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FIBONACCI CORDIAL LABELING OF HERSCHEL GRAPH IN CONTEXT OF VARIOUS GRAPH OPERATIONS

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ABSTRACT

An injective function φ: V (G) → {F_0, F_1, F_2, . . . , F_n}, where F_j is the jth Fibonacci number (j = 1, . . . , n), is said to be Fibonacci product cordial labeling if the induced function φ : E (G) → {0, 1} defined by $\varphi*(uv) = (\varphi(u) + \varphi(v))(\bmod 2)$ satisfies the condition $|e\varphi*(0) - e\varphi*(1)| \leq 1$. A graph which admits Fibonacci product cordial labeling is called Fibonacci product cordial graph. In this paper we investigated the Fibonacci product cordial labeling of Herschel graph and various graph operations on it.*

In this paper we proved the following results:

(1). The Herschel graph Hs is a Fibonacci product cordial graph.

(2). The fusion of any two adjacent vertices of degree 3 in the Herschel graph is a Fibonacci product cordial graph.

(3). The duplication of any vertex in a Herschel graph is a Fibonacci product cordial graph.

(4). The switching of a central vertex v in the Herschel graph Hs is a Fibonacci product cordial graph.

(5). The graph obtained by joint of two copies of Herschel graph Hs is a Fibonacci product cordial graph.

(6). DS(Hs) is Fibonacci product cordial graph.

Keywords: Fibonacci cordial labeling, fusion, duplication, switching, joint sum, degree splitting.

AMS Mathematics Subject Classification (2010): 05C78.

INTRODUCTION

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory we refer to Harary[3] . A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to product cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.

For dynamic survey of various graph labeling, we refer to Gallian [] . The concept of Fibonacci cordial labeling was introduced by A. H. Rokad and G. V. Ghodasara[1].

Definition 1. A Herschel graph H_s is a bipartite undirected graph with 11 vertices and 18 edges.

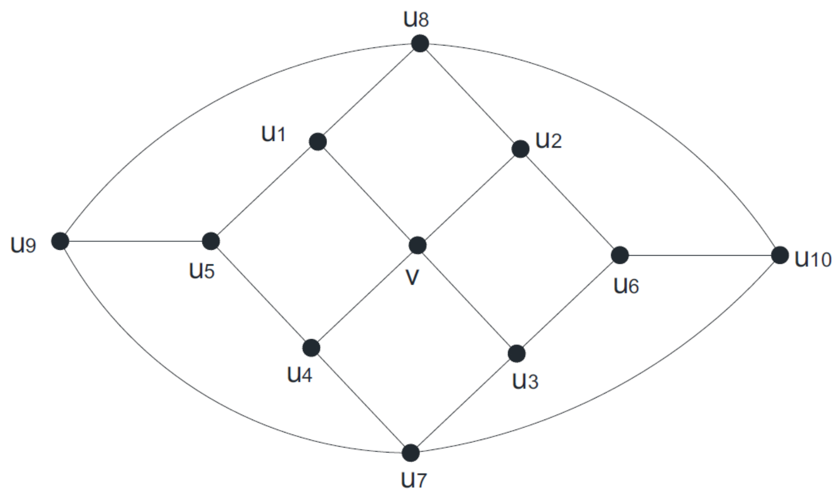

Figure (A)

In this paper, we always fix the position of vertices $v, u_1, u_2, ..., u_{10}$ of H_s as indicated in the above Figure (A), unless or otherwise specified.

Definition 2. Let u and v be two distinct vertices of a graph G . A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 3. Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k) = N(v_k')$. In other words, a vertex v_k' is said to be a duplication of vk if all the vertices which are adjacent to v_k are now adjacent to v_k'.

Definition 4. The graph obtained by connecting a vertex of first copy of a graph G with a vertex of second copy of a graph G is called joint sum of two copies of G.

Definition 5. Let G = (V(G), E(G)) be a graph with $V = S_1 \cup S_2 \cup S_3 \cup \ldots S_i \cup T$ where each Si is a set of vertices having at least two vertices of the same degree and $T = V \setminus \cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of Si for $1 \leq i \leq t$.

