge T_{N_1}(w)$

$d^I_{G_1+G_2}(uv) = \sum_{w \neq v} I_{M_1}(uw) + \sum_{w \neq u} I_{M_1}(wv) + \sum_{uw \in E'} I_{N_1}(u) \wedge I_{N_1}(w) + \sum_{wv \in E'} I_{N_1}(v) \wedge I_{N_1}(w)$

$= d^I_{G_1}(uv) + \sum_{uw \in E'} I_{N_1}(u) \wedge I_{N_1}(w) + \sum_{wv \in E'} I_{N_1}(v) \wedge I_{N_1}(w)$

$d^F_{G_1+G_2}(uv) = \sum_{w \neq v} F_{M_1}(uw) + \sum_{w \neq u} F_{M_1}(wv) + \sum_{uw \in E'} F_{N_1}(u) \wedge F_{N_1}(w) + \sum_{wv \in E'} F_{N_1}(v) \wedge F_{N_1}(w)$

$= d^F_{G_1}(uv) + \sum_{uw \in E'} F_{N_1}(u) \wedge F_{N_1}(w) + \sum_{wv \in E'} F_{N_1}(v) \wedge F_{N_1}(w)$

Similarly for any $uv \in E_2$,

$d^T_{G_1+G_2}(uv) = d^T_{G_2}(uv) + \sum_{uw \in E'} T_{N_1}(u) \wedge T_{N_1}(w) + \sum_{wv \in E'} T_{N_1}(v) \wedge T_{N_1}(w)$

$d^{I}_{G_1+G_2}(uv) = d^{I}_{G_2}(uv) + \sum_{uw\in E'} I_{N_1}(u) \wedge I_{N_1}(w) + \sum_{wv\in E'} I_{N_1}(v) \wedge I_{N_1}(w)$

$d^{F}_{G_1+G_2}(uv) = d^{F}_{G_2}(uv) + \sum_{uw\in E'} F_{N_1}(u) \wedge F_{N_1}(w) + \sum_{wv\in E'} F_{N_1}(v) \wedge F_{N_1}(w)$

Theorem 4.1 Let G_1: (N_1,M_1) and G_2:(N_2,M_2) where $N_1 = (T_{N_1}, I_{N_1}, F_{N_1})$, $M_1 = (T_{M_1}, I_{M_1}, F_{M_1})$, $N_2 = (T_{N_2}, I_{N_2}, F_{N_2})$, $M_2 = (T_{M_2}, I_{M_2}, F_{M_2})$ be two fuzzy graphs.

1. If $T_{N_1} \geq T_{N_2}$, $I_{N_1} \leq I_{N_2}$, $F_{N_1} \leq F_{N_2}$ then

$$d^{T}_{G_1+G_2}(uv) = \begin{cases} d^{T}_{G_1}(uv) + 2O^{T}(G_2) & uv \in E_1 \\ d^{T}_{G_2}(uv) + p_1(T_{N_2}(u) + T_{N_2}(v)) & uv \in E_2 \\ d^{T}_{G_1}(u) + d^{T}_{G_2}(v) + O^{T}(G_2) + (p_1 - 2)T_{N_2}(v) & uv \in E' \end{cases}$$

$$d^{I}_{G_1+G_2}(uv) = \begin{cases} d^{I}_{G_1}(uv) + 2O^{I}(G_2) & uv \in E_1 \\ d^{I}_{G_2}(uv) + p_1(I_{N_2}(u) + I_{N_2}(v)) & uv \in E_2 \\ d^{I}_{G_1}(u) + d^{I}_{G_2}(v) + O^{I}(G_2) + (p_1 - 2)I_{N_2}(v) & uv \in E' \end{cases}$$

$$d^{F}_{G_1+G_2}(uv) = \begin{cases} d^{F}_{G_1}(uv) + 2O^{F}(G_2) & uv \in E_1 \\ d^{F}_{G_2}(uv) + p_1(F_{N_2}(u) + F_{N_2}(v)) & uv \in E_2 \\ d^{F}_{G_1}(u) + d^{F}_{G_2}(v) + O^{F}(G_2) + (p_1 - 2)F_{N_2}(v) & uv \in E' \end{cases}$$

