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# ANALYZING THE RETAILER'S IDEAL REPLENISHMENT STRATEGY, WHICH MINIMIZES THE PRESENT VALUE OF TOTAL INVENTORY COSTS

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## ABSTRACT

*There is too much rivalry in the market, and every company/organization wants to boost sales of its goods. Therefore, the organization/firm contracts with an advertising business to improve the product's popularity among consumers, and as a result, the costs associated with advertising become a cost component of an inventory system. This Paper develops a two-warehouse inventory model for deteriorating goods and examines the impact of inflation while taking time-dependent demand with scarcity into account. In addition to the other cost elements of the inventory system, an advertising cost has also been incorporated in this model. The model is created with shortages in mind, taking into account that a portion of the supply is made available when needed in the next cycle. This study's goal is to determine the retailer's ideal replenishment strategy, which minimizes the present value of total inventory costs. The planned and created model is then validated using numerical examples. The results of a sensitivity analysis are provided, along with potential management ramifications.*

*Keywords: Inflation, Two-Warehouse Inventory Model, Deteriorating, Time, Demand and Shortages*

## INTRODUCTION

A quantity of items kept for use in manufacturing or selling constitutes an inventory. All economic sectors, including business, industry, agriculture, defense, etc., have similar logistical systems that include inventory management of tangible objects and other products or parts. In a completely predictable economy, inventory may be required to take advantage of a certain technology's economic benefits, coordinate human activities, or modify the manufacturing process to accommodate shifting demand patterns. Inventory management serves as a safeguard against the possibility of stockouts when uncertainty is prevalent.

Inventory in a system often denotes the presence of a well-organized, intricate system that involves the input, accumulation, and outflow of various commodities, goods, things, or products. Inventory management and control must take place within the framework of this structured system. Therefore, inventories should be seen as a highly important component, rather than as useless resources, whose research may provide light on how the system as a whole function. The degree of interaction between intake, accumulation, and outflow is defined by the study of the inventory system, which also specifies economic management methods for managing such systems. Continuous review systems and periodic review systems are two primary categories of inventory systems. Systems are continually monitored in continuous review systems, whereas they are periodically checked at discrete, evenly spaced points in time.

A warehouse is a commercial structure used to store goods like food, raw materials, fresh produce, and other items that are purchased in large quantities and degrade over time. Before storing these items, a seller must first separate the degraded item from the good or clean one. Otherwise, the spoilt person will have an impact on the components of the excellent class. In industrialized areas like municipalities, townships, and villages, warehouses are often big, plain structures that suppliers or businesses utilize. On occasion, warehouses are intended for filling rather than simple supply filling because of railroads and airports.

Cranes were regularly employed to hoist the goods. Any raw materials, packaged goods, spare components, or refined goods associated to agriculture, development, and building might be considered as stoke supplies. Because no company can tolerate a situation of stock-outs or shortages, suppliers buy items in bulk. There are many items on the market that have extra features. Therefore, in order to meet market demand, we must buy goods in large quantities. As a result, the provider offers a discount for purchasing large amounts of something within a certain time frame. We first keep the items in a small, own warehouse (OW), and only the surplus is subsequently stored in a larger, leased or borrowed warehouse (RW).

## LITERATURE REVIEW

**Rahaman, Mostafijur et.al (2021)** The novel mathematical approach for choosing the best ordering strategy for industrial and commercial businesses is presented in this study. Numerous inventory models under inflationary situations have been created in earlier studies. These models often assume that the demand rate is steady and well-known, fluctuating over time, stock-dependent, or price-dependent. But in the actual world, the demand rate is often unpredictable. As a result, under stochastic demand circumstances, novel inflationary inventory models have been constructed in this work. The inventory system is currently in a multi-item, budget-constrained condition. Additionally, numerical examples have been provided to support and explain the theoretical findings.

**Kumar, Dr et.al. (2022).** In this work, a two-warehouse inventory model with multivariable demand and an allowed payment delay for decaying goods is created. For the majority of the models, the academics assumed a single warehouse with a set capacity, however this is unrealistic for huge stocks. As a result, we took into account two warehouses with plenty of capacity for this document. The product deterioration rates in the two warehouses are viewed separately. It should be assumed that the inventory expenses in a leased warehouse are higher than those in an own warehouse. We also spoke about how the ideal replenishment strategy is both unique and existent.