Main Results

Theorem 1. The Herschel graph H_S is a Fibonacci cordial graph.

Proof: Let $G = H_S$ be a Herschel graph and let v be the central vertex and u_i $(1 \leq i \leq 10)$ be the remaining vertices of the Herschel graph. Then $|V(G)| = 11$ and $|E(G)| = 18$.

We define the labeling f: V (G) $\rightarrow \{1, 2, ..., |V(G)|\}$ as follows.

$f(v) = F_1$, $f(u_1) = F_{10}$, $f(u_2) = F_6$,

$f(u_3) = F_4$, $f(u_4) = F_7$, $f(u_5) = F_8$,

$f(u_6) = F_2$, $f(u_7) = F_9$, $f(u_8) = F_3$,

$f(u_9) = F_{11}$, $f(u_{10}) = F_5$.

From the above labeling pattern, we have $e_f(0) = e_f(1) = 9$.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Thus Herschel graph H_S is a Fibonacci cordial graph.

Example 1. Fibonacci cordial labeling of Herschel graph H_S is shown in Figure 1.

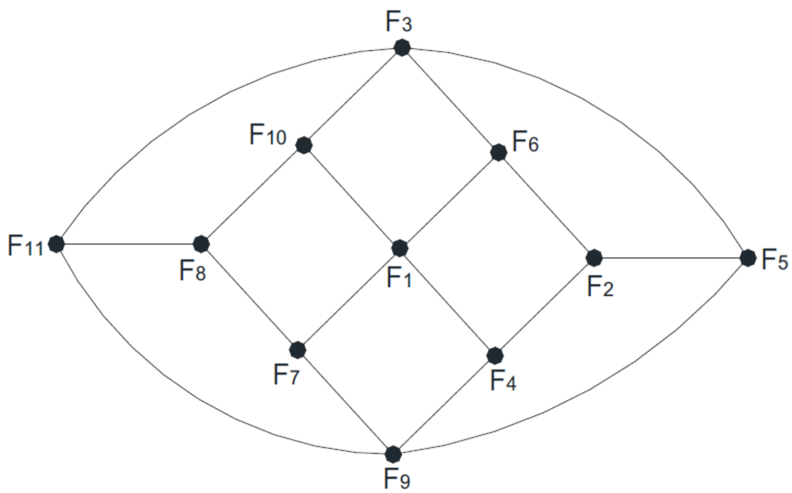

Figure 1

Theorem 2. The fusion of any two adjacent vertices of degree 3 in the Herschel graph is a Fibonacci product cordial graph.

Proof: Let H_S be the Herschel graph with $|V(H_S)| = 11$ and $|E(H_S)| = 18$. Let v be the central vertex of the Herschel graph and it has 3 vertices of degree 4 and 8 vertices of degree 3. Let G be the graph obtained by fusion of two adjacent vertices of degree 3 inthe Herschel graph of H_S.

Then $|V(G)| = 10$ and $|E(G)| = 17$.

We define the vertex labeling

$f: V(G) \rightarrow \{1, 2, ..., |V(G)|\}$ as follows.

Case 1. Fusion of u_2 and u_6.

Suppose that u_2 and u_6 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_2$,

$f(u_1) = F_6$, $f(u_3) = F_9$, $f(u_4) = F_7$,

$f(u_5) = F_8$, $f(u_7) = F_5$, $f(u_8) = F_3$,

$f(u_9) = F_{10}$, $f(u_{10}) = F_4$.

Case 2. Fusion of u_3 and u_6.

Suppose that u_3 and u_6 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_3$,

$f(u_1) = F_8$, $f(u_2) = F_9$, $f(u_4) = F_7$,

$f(u_5) = F_6$, , $f(u_7) = F_3$, $f(u_8) = F_2$,

$f(u_9) = F_{10}$, $f(u_{10}) = F_5$.

Case 3. Fusion of u_6 and u_{10}.