2. If $T_{N_2} \geq T_{N_1}$, $I_{N_2} \leq I_{N_1}$, $F_{N_2} \leq F_{N_1}$ then

$$d^{T}_{G_1+G_2}(uv) = \begin{cases} d^{T}_{G_1}(uv) + p_2(T_{N_1}(u) + T_{N_1}(v)) & uv \in E_1 \\ d^{T}_{G_2}(uv) + 2O^{T}(G_1) & uv \in E_2 \\ d^{T}_{G_1}(u) + d^{T}_{G_2}(v) + O^{T}(G_1) + (p_2 - 2)T_{N_1}(v) & uv \in E' \end{cases}$$

$$d^{I}_{G_1+G_2}(uv) = \begin{cases} d^{I}_{G_1}(uv) + p_2(I_{N_1}(u) + I_{N_1}(v)) & uv \in E_1 \\ d^{I}_{G_2}(uv) + 2O^{I}(G_1) & uv \in E_2 \\ d^{I}_{G_1}(u) + d^{I}_{G_2}(v) + O^{I}(G_1) + (p_2 - 2)I_{N_1}(v) & uv \in E' \end{cases}$$

$$d^{F}_{G_1+G_2}(uv) = \begin{cases} d^{F}_{G_1}(uv) + p_2(F_{N_1}(u) + F_{N_1}(v)) & uv \in E_1 \\ d^{F}_{G_2}(uv) + 2O^{F}(G_1) & uv \in E_2 \\ d^{F}_{G_1}(u) + d^{F}_{G_2}(v) + O^{F}(G_1) + (p_2 - 2)F_{N_1}(v) & uv \in E' \end{cases}$$

Proof. If $T_{N_1} \geq T_{N_2}$, $I_{N_1} \leq I_{N_2}$, $F_{N_1} \leq F_{N_2}$ then for any $uv \in E_1$

$d^{T}_{G_1+G_2}(uv) = d^{T}_{G_1}(uv) + \sum_{w\neq v} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\neq u} T_{N_1}(v) \wedge T_{N_2}(w)$

$= d^{T}_{G_1}(uv) + \sum T_{N_1}(w) + \sum T_{N_2}(w) = d^{T}_{G_1}(uv) + 2O^{T}(G_2)$

$d^{I}_{G_1+G_2}(uv) = d^{I}_{G_1}(uv) + \sum_{w\neq v} I_{N_1}(u) \vee I_{N_2}(w) + \sum_{w\neq u} I_{N_1}(v) \vee I_{N_2}(w)$

$= d^{I}_{G_1}(uv) + \sum I_{N_1}(w) + \sum I_{N_2}(w) = d^{I}_{G_1}(uv) + 2O^{T}(G_2)$

$d^{F}_{G_1+G_2}(uv) = d^{F}_{G_1}(uv) + \sum_{w\neq v} F_{N_1}(u) \vee F_{N_2}(w) + \sum_{w\neq u} F_{N_1}(v) \vee F_{N_2}(w)$

$= d^{F}_{G_1}(uv) + \sum F_{N_1}(w) + \sum F_{N_2}(w) = d^{F}_{G_1}(uv) + 2O^{T}(G_2)$

Similarly for any $v \in E_2$,

$d^{T}_{G_1+G_2}(uv) = d^{T}_{G_2}(uv) + \sum_{w\neq v} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\neq u} T_{N_1}(v) \wedge T_{N_2}(w)$

$= \ d^{T}_{G_2}(uv) + \sum T_{N_2}(u) + \sum T_{N_2}(v) = d^{T}_{G_2}(uv) + p_1 T_{N_2}(u) + p_1 T_{N_2}(v)$

$d^{I}_{G_1+G_2}(uv) = \ d^{I}_{G_2}(uv) + \sum_{w\neq v} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{w\neq u} I_{N_1}(v) \wedge I_{N_2}(w)$