**Agarwal, Anchal et.al. (2018).** In this paper, we suggested a model of economic order quantity for long-lasting degrading goods with exponential demand rates under inflation. Deterioration rate in this research follows the two parameters. Linear dependence exists between Weibull degradation and holding cost. A common presumption among researchers is that the shortfalls are either totally backlogged or lost. However, in this case, paper shortages are accepted and there is some backlog. It is assumed that the backlog rate is an exponential function of time. In this work, lifetime degradation is taken into account. There are provided numerical examples that explain how the theoretical conclusions are applied and how their numerical verifications work. Sensitivity analysis has been used to examine how changing different parameters may affect the results. This model seeks to reduce the overall optimum cost.

**Kumar, Satish et.al. (2018).** In this research, an integrated production/inventory model is developed from the viewpoints of the seller, the supplier, and the purchaser. For the seller, the supplier, and the customer, the demand rate is time-dependent, and the stock-dependent demand rate is assumed. Supplier stores extra inventory in two warehouses (one owned, one leased) in accordance with demand. Only on the part of the buyer are shortages permitted, and some of the unmet demand is backlogged. Also taken into consideration is how defective manufacturing methods affect lot sizes. This whole paradigm is investigated in relation to inflation. The goal is to keep the system's overall cost as low as possible. To discover a nearly ideal solution for the model, a solution process is created. To demonstrate the concept, a numerical example and sensitivity analysis are provided.

**Singhal, Shruti et.al. (2022).** In this study, a two-warehouse inventory strategy with stock- and selling-price-dependent demand has been created. In today's cutthroat commercial transactions, the supplier gives the merchant a specific amount of time to pay the account in order to encourage future orders. The holding cost varies depending on the warehouse and is linearly time dependent. Permitted shortages with some backlog. Two numerical examples are offered as a last step to support the idea. Subject Nomenclature: 90B05

## DEVELOPMENT OF MATHEMATICAL MODEL

A lot size of inventory was present at time t=0 in the beginning. $Q_{max}$ backlogged information is put into the system and provided. W units are retained in OW from the leftover stocks of R units, and the remaining inventory is kept in RW. To lower the overall inventory cost per unit of time, it is assumed that merchandise kept in RW would be used up first. Demand and degradation cause the RW's stock to diminish until it reaches zero within the term. [0 $t_w$] And throughout this time, only deterioration is the cause of the inventory level in OW declining. Due to the combined effects of demand and degradation over time, the stock of OW decreases. [$t_w$ $t_r$]. In figure 1, the model is visually shown.

The following differential equations control how quickly inventory changes during positive stock and the aforementioned times.

$$\frac{dI_r(t)}{dt} = -\alpha I_r(t) - f(t);$$

$$\frac{dI_o(t)}{dt} = -\beta I_o(t);$$

$$\frac{dI_2(t)}{dt} = -\beta I_o(t) - f(t);$$

Using boundary conditions, equations solution is achieved. $I_r$ (t) =0, *at t=* $t_w$, *$I_o(t)$=W at t=0 and $I_o$ (t)= 0 at t= $t_r$* and are given as follows:

$$I_r(t) = \frac{\alpha}{\beta+\lambda}\{(e^{(\beta+\lambda)t_r} - e^{(\beta+\lambda)t}\}e^{-\beta t};$$

$$I_o(t) = W_{e^{-\alpha t}}$$

$$I_o(t) = \frac{\alpha}{\alpha+\lambda}\{(e^{(\alpha+\lambda)t_w} - e^{(\alpha+\lambda)t}\}e^{-\beta t};$$

Continual demand and degradation cause shortages throughout the time period [$t_r$ T] because there is a backlog of demand. The following differential equation governs the process:

$$\frac{dI_s(t)}{dt} = -B(t)f(t);$$

the answer to equation (7) using B.C. $I_o(t) = 0$ at $t = t_w$

$$I_s(t) = \frac{\alpha}{\alpha+\lambda}\{(e^{(\lambda+\sigma)t_w} - e^{(\lambda+\sigma)t}\};$$

