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INFLUENCE OF MAGNETIC FIELD IN A DARCY COUPLE-STRESS NANOFLUID LAYER HEATED AND SOLUTED FROM BELOW

Sudhir Kumar Pundir [a] **, Rahul Kumar** [b,*] **and Rimple Pundir** [c]
Department of Mathematics, S.D. College, Muzaffarnagar-251001 (U.P.), India
$b,*$ Corresponding Author Email: rahulru108@gmail.com

ABSTRACT:
This paper investigates a double-diffusive convection problem in a couple-stress nanofluid that is restricted within two infinite free-free boundaries. A modified Darcy model is employed to inspect the impact of magnetic field. Following the normal mode approach, the subsequent eigenvalue equation is solved by Galerkin method for the stationary as well as for oscillatory state and the Rayleigh number is obtained. The effects of the parameters governing the assumed model have been examined carefully. It is found that at the stationary convection, the parameters L_e, N_A and N_{TC} stabilizes the system whereas L_n , R_S , C_p , Q, R_n and ε destabilizes the system. The presence of oscillatory state is also shown under the specific condition.

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1. INTRODUCTION

A broad inspection of thermal conductivity in a Newtonian fluid heated from under the surface by taking various aspects of hydrodynamics and MHD was proposed by Chandrasekhar [1]. The term "nanofluids" was initiated by Choi and Eastman [2] to express suspensions of copper nanoparticles in water. This study found that thermal conductivity increased by 1.5 and 3.5 times, 5% and 20%, respectively for the fluid as compared to the water containing the lower volume fraction. Further, an experiment was performed by Eastman *et al*. [3]. The reason behind this decrease is the agglomeration or sedimentation of the nanoparticles. Similarly, some other researchers also contribute in this direction and produce some inspiring results. As a result of which the use of nanofluids and magnetic effect to improve heat transfer has since become an active area of research as proposed by Thompson [4], Ghasemi *et al*. [5]. Nield and Kuznetsov [6] have further extended their previous survey of thermally heated nanofluid layer of porous nature in which they used a new boundary condition of realistic nature to determine the nanoparticle fraction by imposing the value of temperature at the boundaries.

Thermal convection on a ferromagnetic fluid by applying Maxwell model in porous medium have been investigated by Pundir *et al*. [7]. Yadav *et al*. [8] worked on a nanofluid layer to observe the out-turn of vertically applied magnetic field on it. Nield and Kuznetsov [9] investigated a double-diffusive instability in nanofluid. The study on a rotatory nanofluid layer with Darcy Brinkman model have been discussed by Chand and Rana [10]. Shukla *et al*. [11] studied a modified model of LTNE effects in a binary nanofluid convection in which the eigenvalue equation consists of three additional LTNE parameters. It was found that these parameters played a major role in onset of convection for layer of nanofluid. Pundir *et al*. [12] considered a problem on a Darcy-Maxwell nanofluid porous layer in an anisotropic medium and discussed the Soret driven instability for this problem. Rana and Chand [13] proposed a double-diffusive instability in a porous nanofluid layer with role of couple stress. Nonlinear and linear analysis of a non-Newtonian couple-stress nanofluid with two thermally insulated plates have been surveyed by Umavati and Beg [14] where the instability is of time dependent nature and medium is porous. Pundir *et al*. [15] investigated the double-diffusive instability in a porous couple-stress nanofluid where the fluid layer subjected to rotation. Choudhary *et al*. [16] carried out stability analysis in a couple-stress nanofluid by taking the viscosity of the fluid as of variable nature.

The magnetic field effect has certain applications in the area of plasma physics, geophysics, medicine, atmospheric science etc. Mahajan and Sharma [17] have discussed linear penetrative instability in a magnetic nanofluid layer by considering the internal heating effect into account. They extended their work to inspect the effects of rotatory porous fluid layer under variable gravity. Influence of magnetic dipole on 2-D ferromagnetic fluid with flat elastic sheet has been explained by Gowda *et al.* [18]. In this study, the flow design has been chosen because it is off times used in magnetic drug systems and bioengineering applications counting engineering. Bishnoi *et al.* [19] scrutinized the effects of hall current on a layer of nanofluid saturated with magnetic field under temperature gradient. For a viscous fluid, the flow (i.e. time dependent) has been performed by Ahmad *et al.* [20] due to a rotating elastic disk by considering the effect of Joule heating on temperature equation and that of magnetic effect in vertical direction.

Kumar and Sarkar [21] explicated the development and scope of composite nanofluids. They discussed how composite nanofluids have achieved such enormous interest and demand in a vast area of applications in engineering field. The main purpose of this assessment is to outline the latest progress in the field of nanofluids and nanocomposite dispersions. Ullah *et al.* [22], Madhukesh *et al.* [23], Pundir *et al.* [24] have scrutinized simulation of a hybrid MHD flow around reliable surfaces and for this, they took two different types of nano particle to check the enhancement in the energy of composite nanofluid. Recently, Pundir *et al.* [25] worked on a double-diffusive instability in a composite nanofluid layer.

The current research deals with the study of a couple stress nanofluid layer heated and soluted in a Darcy porous medium under the existence of magnetic field which is not have been investigated yet. In this study, we assume that the density of nanoparticles in a porous medium is less than the density of the base fluid. For this, we make use of the Galerkin method and investigated the problem for the stationary and the oscillatory convection and observed the effects of the concerned parameters.

2. Problem Formulation

Consider an incompressible layer of a couple-stress fluid having nanoparticles assuming the layer to be enclosed between the boundaries $z=0$ and $z=d$. The temperature at the lower end $z=0$ is $T=T_0$ and at the above end $z=d$ is $T=T_1$. The magnetic field is $\mathbf{H}=(0,0,H_0)$ (vertically upward) i.e., towards Z-axis. The fluid is likely to be heated and salted from beneath the surface while the nanoparticle flux is supposed as zero at the boundary.