$= \ d^{I}_{G_2}(uv) + \sum I_{N_2}(u) + \sum I_{N_2}(v) = d^{I}_{G_2}(uv) + p_1 I_{N_2}(u) + p_1 I_{N_2}(v)$

$d^{F}_{G_1+G_2}(uv) = \ d^{F}_{G_2}(uv) + \sum_{w\neq v} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{w\neq u} F_{N_1}(v) \wedge F_{N_2}(w)$

$= \ d^{F}_{G_2}(uv) + \sum F_{N_2}(u) + \sum F_{N_2}(v) = d^{F}_{G_2}(uv) + p_1 F_{N_2}(u) + p_1 F_{N_2}(v)$

For any $v \in E'$,

$$d^{T}_{G_1+G_2}(uv) = d^{T}_{G_1}(u) + d^{T}_{G_2}(v) + \sum_{w\neq v} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\neq u} T_{N_1}(v) \wedge T_{N_2}(w)$$

$= d^{T}_{G_1}(u) + d^{T}_{G_2}(v) + \sum_{w\in V_1} T_{N_2}(w) + \sum_{w\in V_1} T_{N_2}(v) - 2T_{N_2}(v)$

$= d^{T}_{G_1}(u) + d^{T}_{G_2}(v) + O^{T}(G_2) + (p_1 - 2)T_{N_2}(v)$

$$d^{I}_{G_1+G_2}(uv) = d^{I}_{G_1}(u) + d^{I}_{G_2}(v) + \sum_{w\neq v} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{w\neq u} I_{N_1}(v) \wedge I_{N_2}(w)$$

$= d^{I}_{G_1}(u) + d^{I}_{G_2}(v) + \sum_{w\in V_1} I_{N_2}(w) + \sum_{w\in V_1} I_{N_2}(v) - 2I_{N_2}(v)$

$= d^{I}_{G_1}(u) + d^{I}_{G_2}(v) + O^{I}(G_2) + (p_1 - 2)I_{N_2}(v)$

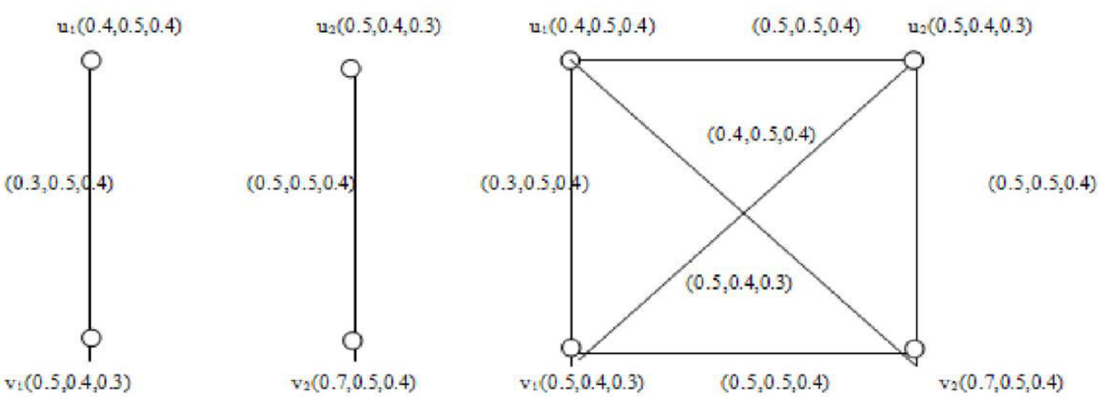
$$d^{F}_{G_1+G_2}(uv) = d^{F}_{G_1}(u) + d^{F}_{G_2}(v) + \sum_{w\neq v} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{w\neq u} F_{N_1}(v) \wedge F_{N_2}(w)$$

$= d^{F}_{G_1}(u) + d^{F}_{G_2}(v) + \sum_{w\in V_1} F_{N_2}(w) + \sum_{w\in V_1} F_{N_2}(v) - 2F_{N_2}(v)$

$= d^{F}_{G_1}(u) + d^{F}_{G_2}(v) + O^{F}(G_2) + (p_1 - 2)F_{N_2}(v)$

Proof of (2) is similar to (1).