Continuousness at $t = t_r$ $i.e I_o(t) = I_o(t)$ yields

$$t_w = \frac{1}{\alpha+\lambda}\log\{\frac{W(\alpha+\lambda)}{\alpha} + e^{(\alpha+\lambda)t_r}$$

it suggests that $t_w$ is a function of $t_r$ i.e.

$$t_w = f(t_r)$$

On eq differentiation. w. r. t. $t_r$, we get

$$\frac{df(t_r)}{dt_r} = \frac{We^{(\alpha+\lambda)t_r}}{(\alpha+\lambda)^2\{W(\alpha+\lambda) + ae^{(\alpha+\lambda)t_r}\}} < 1$$

At t = 0 we have, $I_r$ (0) = R -W which gives

$$R = W + \frac{\alpha}{\beta+\lambda}\{e^{(\alpha+\lambda)t_r}-1\};$$

Maximum inventory units requested at the conclusion of the whole cycle are provided by

$$Q_{max} = I_o(t) + W + (-I_S(T))$$

$$W + \frac{a}{\beta+\lambda}\{e^{(\beta+\lambda)t_r}-1\} + \frac{a}{\lambda+\sigma}\{e^{(\lambda+\sigma)T}-e^{(\lambda+\sigma)t_r}\}$$

The quantity of inventories that degraded in RW throughout the time of positive stock is

$$D_r = \alpha \int_0^{t_r} I_r(t)\,dt$$

$$= \frac{a}{\beta+\lambda}\{e^{(\beta+\lambda)t_r}-1\}-\left(\frac{a}{\lambda}\{e^{\lambda t_r}-1\}\right)$$

When OW had good stock levels, the quantity of inventories that had deteriorated was

$$D_o = \beta \int_0^{t_r} I_o(t)\,dt + \int_0^{t_w} I_o(t)\,dt$$

$$= W - \left(\frac{a}{\lambda}\{(e^{\lambda t_W}-e^{\lambda r}\}\right)$$

The following elements make up the inventory system's current worth inventory cost per unit time:

(i) The ordering cost's current value is provided by

$C_o = A\ e^{-rt}$

The current worth inventory holding cost in RW is calculated as follows:

$$H_r = \int_0^{t_r} h_r I_r(t)\, e^{-rt} dt$$

$$= \int_0^{t_r} h_r \frac{a}{\beta+\lambda}\{\left(e^{(\beta+\lambda)t_r} - e^{(\beta+\lambda)t}\right\}e^{-\beta t}e^{-rt}dt$$

Given is the current value inventory holding cost in OW.

$$H_o = \int_0^{t_r} h_o I_o(t)\, e^{-rt}dt \int_0^{t_r} h_o I_o(t)\, e^{-rt}dt$$

$$= \int_0^{t_r} h_o W\, e^{-at} e^{-rt}dt + \int_0^{T} \frac{a}{\alpha+\lambda}\left(e^{(\alpha+\lambda)t_W} - e^{(\alpha+\lambda)t}\right\}\, e^{\beta t}\, e^{-rt}\,dt$$

(ii) The following is the current value inventory degradation cost in RW:

$$H_{rc} = d_r e^{-rt}\left(\alpha \int_0^{t_r} I_r(t)dt\right)$$

$$= d_r e^{-rt}\left(\alpha \int_0^{t_r} \frac{a}{\alpha+\lambda}\{(e^{(\beta+\lambda)t_r} - e^{(\beta+\lambda)t}\}e^{-rt}dt\right)$$

(iii) In OW, the cost of current worth inventory degradation is computed as

$$H_{rc} = d_o e^{-rt}\beta\left\{\int_0^{t_r} I_o(t)dt + \int_0^{t_w} I_o(t)dt\right\}$$

$$= d_o e^{-rt}\beta\left\{\int_0^{t_r} We^{-at}dt + \int_0^{t_w}\frac{a}{\alpha+\lambda_o}\{e^{(\alpha+\lambda)t_w} - e^{(\alpha+\lambda)}\}e^{\beta t}dt\right\}$$