Suppose that u_6 and u_{10} are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_4$,

$f(u_1) = F_6$, $f(u_2) = F_2$, $f(u_3) = F_9$,

$f(u_4) = F_7$, $f(u_5) = F_8$, $f(u_7) = F_5$,

$f(u_8) = F_3$, $f(u_9) = F_{10}$.

Case 4. Fusion of u_5 and u_9.

Suppose that u_5 and u_9 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_4$,

$f(u_1) = F_2$, $f(u_2) = F_6$, $f(u_3) = F_7$,

$f(u_4) = F_9$, $f(u_6) = F_8$, $f(u_7) = F_5$,

$f(u_8) = F_3$, $f(u_{10}) = F_{10}$.

Case 5. Fusion of u_4 and u_5.

Suppose that u_4 and u_5 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_4$,

$f(u_1) = F_9$, $f(u_2) = F_8$, $f(u_3) = F_7$,

$f(u_6) = F_6$, $f(u_7) = F_3$, $f(u_8) = F_2$,

$f(u_9) = F_5$, $f(u_{10}) = F_{10}$.

Case 6. Fusion of u_1 and u_5.

Suppose that u_5 and u_1 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_2$,

$f(u_2) = F_6$, $f(u_3) = F_7$, $f(u_4) = F_9$,

$f(u_6) = F_8$, $f(u_7) = F_5$, $f(u_8) = F_3$,

$f(u_9) = F_4$, $f(u_{10}) = F_{10}$.

Hence $|e_f(0) – e_f(1)| \leq 1$.

Hence, the fusion of any two adjacent vertices of degree 3 in the Herschel graphis a Fibonacci product cordial graph.

Example 2. Fibonacci product cordial labeling of fusion of u_6 and u_{10} in H_S is shown in Figure 2.

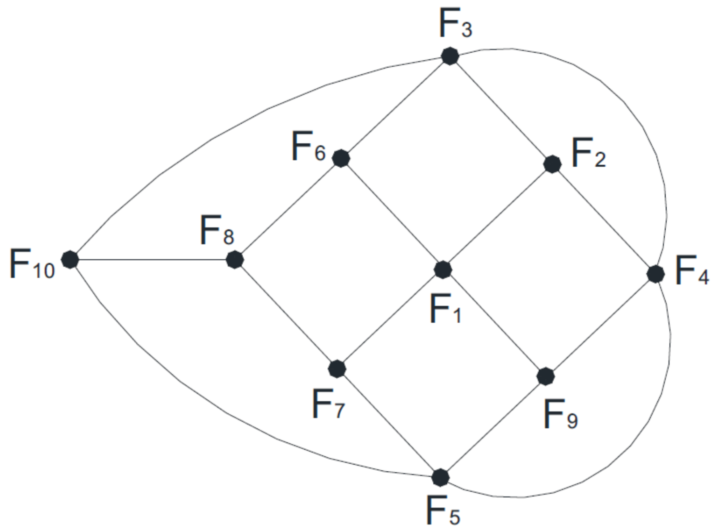

Figure 2

Theorem 3. The duplication of any vertex in a Herschel graph is a Fibonacci product cordial graph.

Proof: Let H_S be the Herschel graph with $|V(H_S)|= 11$ and $|E(H_S)|= 18$. Let v be the central vertex and $u_k{}'$ be the duplication of the vertex u_k in the Herschel graph H_s. Let G be the graph obtained by duplicating the vertex u_k of degree 3 and degree 4 in H_s.

Then $|V(G)| = 12$ and $|E(G)| = 21$.

We define the vertex labeling f: V (G) → {1, 2,...,|V (G)|} as follows.

Case 1. Duplication of vertex u_k, where $k = 1,2,3,4,6,10$.

$f(v) = F_1, f(u_k{}') = F_{12}$,

$f(u_1) = F_{10}$, $f(u_2) = F_6$,

$f(u_3) = F_4$, $f(u_4) = F_7$, $f(u_5) = F_8$,

$f(u_6) = F_2$, $f(u_7) = F_9$, $f(u_8) = F_3$,

$f(u_9) = F_{11}$, $f(u_{10}) = F_5$.