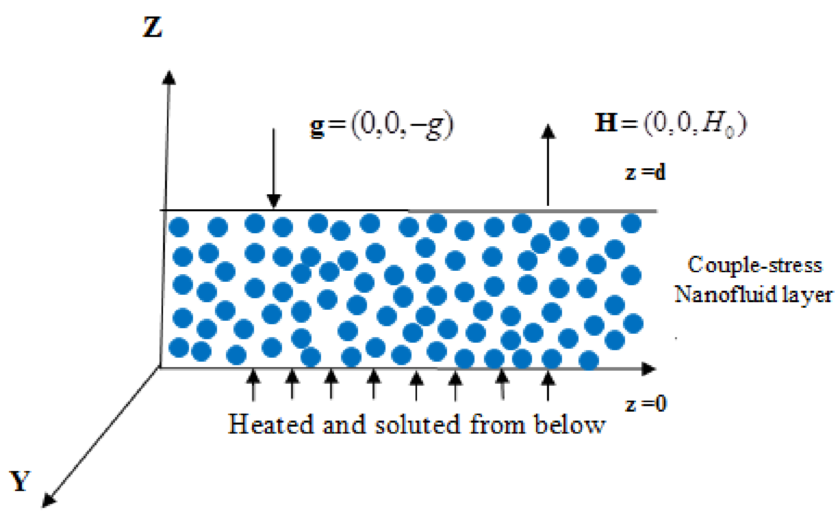

Figure 1: Physical Configuration

The constitutive equations governing the system are as follows:

$$\nabla\cdot q=0 \tag{1}$$

$$0=-\nabla p-\frac{1}{k_1}\left(\mu-\mu_c\nabla^2\right)q+\frac{\mu_e}{4\pi}\left(H\cdot\nabla\right)H+$$
$$\left(\phi\rho_p+\left(1-\phi\right)\rho_0\left\{1-\alpha_T\left(T-T_0\right)-\alpha_C\left(C-C_0\right)\right\}\right)g \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\left(q\cdot\nabla\right)\right]H=\left(H\cdot\nabla\right)H+\eta'\nabla^2H \tag{3}$$

$$\nabla\cdot H=0 \tag{4}$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla\phi=D_B\nabla^2\phi+\frac{D_T}{T_1}\nabla^2T \tag{5}$$

$$\left(\rho c\right)_m\frac{\partial}{\partial t}+\left(\rho c\right)_f q\cdot\nabla T=k_m\nabla^2T+\varepsilon\left(\rho c\right)_p\left(D_B\nabla\phi\cdot\nabla T+\frac{D_T}{T_1}\nabla T\cdot\nabla T\right)+\left(\rho c\right)D_{TC}\nabla^2C \tag{6}$$

$$\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla C=D_S\nabla^2C \tag{7}$$

where

$\eta'=\dfrac{1}{\left(4\pi\mu_e\sigma\right)}$ is electrical resistivity and σ is electrical conductivity of the nanofluid.

Here, the temperature is assumed as constant and the flux of thermophoretic nanoparticles is assumed as zero within the limits as stated by [19].

The boundary conditions compatible to the problem [8] are

$$w=0,\ \frac{\partial w}{\partial z}=0,\ \mathrm{T}=\mathrm{T}_0,\ \phi=\phi_0,\ C=C_0,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0\quad at\ z=0 \tag{8}$$

$$\mathrm{w}=0,\ \frac{\partial w}{\partial z}=0,\ \mathrm{T}=\mathrm{T}_1,\ \phi=\phi_1,\ C=C_1,\ D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0\quad at\ z=1 \tag{9}$$

Now let us non-dimensionalizing the variables as

$$\left(x_a,y_a,z_a\right)=\left(\frac{x,y,z}{d}\right),\left(u_a,v_a,w_a\right)=\left(\frac{u,v,w}{\alpha}\right)d,\left(H_x',H_y',H_z'\right)=\left(\frac{H_x,H_y,H_z}{H_0}\right),t'=\frac{t\alpha}{d^2},$$

$$p'=\frac{pk_1}{\mu\alpha},\phi'=\frac{\phi-\phi_0}{\phi_0},T'=\frac{T-T_0}{T_0-T_1},C'=\frac{C-C_0}{C_0-C_1} \tag{10}$$

where

$\alpha=\dfrac{k}{\left(\rho c\right)_f}$ is nanofluid thermal diffusivity.

On omitting the dashes for convenience, we have

Now the eq. (1)-(7) takes the form

$$\nabla \cdot q = 0 \tag{11}$$

$$0 = -\nabla p - \left(1 - C_P \nabla^2\right) q + \frac{P_r}{P_{rm}} Q\left(H \cdot \nabla\right) H - R_m e_z - R_n \phi e_z + R_a T e_z + \frac{R_s}{L_e} C e_z \tag{12}$$

$$\varepsilon \frac{\partial H}{\partial t} = \left(H \cdot \nabla\right) q + \varepsilon \frac{P_r}{P_{rm}} \nabla^2 H \tag{13}$$

$$\nabla \cdot H = 0 \tag{14}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} q \cdot \nabla \phi = \frac{1}{L_n} \nabla^2 \phi + \frac{N_A}{L_n} \nabla^2 T \tag{15}$$

$$\left(\frac{\partial T}{\partial t} + q \cdot \nabla T\right) = \nabla^2 T + \frac{N_B}{L_n} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{L_n} \nabla T \cdot \nabla T + N_{TC} \nabla^2 C \tag{16}$$