Provided

$$\left(\frac{\partial^2 \hat{S}(t_r,T)}{\partial t_r^2}\right)\left(\frac{\partial^2 \hat{S}(t_r,T)}{\partial T^2}\right) - \left(\frac{\partial^2 \hat{S}(t_r,T)}{\partial t_r \partial T}\right)^2 > 0$$

$$\left(\frac{\partial^2 \hat{S}(t_r,T)}{\partial t_r^2}\right) > 0; \left(\frac{\partial^2 \hat{S}(t_r,T)}{\partial T^2}\right) > 0\ at(\overline{T_1},\ \overline{T_1})$$

Two instances are taken into consideration and shown individually in Table-1 to demonstrate the applicability of the approach. The parameter values are picked at random, and the outcomes are shown in Table 2.

**Table-1:** Representing Examples with Different Value of Parameters

| Parameter | A | $\lambda$ | W | $C_o$ | $S_c$ | $L_c$ | $d_r$ | $d_o$ | $\beta$ | $\alpha$ | $\sigma$ | $h_r$ | $h_o$ | $C_a$ | $t_v$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Example-1 | 50 | 0.3 | 300 | 1000 | 5.0 | 15 | 2 | 3 | 0.003 | 0.013 | 0.6 | 30 | 25 | 500 | 10 |
| Example-2 | 50 | 0.3 | 300 | 1200 | 4.5 | 15 | 3 | 5 | 0.004 | 0.017 | 1.5 | 50 | 35 | 600 | 8.0 |

**Table-2:** Representing Value of Decision Variables; Ordered Quantity and Cost Function

| | $t_r$ | $t_w$ | $\tilde{T}$ | $Q_{max}$ | $\hat{S}(t_r, T)$ |
|---|---|---|---|---|---|
| Example-6.1 | 0.2712 | 1.5135 | 7.6809 | 622 | 928.0 |
| Example-6.2 | 0.6240 | 0.9452 | 8.6181 | 715 | 244.13 |

The Constructed Model Is Highly Non-Linear Therefore It Is Solved With The Help Of The Values Of The Choice Variables Are Derived Using The Mathematica-9.0 Program. Figures 2(A)-2(D) And 3(A)-3(D) Respectively, Which Use Time-Inventory Cost Graphs, Illustrate The Convexity Of The Model For Examples 1 And Example 2.

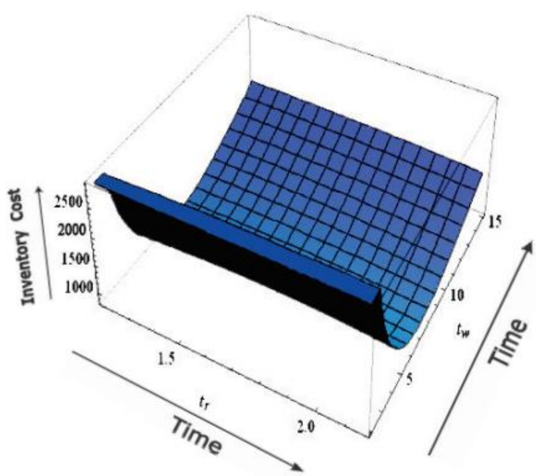


**Figure-2a:** Graphical representation of cost function depicting convexityw.r.t$t_r$ $_{and}$ $t_w$ for Inventory System