Case 2. Duplication of vertex u_k, where $k = 5, 9$.

$f(v) = F_1, f(u_k{}') = F_{12}$,

$f(u_1) = F_6$, $f(u_2) = F_{10}$, $f(u_3) = F_7$,

$f(u_4) = F_4$, $f(u_5) = F_2$, $f(u_6) = F_8$,

$f(u_7) = F_9$, $f(u_8) = F_3$, $f(u_9) = F_5$,

$f(u_{10}) = F_{11}$.

Case 3. Duplication of vertex v and u_7.

$f(v) = F_1, f(u_k{}') = F_{12}$,

$f(u_1) = F_{10}$, $f(u_2) = F_6$, $f(u_3) = F_4$,

$f(u_4) = F_9$, $f(u_5) = F_7$, $f(u_6) = F_2$,

$f(u_7) = F_3$, $f(u_8) = F_{11}$, $f(u_9) = F_8$,

$f(u_{10}) = F_5$.

Case 4. Duplication of vertex u_8.

$f(v) = F_1, f(u_k') = F_{12}$,

$f(u_1) = F_6$, $f(u_2) = F_{10}$, $f(u_3) = F_9$,

$f(u_4) = F_4$, $f(u_5) = F_2$, $f(u_6) = F_7$,

$f(u_7) = F_{11}$, $f(u_8) = F_3$, $f(u_9) = F_8$,

$f(u_{10}) = F_5$.

From all the above cases we have $|e_f(0) - e_f(1)| \leq 1$.

Hence, the duplication of any vertex in a Herschel graph is a Fibonacci cordial graph.

Example 3. Fibonacci cordial labeling of the duplication of the vertex u_{10} in H_S is shown in Figure 3.

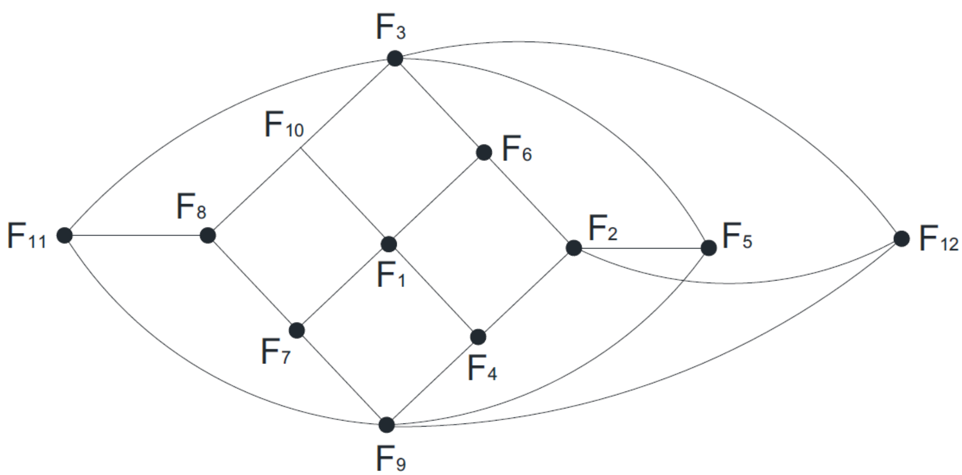

Figure 3

Theorem 4. The switching of a central vertex v in the Herschel graph H_s is a Fibonacci cordial graph.

Proof: Let H_S be the Herschel graph with $|V(H_S)| = 11$ and $|E(H_S)| = 18$. Let v be the central vertex and G be the new graph obtained by switching the central vertex v. Then $|V(G)| = 11$ and $|E(G)| = 20$.

We define the labeling $f: V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v) = F_1$, $f(u_1) = F_{10}$, $f(u_2) = F_6$,

$f(u_3) = F_4$, $f(u_4) = F_7$, $f(u_5) = F_8$,

$f(u_6) = F_2$, $f(u_7) = F_9$, $f(u_8) = F_3$,

$f(u_9) = F_{11}$, $f(u_{10}) = F_5$.