Example 4.2 Consider a neutrosophic fuzzy graph on $G^*(V,E)$.



Theorem 4.3. Let G_1: (N_1,M_1) and G_2:(N_2,M_2) where $N_1 = (T_{N_1}, I_{N_1}, F_{N_1})$, $M_1 = (T_{M_1}, I_{M_1}, F_{M_1})$, $N_2 = (T_{N_2}, I_{N_2}, F_{N_2})$, $M_2 = (T_{M_2}, I_{M_2}, F_{M_2})$ be two fuzzy graphs. If $T_{N_1} \wedge T_{N_2}$ and $T_{N_1} \vee T_{N_2}$ are constant functions then

If $d^{T}_{G_1+G_2}(uv) = \begin{cases} d^{T}_{G_1}(uv) + 2cp_2 & uv \in E_1 \\ d^{T}_{G_2}(uv) + 2cp_1 & uv \in E_2 \\ d^{T}_{G_1}(u) + d^{T}_{G_1}(v) + c(p_1 + p_2 - 2) & uv \in E' \end{cases}$

If $d^I_{G_1+G_2}(uv) = \begin{cases} d^I_{G_1}(uv) + 2kp_2 & uv \in E_1 \\ d^I_{G_2}(uv) + 2kp_1 & uv \in E_2 \\ d^T_{G_1}(u) + d^T_{G_1}(v) + k(p_1 + p_2 - 2) & uv \in E' \end{cases}$

If $d^F_{G_1+G_2}(uv) = \begin{cases} d^F_{G_1}(uv) + 2rp_2 & uv \in E_1 \\ d^F_{G_2}(uv) + 2rp_1 & uv \in E_2 \\ d^F_{G_1}(u) + d^F_{G_1}(v) + r(p_1 + p_2 - 2) & uv \in E' \end{cases}$

Proof.

Let $T_{N_1} \wedge T_{N_2}$ = c, $I_{N_1} \vee I_{N_2}$=k, $F_{N_1} \vee F_{N_2}$ = r for any $uv \in E_1$

$d^T_{G_1+G_2}(uv) = d^T_{G_1}(uv) + \sum_{uw\in E'} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{wv\in E'} T_{N_1}(w) \wedge T_{N_2}(v)$

$= d^T_{G_1}(uv) + \sum_{w\neq v} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\neq u} T_{N_1}(v) \wedge T_{N_2}(w)$

$= d^T_{G_1}(uv) + \sum_{w\in V_2} c + \sum_{w\in V_2} c = d^T_{G_1}(uv) + cp_2 + cp_2$

$= d^T_{G_1}(uv) + 2\,cp_2$

$d^I_{G_1+G_2}(uv) = d^I_{G_1}(uv) + \sum_{uw\in E'} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{wv\in E'} I_{N_1}(w) \wedge I_{N_2}(v)$

$= d^I_{G_1}(uv) + \sum_{w\neq v} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{w\neq u} I_{N_1}(v) \wedge I_{N_2}(w)$

$= d^I_{G_1}(uv) + \sum_{w\in V_2} c + \sum_{w\in V_2} c = d^I_{G_1}(uv) + kp_2 + kp_2$

$= d^I_{G_1}(uv) + 2\,kp_2$

$d^F_{G_1+G_2}(uv) = d^F_{G_1}(uv) + \sum_{uw\in E'} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{wv\in E'} F_{N_1}(w) \wedge F_{N_2}(v)$

$= d^F_{G_1}(uv) + \sum_{w\neq v} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{w\neq u} F_{N_1}(v) \wedge F_{N_2}(w)$

$= d^F_{G_1}(uv) + \sum_{w\in V_2} r + \sum_{w\in V_2} r = d^F_{G_1}(uv) + rp_2 + rp_2$