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon} q \cdot \nabla C = \frac{1}{L_e} \nabla^2 C \tag{17}$$

$$at\ z = 0, w = 0, C = 1, \phi = 0, \frac{\partial w}{\partial z} = 0, T = 1. \tag{18}$$

$$at\ z = 1, w = 0, C = 0, \phi = 1, \frac{\partial w}{\partial z} = 0, T = 0. \tag{19}$$

where,

$P_r = \frac{\mu}{\rho_0 \alpha}$; (Nanofluid Prandtl parameter), $P_{rm} = \frac{\mu}{\rho_0 \eta'}$; (Nanofluid magnetic Prandtl parameter), $C_p = \frac{\mu_c}{\mu d^2}$; (Couple stress parameter), $Q = \left(\frac{\mu_e H_0{}^2 K_1}{4\pi \rho_0 \upsilon \eta'}\right)$; (Nanofluid Magnetic parameter), $R_m = \frac{\left(\rho_p \phi_0 + \rho_0 \left(1 - \phi_0\right)\right) k_1 g d}{\mu \alpha}$; (Basic Density Rayleigh number) , $R_n = \frac{\left(\rho_p - \rho_0\right) k_1 \phi_0 g d}{\mu \alpha}$; (Concentration Rayleigh Number), $R_a = \frac{\left(1 - \phi_0\right)\left(T_0 - T_1\right) \rho_0 \alpha_T k_1 d}{\mu \alpha}$; (Rayleigh Number), $R_S = \frac{\left(1 - \phi_0\right)\left(C_0 - C_1\right) \rho_0 \alpha_T d k_1 g}{\mu D_s}$; (Solute Rayleigh Number), $L_e = \frac{\alpha}{D_S}$; (Thermo-solute specify Lewis Number), $L_n = \frac{\alpha}{D_B}$; (Thermo-nanofluid specify Lewis Number), $N_A = \frac{D_T \left(T_0 - T_1\right)}{D_B T_1 \phi_0}$; (Modify diffusivity ratio), $N_B = \frac{\varepsilon \left(\rho c\right)_p \phi_0}{\left(\rho c\right)_f}$ (Modify nano particle density increment), $N_{TC} = \frac{D_{TC}\left(C_0 - C_1\right)}{\alpha \left(T_0 - T_1\right)}$; (Dufour parameter).

3. BASIC STATE AND PERTURBATION SOLUTIONS

Succeeding Chand and Rana [13], we assume that

$$u = v = w = 0, p = p_b(z), C = C_b(z), T = T_b(z), \phi = \phi_b(z), H = \hat{e}_z \quad (20)$$

which reduces the basic solution as $T_b = 1 - z,\ C_b = 1 - z,\ \phi_b = z$ (21)

We superimpose infinitesimal small perturbations into the basic state as

$$q(u,v,w) = q''(u,v,w), H = \hat{e}_z + H'', T = T_b + T'', C = C_b + C'', \phi = \phi_b + \phi'', p = p_b + p'' \quad (22)$$

Using eq. (22) in eq. (11) to (17) and linearizing the subsequent equations we obtain,

$$\nabla \cdot q = 0 \quad (23)$$

$$0 = -\nabla p - \left(1 - C_p \nabla^2\right) q + \frac{P_r}{P_{rm}} Q \frac{\partial H}{\partial z} + R_a T \hat{e}_z + \frac{R_S}{L_e} C \hat{e}_z - R_n \phi \hat{e}_z \quad (24)$$

$$\varepsilon \frac{\partial H}{\partial T} = \frac{\partial w}{\partial z} \hat{e}_z + \varepsilon \frac{P_r}{P_{rm}} \nabla^2 H \quad (25)$$

$$\nabla \cdot H = 0 \quad (26)$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{L_n} \nabla^2 \phi + \frac{N_A}{L_n} \nabla^2 T \quad (27)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{L_n}\left(\frac{\partial T}{\partial z} - \frac{\partial \phi}{\partial z}\right) - \frac{2 N_A N_B}{L_n} \frac{\partial T}{\partial z} + N_{TC} \nabla^2 C \quad (28)$$

$$\frac{\partial C}{\partial t} - \frac{1}{\varepsilon} w = \frac{1}{L_e} \nabla^2 C \quad (29)$$

$$at\ \ z = 0\ \ and\ \ z = 1,\ \frac{\partial \phi}{\partial z} + \frac{\partial T}{\partial z} = 0, C = 0, w = 0, \phi = 0, T = 0. \quad (30)$$

It is noticed that parameter Rm is absent in Eq. (23) - (29), being an estimate of basic static pressure gradient. Applying $\hat{e}_z \cdot$ twice of curl on eq. (24) and by using twice of curl = $\nabla\nabla - \nabla^2$ in both eq's. (23) and (26), the z-component of the momentum equation becomes,

$$\left(1 - C_p \nabla^2\right) \nabla^2 w - R_a \nabla_H{}^2 T - \frac{R_S}{L_e} \nabla_H{}^2 C + R_n \nabla_H{}^2 \phi + Q \frac{P_r}{P_{rm}} \nabla_H{}^2 \left(\frac{\partial H_z}{\partial t}\right) = 0 \quad (31)$$

where $\nabla_H{}^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$ Eliminating H_z from Eq. (25) and (31), we get

$$\left(1 - C_p \nabla^2\right) \nabla^2 w - R_a \nabla_H{}^2 T - \frac{R_S}{L_e} \nabla_H{}^2 C + R_n \nabla_H{}^2 \phi - Q \frac{P_r}{P_{rm}} \left\{\varepsilon \frac{\partial}{\partial t} - \frac{P_r}{P_{rm}}\right\}^{-1} \nabla_H{}^2 \frac{\partial^2 w}{\partial z^2} = 0 \quad (32)$$

4. NORMAL MODE ANALYSIS

The perturbation into the normal modes takes the form $\left[w,\phi,T,C\right]=\left[W''(z),\varphi''(z),\Theta''(z),\Gamma''(z)\right]\exp(ilx+imy+nt)$ (33)

where l and m are the wave numbers in the x and y directions, respectively and n denote the growth rate of the perturbation in nanofluid and leave " for convenience.