From the above labeling pattern, we observe that $e_f(0) = e_f(1) = 10$.

Hence, $|e_f(0) - e_f(1)| \leq 1$.

Thus, the switching of a central vertex v in the Herschel graph H_s is a Fibonacci product cordial graph.

Example 4. Fibonacci product cordial labeling of switching of a central vertex v in H_S is shown in Figure 4.

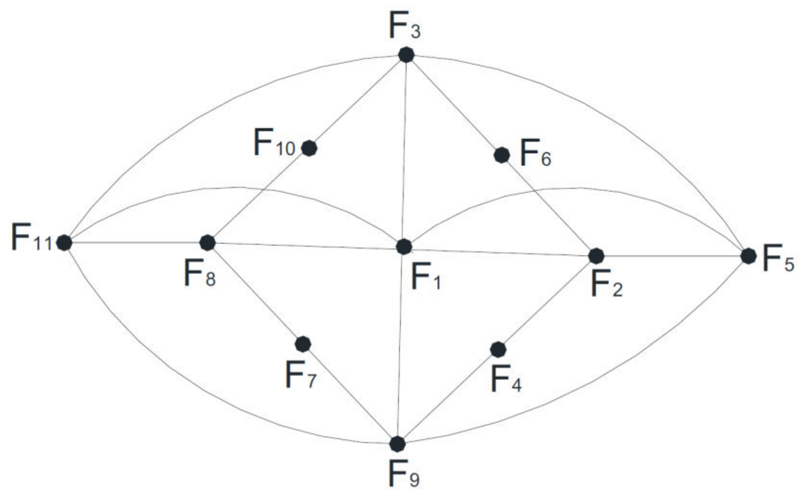

Figure 4

Theorem 5. The graph obtained by joint of two copies of Herschel graph H_s is a Fibonacci product cordial graph.

Proof . Let G be the joint of two copies of Herschel graph H_s. Let $\{u, u_1, u_2, \ldots, u_{10}\}$ and $\{v, v_1, v_2, \ldots, v_{10}\}$ be the vertices of first and second copy of Herschel graph H_s respectively. Then $|V(G)| = 22$ and $E(G) = 37$.

We define the vertex labeling

$f: V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ as follows.

$f(v) = F_1$, $f(u_1) = F_{10}$, $f(u_2) = F_6$,

$f(u_3) = F_4$, $f(u_4) = F_7$, $f(u_5) = F_8$,

$f(u_6) = F_2$, $f(u_7) = F_9$, $f(u_8) = F_3$,

$f(u_9) = F_{11}$, $f(u_{10}) = F_5$

and

$f(v) = F_{22,}$

$f(v_1) = F_{17}$, $f(v_2) = F_{15}$, $f(v_3) = F_{13}$,

$f(v_4) = F_{14}$, $f(v_5) = F_{16}$, $f(v_6) = F_{18}$,

$f(v_7) = F_{21}$, $f(v_8) = F_{12}$, $f(v_9) = F_{20}$,

$f(v_{10}) = F_{19}$.

From the above labeling pattern, we observe that $e_f(0) = 18$, $e_f(1) = 19$.

Hence, $|e_f(0) - e_f(1)| \leq 1$.

Thus, the graph obtained by joint of two copies of Herschel graph H_s is a Fibonacci cordial graph.

Example 5. Fibonacci cordial labeling of the joint of two copies of Herschel graph H_s is shown in Figure 5.