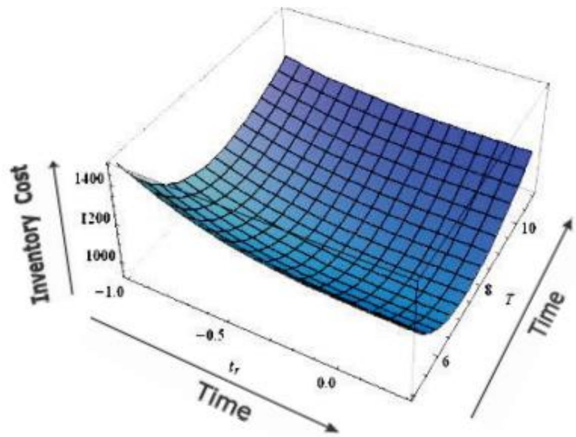


**Figure-2b:** Graphical representation of cost function depicting convexity w.r.t $t_{r\ and}$ $T$ for Inventory System

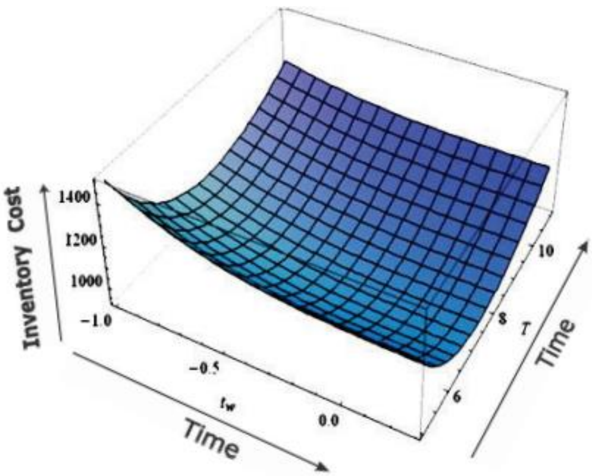


**Figure-2c:** Graphical representation of cost function depicting convexity w.r.t $t_{W\ and}$ $T$ for Inventory System

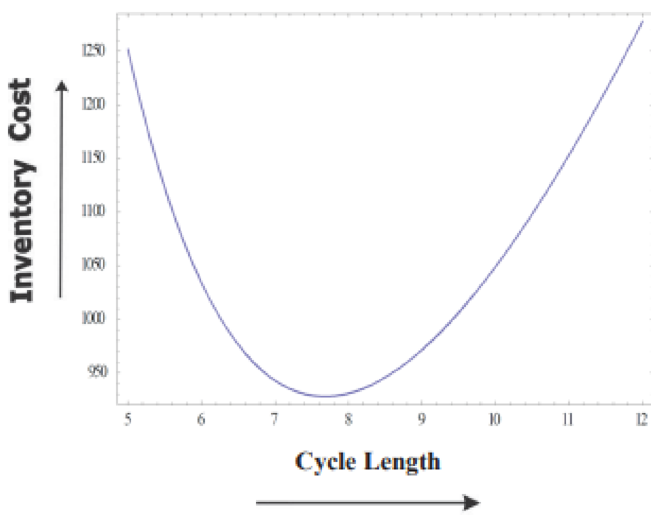


**Figure-2d:** Graphical representation depicting Inventory Level Vs. Cycle Length

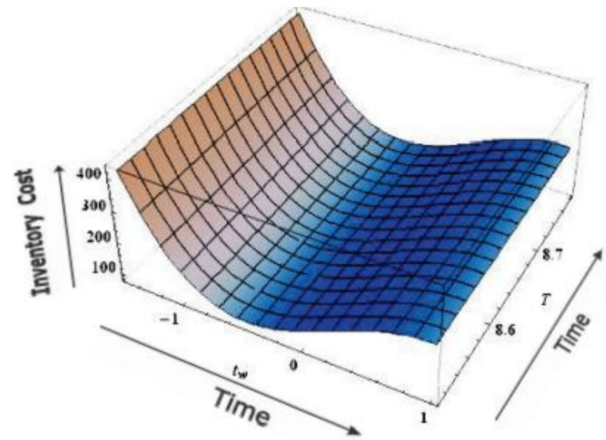


**Figure-3a:** Graphical representation of cost function depicting convexity w.r.t $t_{W\ and}$ $T$ for Inventory System

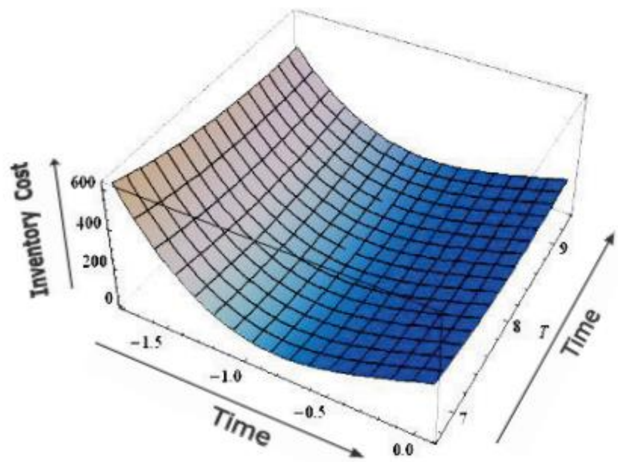


**Figure-3b:** Graphical representation of cost function depicting convexity w.r.t $t_{r\ and}$ $T$ for Inventory System

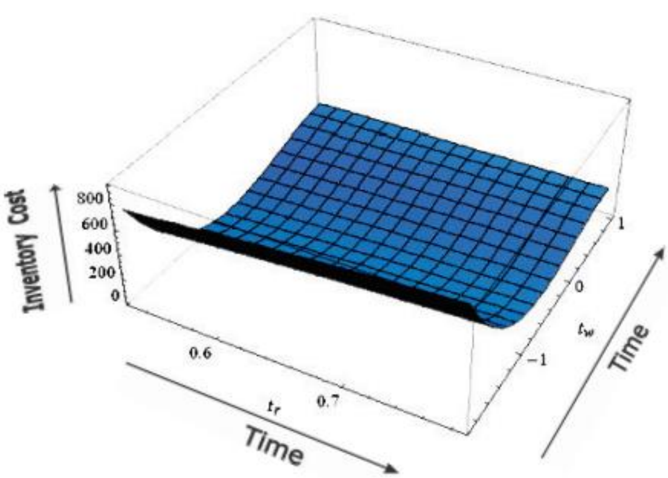