Using eq. (33) into eq. (32), (25)-(29), the subsequent eigenvalue problem takes place as follows

$$\left[\left(\varepsilon n-\frac{P_r}{P_{rm}}\right)\left\{1-C_p\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)+Q\frac{P_r}{P_{rm}}a^2D^2\right]W(z)+a^2R_a\left(\varepsilon n-\frac{P_r}{P_{rm}}\right)\Theta(z)$$

$$+\frac{R_S}{L_e}a^2\left(\varepsilon n-\frac{P_r}{P_{rm}}\right)\Gamma(z)-a^2R_n\left(\varepsilon n-\frac{P_r}{P_{rm}}\right)\varphi(z)=0 \quad (34)$$

$$\frac{1}{\varepsilon}W(z)+\left\{\frac{1}{L_e}\left(D^2-a^2\right)-n\right\}\Gamma(z)=0 \quad (35)$$

$$W(z)+\left(D^2-a^2-n+\frac{N_B}{L_n}D-\frac{2N_AN_B}{L_n}D\right)\Theta(z)-\frac{N_B}{L_n}D\varphi(z)+N_{TC}\left(D^2-a^2\right)\Gamma(z)=0 \quad (36)$$

$$\frac{1}{\varepsilon}W(z)-\frac{N_A}{L_n}\left(D^2-a^2\right)\Theta(z)-\left(\frac{1}{L_n}\left(D^2-a^2\right)-n\right)\varphi(z)=0 \quad (37)$$

$$W=0, D^2W=0, \Gamma=0, \Theta=0, \varphi=0 \quad at \quad z=0 \quad and \quad z=1 \quad (38)$$

where $D=\frac{d}{dz}$ and $a^2=l^2+m^2$ is the dimensionless horizontal wave number.

5. METHOD OF SOLUTION

Using the GWR method, we consider the solution of the form

$$[W,\Theta,\Gamma,\varphi]=[W_0,\Theta_0,\Gamma_0,\varphi_0]\sin(\pi z) \quad (39)$$

Now using eq. (39) in eq's. (34)-(37) and integrating them from 0 to 1, we obtain the matrix specified in eq. (40).

$$\begin{pmatrix} z_3\left(J+C_pJ^2\right)+\frac{Q}{\varepsilon}\frac{P_r}{P_{rm}}a^2\pi^2 & -a^2R_az_3 & -\frac{R_S}{L_e}a^2z_3 & a^2R_nz_3 \\ \frac{1}{\varepsilon} & \frac{N_A}{L_n}J & 0 & \frac{J}{L_n}+n \\ 1 & -\left(J^2+n\right) & -JN_{TC} & 0 \\ \frac{1}{\varepsilon} & 0 & -\left(\frac{J}{L_e}+n\right) & 0 \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \varphi_0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (40)$$

where total wave number $\left(J^2\right)=\pi^2+a^2$.

6. THE STATIONARY CONVECTION

We have considered the case where density of nanoparticles is less than density of base fluid $(\rho_p < \rho_0)$. For the case of non- oscillatory state, putting $n=0$ and solving, we get $R_a^{st} = \frac{1}{(N_{TC}L_e - \varepsilon)}\left[R_n\left\{JL_n - N_A\left(N_{TC}L_e - \varepsilon\right)\right\} - \left\{\frac{\left(J + C_p J^2\right)J^2\varepsilon}{a^2} + Q\pi^2 J\right\} + R_S J\right]$ (41) The Rayleigh number R_a as stated by (41) is a function of non-dimensional parameters namely solutal specify Rayleigh number(R_S), nanofluid magnetic number (Q), Dufour parameter (N_{TC}),thermo-solute Lewis Number(L_e),resultant wave number a, the medium porosity (ε),concentration specify Rayleigh number (R_n), thermo-nanofluid Lewis number (L_n), modified diffusivity ratio (N_A), Couple-stress parameter (C_p).

Since (41) contains modified diffusivity ratio (N_A) absorption with the nano particle Rayleigh number, but does not contain the modify nano particle density increment (N_B). This proves that the terms of cross-diffusion in nanofluid is constrained by the regular cross- diffusion term.

Now if $Q=0$, that is absence of magnetic field, equation (41) becomes

$$R_a^{st} = \frac{1}{(N_{TC}L_e - \varepsilon)}\left[R_n\left\{JL_n - N_A\left(N_{TC}L_e - \varepsilon\right)\right\} - \left\{\frac{\left(J + C_p J^2\right)J^2\varepsilon}{a^2}\right\} + R_S J\right] \quad (41.1)$$

If $C_p = 0$ and $Q=0$, that is the absence of couple-stress parameter and magnetic field, equation (41) becomes

$$R_a^{st} = \frac{1}{(N_{TC}L_e - \varepsilon)}\left[R_n\left\{JL_n - N_A\left(N_{TC}L_e - \varepsilon\right)\right\} - \frac{J^3\varepsilon}{a^2} + R_S J\right] \quad (41.2)$$

which is in accordance with the previous result of Rana [13].