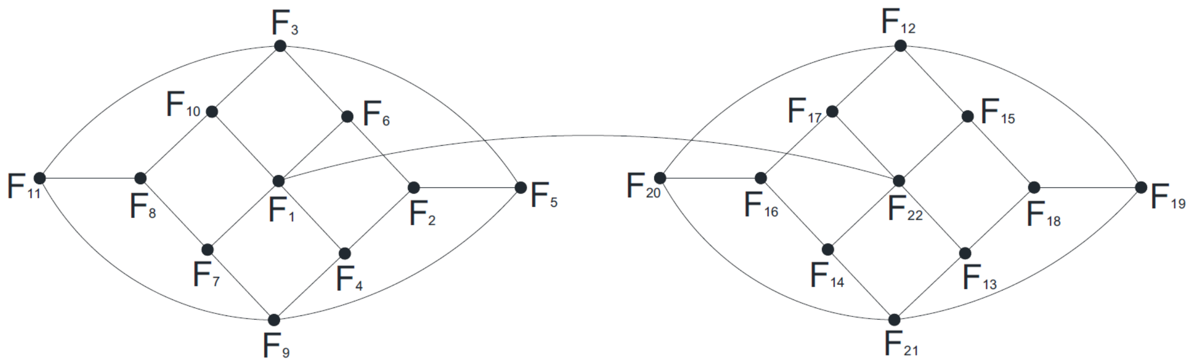

Figure 5

Theorem 6. DS(H_s) is Fibonacci product cordial graph.

Proof . Consider H_s with V (H_s) = {v, u_i : $1 \leq i \leq 10$}. Here V (K1,n) = V1 ∪ V2, where V_1 = vertices of degree 3 and V_2 = vertices of degree 4 . Now in order to obtain DS(H_s) from G, we add w_1 and w_2 corresponding to V_1 and V_2. Then $|V(DS(H_s))| = 13$ and $|E(DS(K_{1,n}))| = 28$.

We define the labeling

f: V (G) → {1, 2,...,|V (G) |} as follows.

$f(v) = F_3$, $f(w_1) = F_{11}$, $f(w_1) = F_{13}$.

$f(u_1) = F_1$, $f(u_2) = F_2$, $f(u_3) = F_4$,

$f(u_4) = F_5$, $f(u_5) = F_9$, $f(u_6) = F_7$,

$f(u_7) = F_{12}$, $f(u_8) = F_6$, $f(u_9) = F_8$,

$f(u_{10}) = F_{10}$.

From the above labeling pattern, we observe that $e_f(1) = 15$, $e_f(0) = 14$.

Hence, $|e_f(0) - e_f(1)| \leq 1$.

Thus, DS(H_s) is Fibonacci cordial graph.

Example 6. Fibonacci cordial labeling of DS(H_s) is shown in Figure 6.

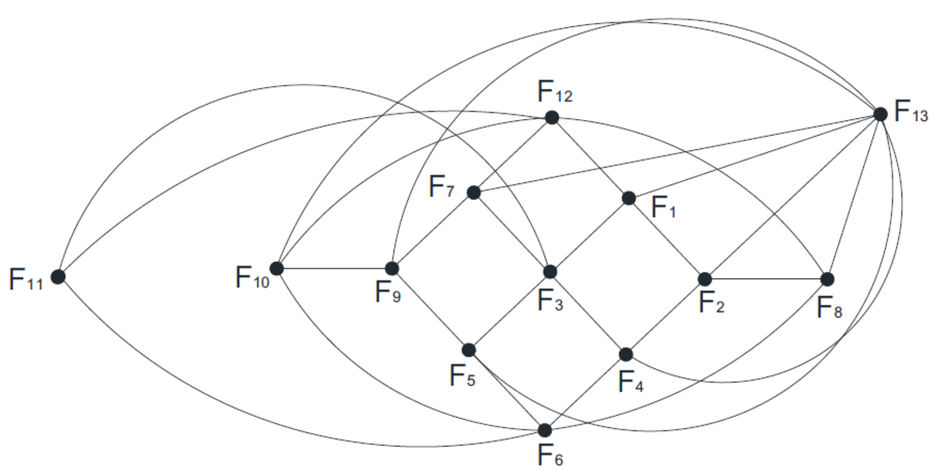

Figure 6

CONCLUSION

In this paper we have proved six new results of herschel graph. To explore some new fibonacci cordial graph is an open problem.

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