**Figure-3c:** Graphical representation of cost function depicting convexity w.r.t $t_{r\ and}$ $t_w$ for Inventory System

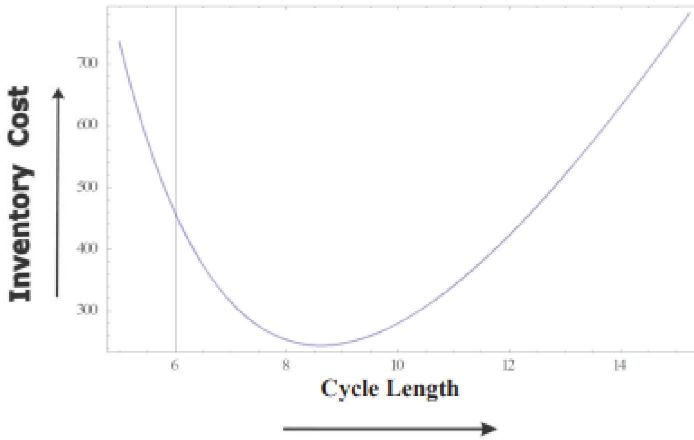


**Figure-3d:** Graphical representation depicting Inventory Level Vs. Cycle Length

## SENSITIVITY ANALYSIS

The sensitivity analysis is performed with respect to each parameter changing the value up to ∓ 10 % in one parameter at a time and keeping the value of remaining parameters fixed. The corresponding percentage change in the cycle length and the total relevant inventory cost per unit of time is obtained and is represented in Table-3.

**Table-3:** Representing Sensitivity Analysis with respect to parameters to study percentage change in cycle length and cost function