If $R_n = 0$, $R_S = 0$ and $N_A = 0$ (in absence of nanoparticles and solute gradient), equation (41) becomes

$$R_a^{st} = \frac{J^3\varepsilon}{a^2\left(N_{TC}L_e - \varepsilon\right)} \quad (41.3)$$

which is approximately same as given by Chandrasekhar [1] for ordinary fluid in porous medium. Now we find some derivatives of Rayleigh number w.r.t some dimensionless resultant parameters for analyzing the behavior analytically.

$$\frac{\partial R_a^{st}}{\partial R_S} = \frac{J}{N_{TC}L_e - \varepsilon} \quad (42)$$

$$\frac{\partial R_a^{st}}{\partial R_n} = \left\{\frac{JL_n}{N_{TC}L_e - \varepsilon} - N_A\right\} \quad (43)$$

$$\frac{\partial R_a^{st}}{\partial Q} = \left\{\frac{\pi^2 J}{\varepsilon - N_{TC}L_e}\right\} \quad (44)$$

$$\frac{\partial R_a^{\ st}}{\partial C_p}=\left\{\frac{J^4\varepsilon}{\left(\varepsilon-N_{TC}L_e\right)a^2}\right\} \quad (45)$$

$$\frac{\partial R_a^{\ st}}{\partial N_A}=\left\{-R_n\right\} \quad (46)$$

$$\frac{\partial R_a^{\ st}}{\partial L_n}=\left\{\frac{R_nJ}{\left(N_{TC}L_e-\varepsilon\right)}\right\} \quad (47)$$

$$\frac{\partial R_a^{\ st}}{\partial N_{TC}}=-\left\{\frac{L_eN_AR_n}{\left(N_{TC}L_e-\varepsilon\right)}-\frac{L_e\left\{J\pi^2Q+\frac{J^3\varepsilon\left(1+JC_p\right)}{a^2}-\left\{JL_n-N_A\left(N_{TC}L_e-\varepsilon\right)\right\}R_n+JR_s\right\}}{\left(N_{TC}L_e-\varepsilon\right)^2}\right\} \quad (48)$$

$$\frac{\partial R_a^{\ st}}{\partial L_e}=-\left\{\frac{N_{TC}N_AR_n}{\left(N_{TC}L_e-\varepsilon\right)}+\frac{N_{TC}\left\{-J\pi^2Q-\frac{J^3\varepsilon\left(1+JC_p\right)}{a^2}+\left\{JL_n-N_A\left(N_{TC}L_e-\varepsilon\right)\right\}R_n+JR_s\right\}}{\left(N_{TC}L_e-\varepsilon\right)^2}\right\} \quad (49)$$

$$\frac{\partial R_a^{\ st}}{\partial \varepsilon}=-\left\{\frac{-\frac{J^3\left(1+JC_p\right)}{a^2}+N_AR_n}{\left(N_{TC}L_e-\varepsilon\right)}+\frac{\left\{\begin{array}{l}-J\pi^2Q-\frac{J^3\varepsilon\left(1+JC_p\right)}{a^2}+\\ \left\{JL_n-N_A\left(N_{TC}L_e-\varepsilon\right)\right\}R_n+JR_s\end{array}\right\}}{\left(N_{TC}L_e-\varepsilon\right)^2}\right\} \quad (50)$$

7. OSCILLATORY CONVECTION

Considering that at the marginal state, the oscillatory convection, i.e., $n=n'i(n'>0)$ and we write n in place of n' for convenience, then from (40) Rayleigh number is obtained as

$$R_a^{\ osc}=\frac{(M_1-M_2+M_3)+i(L_1-L_2+L_3)}{C+iD} \quad (51)$$

where,

$$M_1=R_na^2J\left[\left\{JL_n-N_A\left(N_{TC}L_e-\varepsilon\right)\right\}J^2P_r-\left\{L_nP_{rm}\left(JL_e+1\right)+N_A\varepsilon L_eP_{rm}+L_eL_nP_r\right\}n^2\right],$$

$$M_2=\left[\begin{array}{l}\left\{\varepsilon P_rJ^6\left(\eta J+1\right)+a^2Q\pi^2P_rJ^4\right\}+\left\{\begin{array}{l}\varepsilon P_{rm}J^3\left(\eta J+1\right)\left(2JL_e+1\right)+L_eJ\left(2+J\right)\\ \left(\varepsilon P_rJ^2\left(\eta J+1\right)+a^2Q\pi^2P_r\right)\end{array}\right\}n^2\\ +\left\{\left(\eta J+1\right)\varepsilon P_{rm}L_e^{\ 2}\right\}n^4\end{array}\right],$$

$M_3 = R_S a^2 J\left[\left\{J^3 P_r - \left\{L_e P_r + (JL_e + 1)P_{rm}\right\} n^2\right\}\right],$

$L_1 = R_n a^2\left[J^2\left\{L_n J P_{rm} - P_{rm} N_A\left(N_{TC} L_e - \varepsilon\right) + L_n P_r\left(JL_e + 1\right) + P_r N_A \varepsilon L_e\right\} n - \left\{L_e L_n P_r\right\} n^3\right],$

$$L_2 = \begin{bmatrix} \left\{\varepsilon P_{rm} J^5\left(\eta J + 1\right) + \left(\varepsilon P_r J^2\left(\eta J + 1\right) + a^2 Q \pi^2 P_r\right) J^2\left(2JL_e + 1\right)\right\} n \\ -\left\{\varepsilon P_{rm} J^2 L_e\left(\eta J + 1\right)\left(2 + J\right) + N_{TC} L_e - \varepsilon\left(\varepsilon P_r J^2\left(\eta J + 1\right) + a^2 Q \pi^2 P_r\right) L_e^{\,2}\right\} n^3 \end{bmatrix},$$