$= d^F_{G_1}(uv) + 2\,rp_2$

for any $uv \in E_2$

$d^T_{G_1+G_2}(uv) = d^T_{G_2}(uv) + \sum_{uw\in E'} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{wv\in E'} T_{N_1}(w) \wedge T_{N_2}(v)$

$= d^T_{G_2}(uv) + \sum_{w\neq v} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\neq u} T_{N_1}(v) \wedge T_{N_2}(w)$

$= d^T_{G_2}(uv) + \sum_{w\in V_1} c + \sum_{w\in V_1} c = d^T_{G_2}(uv) + cp_1 + cp_1$

$= d^T_{G_2}(uv) + 2\,cp_1$

$d^I_{G_1+G_2}(uv) = d^I_{G_2}(uv) + \sum_{uw\in E'} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{wv\in E'} I_{N_1}(w) \wedge I_{N_2}(v)$

$= d^I_{G_2}(uv) + \sum_{w\neq v} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{w\neq u} I_{N_1}(v) \wedge I_{N_2}(w)$

$= d^I_{G_2}(uv) + \sum_{w\in V_1} k + \sum_{w\in V_1} k = d^T_{G_2}(uv) + kp_1 + kp_1$

$= d^I_{G_2}(uv) + 2\,kp_1$

$d^F_{G_1+G_2}(uv) = d^F_{G_2}(uv) + \sum_{uw\in E'} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{wv\in E'} F_{N_1}(w) \wedge F_{N_2}(v)$

$= d^F_{G_2}(uv) + \sum_{w\neq v} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{w\neq u} F_{N_1}(v) \wedge F_{N_2}(w)$

$= d^F_{G_2}(uv) + \sum_{w\in V_1} r + \sum_{w\in V_1} r = d^T_{G_2}(uv) + rp_1 + rp_1$

$= d^F_{G_2}(uv) + 2\,rp_1$

For any $uv \in E'$,

$d^T_{G_1+G_2}(uv) = d^T_{G_1}(u) + d^T_{G_2}(v) + \sum_{uw\in E'} T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{wv\in E'} T_{N_1}(w) \wedge T_{N_2}(v)$

$= d^T_{G_1}(u) + d^T_{G_2}(v) + \sum_{w\in V_2} c - T_{N_1}(u) \wedge T_{N_2}(w) + \sum_{w\in V_1} c - T_{N_1}(v) \wedge T_{N_2}(w)$

$= d^T_{G_1}(u) + d^T_{G_2}(v) + p_1 c + p_2 c - 2c = d^T_{G_1}(u) + d^T_{G_2}(v) + (p_1 + p_2 - 2c)$

$d^I_{G_1+G_2}(uv) = d^I_{G_1}(u) + d^I_{G_2}(v) + \sum_{uw\in E'} I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{wv\in E'} I_{N_1}(w) \wedge I_{N_2}(v)$

$= d^I_{G_1}(u) + d^I_{G_2}(v) + \sum_{w\in V_2} k - I_{N_1}(u) \wedge I_{N_2}(w) + \sum_{w\in V_1} k - I_{N_1}(v) \wedge I_{N_2}(w)$

$= d^I_{G_1}(u) + d^I_{G_2}(v) + p_1 k + p_2 k - 2k = d^I_{G_1}(u) + d^I_{G_2}(v) + (p_1 + p_2 - 2k)$

$d^F_{G_1+G_2}(uv) = d^F_{G_1}(u) + d^F_{G_2}(v) + \sum_{uw\in E'} F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{wv\in E'} F_{N_1}(w) \wedge F_{N_2}(v)$

$= d^F_{G_1}(u) + d^F_{G_2}(v) + \sum_{w\in V_2} r - F_{N_1}(u) \wedge F_{N_2}(w) + \sum_{w\in V_1} r - F_{N_1}(v) \wedge F_{N_2}(w)$

$= d^F_{G_1}(u) + d^F_{G_2}(v) + p_1 r + p_2 r - 2r = d^F_{G_1}(u) + d^F_{G_2}(v) + (p_1 + p_2 - 2r)$

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