| a=50 | $t_{\tilde{r}} =$ | $t_{\tilde{w}} =$ | $\tilde{T} =$ | % change in $\tilde{T}$ | $\hat{S}(t_r, T)$ | % change in $\hat{S}(t_r, T)$ |
|---|---|---|---|---|---|---|
| 40 | Infeasible solution | | | | | |
| 60 | | | | | | |
| r=0.4 | | | | | | |
| 0.38 | 0.5499 | 1.4419 | 7.7165 | 0.46 | 1081.92 | 16.59 |
| *0.44* | *0.0121* | *1.4672* | *7.5016* | -2.33 | *618.91* | -33.31 |
| $C_o = 1000$ | | | | | | |
| 900 | 0.2745 | 1.5135 | 7.6518 | -0.38 | 954.96 | 2.91 |
| 1100 | 0.2679 | 1.5135 | 7.7097 | 0.38 | 900.99 | -2.91 |
| $h_r = 30$ | | | | | | |
| 27 | 0.2145 | 1.5527 | 7.7062 | 0.33 | 992.61 | 0.17 |
| 32.5 | 0.3992 | 1.4228 | 7.6250 | -0.73 | 925.54 | -0.27 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $h_o = 25$ | | | | | | |
| 22.5 | 0.2365 | 1.4699 | 7.4737 | -2.69 | 845.69 | -8.87 |
| 27.5 | 0.2844 | 1.5588 | 7.8878 | 2.69 | 1009.80 | 8.82 |
| $S_c = 5$ | | | | | | |
| 4.5 | 0.3474 | 1.5078 | 7.6481 | -0.43 | 827.40 | -10.84 |
| 5.5 | 0.2083 | 1.5110 | 7.7090 | 0.37 | 928.53 | 0.057 |
| $L_c = 15$ | | | | | | |
| 13.5 | 0.2535 | 1.5134 | 8.1120 | 5.61 | 756.99 | -18.43 |
| 16.5 | 0.2827 | 1.5133 | 7.3041 | -4.91 | 1089.66 | 17.42 |
| $t_a = 500$ | | | | | | |
| 400 | 0.3067 | 1.5121 | 7.3851 | -3.85 | 1195.26 | 28.79 |
| 600 | 0.2401 | 1.5131 | 7.9647 | 3.69 | 655.834 | -29.33 |
| $t_v = 10$ | | | | | | |
| 9 | 0.2883 | 1.5131 | 7.5346 | -1.90 | 1062.28 | 14.47 |
| 11 | 0.2552 | 1.5135 | 7.8242 | 1.87 | 792.50 | -14.60 |
| $W = 300$ | | | | | | |
| 270 | Infeasible | | | | | |
| 315 | 0.0799 | 1.6429 | 7.7838 | 1.34 | 941.98 | 1.51 |
| $\alpha = 0.013$ | | | | | | |
| 0.0117 | 0.2926 | 1.5051 | 7.6826 | 0.02 | 930.89 | 0.31 |
| 0.0143 | 0.2512 | 1.5210 | 7.6785 | -0.03 | 925.07 | -0.32 |
| $\beta = 0.003$ | | | | | | |
| 0.0027 | 0.2713 | 1.5134 | 7.6808 | -0.0013 | 928.00 | 0 |
| 0.0033 | 0.2711 | 1.5136 | 7.6809 | 0 | 928.01 | 0.001 |
| $\sigma = 0.6$ | Infeasible | | | | | |
| 0.54 | | | | | | |
| 0.66 | | | | | | |
| $\lambda$=0.3 | \ | | | | | |
| 0.27 | | | Infeasible | | | |
| 0.33 | 0.1219 | 1.6491 | 7.7400 | 0.769441 | 922.44 | -0.59 |

Note: Because W=300 and $h\%$ = 10, (+10%), these two cases are unreasonable, therefore sensitivity analysis is performed by changing +5% at a time

The following conclusions may be drawn from Table 3: It has been shown that the model is very sensitive to and closely tied to both the rate of inflation and the price of advertising. The holding cost in RW, ordering cost, and deterioration rate in both warehouses are only slightly sensitive to the model and only moderately sensitive to other variables. The length of the ordering cycle is only marginally responsive to the other components, but it is quite sensitive to the cost of a missed sale. The model is significantly influenced by the demand parameter and the potential cost of lost sales.

## CONCLUSION

The author suggests an inventory model for deteriorating items in this Paper that may be used to calculate the ideal replenishment cycle duration for two-warehouse management with inflation. Backlogs of shortages are in part cleared. The model accounts for time-dependent advertising costs. Since inventory costs fall when inflation and advertising spending rise, the model is realistically beneficial.

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