$L_3 = \left[R_S a^2 J^2\left\{J P_{rm} + \left(JL_e + 1\right) P_r\right\} n - \left\{L_e P_{rm}\right\} n^3\right],$

$C = a^2\left[P_r J^3\left(N_{TC} L_e - \varepsilon\right) - JL_e\left\{\left(N_{TC} L_e - \varepsilon\right) P_{rm} - \left(P_{rm} + L_e P_r\right)\varepsilon\right\} n^2\right],$

$D = a^2\left[\left\{\left(P_{rm} + L_e P_r\right)\left(N_{TC} L_e - \varepsilon\right) - P_r \varepsilon L_e\right\} n + \left\{P_r \varepsilon L_e^{\,2}\right\} n^3\right],$

The frequency of oscillation is designated by the following third order equation in ζ i.e.

$$\left(P_1 P_2\right)\zeta^3 + \left(P_9 P_7 - P_1 P_5\right)\zeta^2 + \left(P_7 P_8 + P_6 P_9 - P_3 P_4 - P_5 P_2\right)\zeta + \left(P_6 P_8 - P_3 P_5\right) = 0 \qquad (52)$$

here,

$P_1 = \left(1 + \eta J\right) J \varepsilon P_{rm} L_e^{\,2},$

$P_2 = J^3 \varepsilon P_{rm}\left(1 + \eta J\right)\left(2JL_e + 1\right) + \left(J^2 \varepsilon P_r\left(1 + \eta J\right) + a^2 \pi^2 P_r Q\right)\left(2 + J\right) L_e J,$
$\quad - R_n a^2 J\left\{L_n P_{rm}\left(JL_e + 1\right) + N_A \varepsilon P_{rm} L_e + L_e L_n P_r\right\} - R_s a^2 J\left\{P_{rm}\left(JL_e + 1\right) + L_e P_r\right\},$

$P_3 = R_n a^2 J^3 P_r\left\{JL_n - \left(N_{TC} L_e - \varepsilon\right)\right\} - P_r J^4\left\{J^2 \varepsilon\left(1 + \eta J\right) + a^2 Q \pi^2\right\} + P_r J^4 a^2 P_r,$

$P_4 = a^2 P_r \varepsilon L_e^{\,2},$

$P_5 = \left\{\left(N_{TC} L_e - \varepsilon\right)\left(P_{rm} + L_e P_r\right) - L_e P_r \varepsilon\right\} J^2,$

$P_6 = \left\{JL_n P_{rm} - N_A P_{rm}\left(N_{TC} L_e - \varepsilon\right) + \left(JL_e + 1\right) L_n P_r + N_A P_r \varepsilon L_e\right\} R_n a^2 J^2 + \left\{P_{rm} J + P_r\left(JL_e + 1\right)\right\} R_s a^2 J$
$\quad - \left\{\left(1 + \eta J\right) J^3 \varepsilon P_{rm} + \left(\varepsilon J^2 + \eta \varepsilon J^3 + a^2 Q \pi^2\right)\left(2JL_e + 1\right) P_r J^2\right\},$

$P_7 = \left\{\left(1 + \eta J\right)\left(2 + J\right) J^2 \varepsilon P_{rm} L_e + \left(J^2 \varepsilon + J^3 \eta \varepsilon + a^2 Q \pi^2\right) L_e^{\,2} P_r\right\} - P_{rm} L_e - R_n a^2 P_r L_e L_n,$

$P_8 = a^2 P_r J^3\left(N_{TC} L_e - \varepsilon\right),$

$P_9 = \left(P_{rm} + L_e P_r\right)\varepsilon J a^2 L_e - P_{rm}\left(N_{TC} L_e - \varepsilon\right),$

Clearly, eq. (52) is a cubic equation in $\zeta(=\mathrm{n}^2)$, so it has three values or roots which must be real and positive. For equation (52), the sum of roots is given by $\dfrac{-\left(P_9 P_7 - P_1 P_5\right)}{P_1 P_2}$, therefore for oscillatory analysis to arise, either

$P_9P_7 - P_1P_5$ or P_1P_2 must be negative. If both signs are equally positive or negative, the possibility of oscillatory convection is excluded.

8. RESULTS AND DISCUSSION

We observe the effects of couple-stress parameter (C_p), concentration specify Rayleigh number (R_n), solute specify Rayleigh number (R_s), nanofluid magnetic number (Q), thermo-solutal Lewis number (L_e), thermo-nanofluid Lewis number (L_n), Dufour parameter (N_{TC}), medium porosity (ε) and modified diffusivity ratio (N_A) on stationary Rayleigh number analytically and graphically. We discuss the behavior of $\frac{\partial R_a^{st}}{\partial R_S}$, $\frac{\partial R_a^{st}}{\partial R_n}$, $\frac{\partial R_a^{st}}{\partial Q}$, $\frac{\partial R_a^{st}}{\partial C_p}$, $\frac{\partial R_a^{st}}{\partial N_A}$, $\frac{\partial R_a^{st}}{\partial L_n}$, $\frac{\partial R_a^{st}}{\partial L_e}$, $\frac{\partial R_a^{st}}{\partial \varepsilon}$ and $\frac{\partial R_a^{st}}{\partial N_{TC}}$ analytically.

From equation (42), $\frac{\partial R_a^{st}}{\partial R_S}>0$ since the value of $N_{TC}L_e >> \varepsilon$, so R_S stabilizes the system. From equation (43), $\frac{\partial R_a^{st}}{\partial R_n} > 0$ if $\frac{JL_n}{N_{TC}L_e - \varepsilon} > N_A$ and $\frac{\partial R_a^{st}}{\partial R_n} < 0$ if $\frac{JL_n}{N_{TC}L_e - \varepsilon} < N_A$ thereby the parameter R_n showing the stabilizing and destabilizing effect respectively. Equation (44) clearly shows that $\frac{\partial R_a^{st}}{\partial Q} < 0$ which implies that the nanofluid magnetic parameter Q destabilizes the system. Similarly, equation (45) yields that $\frac{\partial R_a^{st}}{\partial C_p} < 0$ implying that C_p shows a destabilizing effect on the system. From equation (46), we find that $\frac{\partial R_a^{st}}{\partial N_A} > 0$ since R_n is taken to be negative indicating N_A to stabilize the system. Equation (47) demonstrates that $\frac{\partial R_a^{st}}{\partial L_n} < 0$ if $R_n < 0$ showing destabilizing effect of L_n on the system. From eq. (48) it is found that $\frac{\partial R_a^{st}}{\partial N_{TC}} > 0$ if $\frac{L_e N_A R_n}{(N_{TC}L_e - \varepsilon)} < \frac{L_e\left\{J\pi^2 Q + \frac{J^3\varepsilon\left(1+JC_p\right)}{a^2} - \left\{JL_n - N_A\left(N_{TC}L_e - \varepsilon\right)\right\}R_n + JR_s\right\}}{\left(N_{TC}L_e - \varepsilon\right)^2}$ so that the system becomes stable for parameter N_{TC}. Eq.(49) implies that $\frac{\partial R_a^{st}}{\partial L_e} < 0$ if $J\pi^2 Q + \frac{J^3\varepsilon\left(1+JC_p\right)}{a^2} < JR_s$ therefore malign L_e to destabilize the system. Equation (50) demonstrates that $\frac{\partial R_a^{st}}{\partial \varepsilon} < 0$ if

$\left[-J\pi^2 Q-\frac{J^3\varepsilon\left(1+JC_p\right)}{a^2}+\left\{JL_n-N_A\left(N_{TC}L_e-\varepsilon\right)\right\}R_n\right]>JR_s$ which predicts that the porosity parameter ε destabilizes the system.

The solution of the eigenvalue problem is carried out numerically by plotting the graphs between the stationary Rayleigh number R_a^{st} and wavenumber a for different values of parameters $L_e,\ L_n,\ R_S,\ C_p,\ Q,\ N_A,\ N_{TC},\ R_n,\ \varepsilon$. Figure 2 represents that for fixed values of $N_A=5,\ N_{TC}=35,\ R_S=2000,\ R_n=-70,\ L_n=1500,\ Q=50,\ C_p=0.5,\ \varepsilon=0.5$ and for different values of $L_e=1000, 1020, 1040$, it is clearly seen that on increasing L_e, the stationary Rayleigh number R_a^{st} increases. This clearly implies that parameter L_e stabilizes the system. Figure 3 and 4 represents that for fixed values of $N_A=5,\ N_{TC}=35,\ R_n=-70,\ L_e=1200,\ Q=50,\ C_p=0.5,\ \varepsilon=0.5$ and for different values of L_n=1300, 1500, 1700 and that of $R_s=5000, 10000, 15000$, R_a^{st} decreases as L_n and R_S increases thus the parameters L_n and R_S destabilizes the system. Figure 5 and 6 represents that on increasing the values of $C_p=0.2,\ 0.4,\ 0.6$ and $Q=50, 100, 150$ with fixed $N_A=5,\ N_{TC}=35,\ L_n=1500,\ R_n=-70,\ L_e=1200,\ R_S=2000,\ \varepsilon=0.5$, a slight decrease is noticed in stationary Rayleigh number R_a^{st} .Thus C_p and Q produces a destabilizing effect.

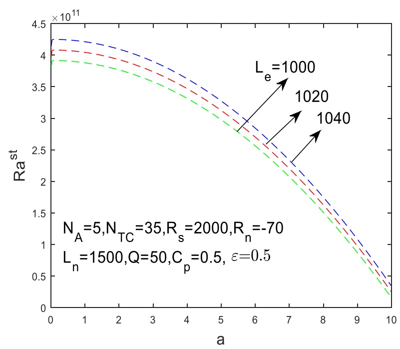

Figure 2: Variation of Rayleigh number R_a^{st} with wave number a for different values of Lewis number L_e

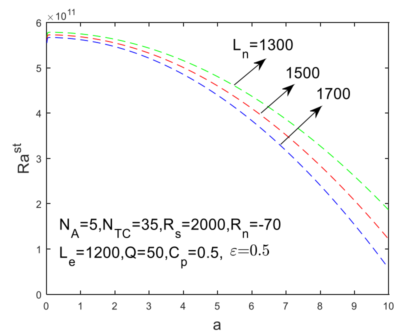

Figure 3: Variation of Rayleigh number R_a^{st} with wave number a for different values of thermo-nanofluid Lewis Number L_n

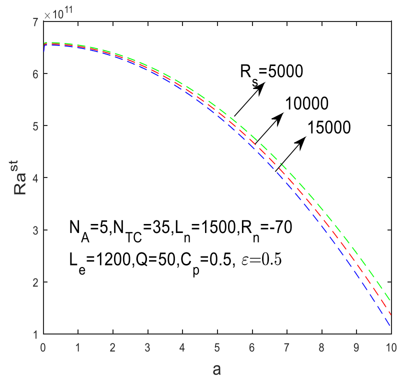

Figure 4: Variation of Rayleigh number R_a^{st} with wave number a for different values of solute Rayleigh Number R_S

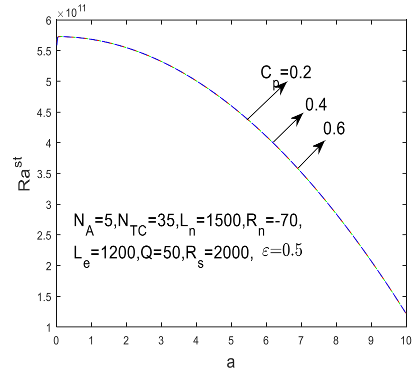

Figure 5: Variation of Rayleigh number R_a^{st} with wave number a for different values of Couple stress parameter C_p

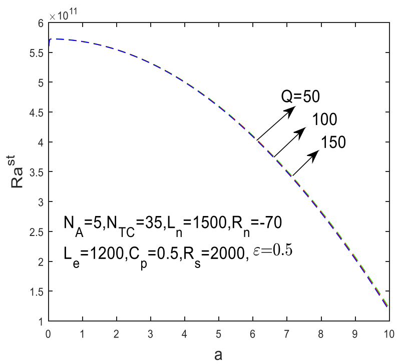

Figure 6: Variation of Rayleigh number R_a^{st} with wave number a for different values of nanofluid magnetic parameter Q

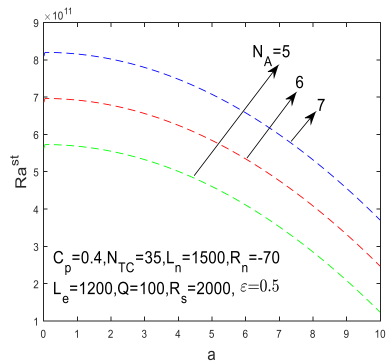

Figure 7: Variation of Rayleigh number R_a^{st} with wave number a for different values of modify diffusivity ratio N_A

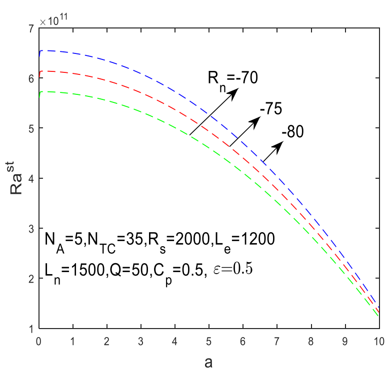

Figure 8: Variation of Rayleigh number R_a^{st} with wave number a for different values of concentration Rayleigh number R_n

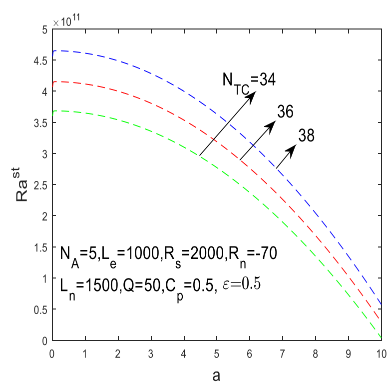

Figure 9: Variation of Rayleigh number R_a^{st} with wave number a for different values of Dufour parameter N_{TC}

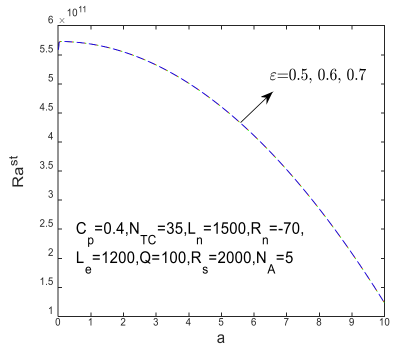

Figure 10: Variation of Rayleigh number R_a^{st} with wave number a for different values of porosity parameter ε

Figure 7 represents that for fixed values of $C_p = 0.4,\ N_{TC} = 35,\ L_n = 1500,\ R_n = -70,\ L_e = 1200,\ Q = 100,\ R_S = 2000,\ \varepsilon = 0.5$, Ra^{st} increases as we keep on increasing N_A which indicates the stabilizing effect of N_A on the system. Figure 8 represents that Ra^{st} increases as we decrease the value of R_n for $N_A = 5,\ N_{TC} = 35,\ R_S = 2000,\ L_e = 1200,\ L_n = 1500,\ Q = 50,\ C_p = 0.5,\ \varepsilon = 0.5$ and $R_n = -70, -75, -80$. Thus R_n destabilizes the system. Figure 9 represents that for $N_A = 5,\ L_e = 1000,\ R_S = 2000,\ R_n = -70,\ L_n = 1500,\ Q = 50,\ C_p = 0.5,\ \varepsilon = 0.5$, the value of Ra^{st} increases on increasing N_{TC} say $N_{TC} = 34, 36, 38$ showing the stabilizing effect on system. Figure 10 represents that for fixed $C_p = 0.4,\ N_{TC} = 35,\ L_n = 1500,\ R_n = -70,\ L_e = 1200,\ Q = 100,\ R_S = 2000,\ N_A = 5$, we observe that as we slightly increase the porosity parameter ε, a slight decrease is observed in the value of Ra^{st}. Thus, for the parameter ε, the system becomes destabilize.

9. CONCLUSION

A study in a Darcy Couple-stress nanofluid has been performed in appearance of magnetic field. At boundaries, the thermophoretic nanoparticles flux is assumed as zero. The nanoparticle density is considered to be less than that of the base fluid i.e.,$(\rho_p < \rho_0)$. The problem is solved for the stationary and the oscillatory convection analytically and the corresponding stationary and oscillatory Rayleigh numbers have been obtained. The variation between Ra^{st} and wavenumber a has been shown graphically for different parameters governing the system. It is noticed that parameters L_e , N_A and N_{TC} has a stabilizing effect whereas L_n ,R_S , C_p , Q , R_n and ε found to destabilizes the system. The existence of oscillatory convection is possible under certain conditions.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest among them.

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