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# STRONG SPLIT AND NON-SPLIT DOMINATION NUMBER OF GRAPHS

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## ABSTRACT
*A strong dominating set D of a graph G= (V, E) is a strong split (nonsplit) dominating set if the induced sub-graph <V-D> is disconnected (connected). The strong split (nonsplit) domination number* $\gamma_{ss}(G)(\gamma_{sns}(G))$ *of G is the minimum cardinality of a strong split (nonsplit) dominating set. In this work, we obtained some bounds on* $\gamma_{ss}(G)$ *and* $\gamma_{sns}(G)$.

*Keywords: Split; Nonsplit; Number; Graph.*

## 1 INTRODUCTION
Let G= (V, E) be a graph and $uv \in E$, then u and v dominate each other. Further, u strongly dominates v and v weakly dominates u if deg (u) $\geq$ deg (v). A set $D \subseteq V$ is a strong dominating set of G. If every vertex in V-D is strongly dominated by at least one vertex in D. The strong domination number $\gamma_s(G)$ of G is the minimum cardinality of such a set. This concept was introduced by Sampath Kumar and Pushpa Latha [26]. Recently, Kulli and Janakiram introduced the concept of split domination and nonsplit domination, see, [18, 19]. Analogously, we define the following concept.

A strong dominating set D of G is a strong split (nonsplit) dominating set if the induced sub-graph <V-D> is disconnected (connected). The strong split (nonsplit) domination number $\gamma_{ss}(G)(\gamma_{sns}(G))$ of G is the minimum cardinality of a strong split (nonsplit) dominating set.

Example: For the graph G is given in **Figure 1**, {$v_2$} is a minimum strong nonsplit dominating set and {$v_2$, $v_4$, $v_7$} is a minimum strong split dominating set.

so $\gamma_{sns}(G)=1$ and $\gamma_{ss}(G)=3$

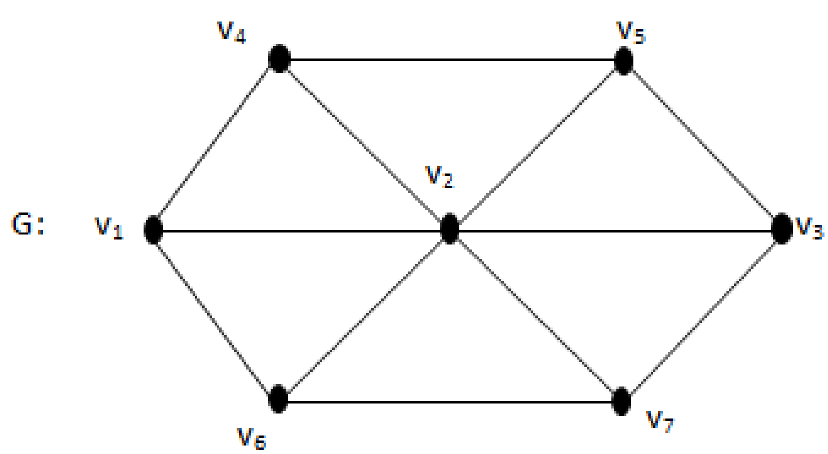


**Figure 1**

**Results on strong split domination number:**

**Observation 2.** For any cycle $C_p$ with $p \geq 4$ vertices, $\gamma_{ss}(C_p) = \left\lceil \frac{p}{3} \right\rceil$.

**Observation 3.** For any wheel $W_p$ with $p \geq 5$ vertices, $\gamma_{ss}(W_p) = 3$.

**Observation 4.** For any complete bipartite graph $K_{r,t}$ with $r \leq t$ and $t \geq 2$,

$\gamma_{ss}(K_{r,t}) = r$

**Theorem 5.** For any graph G,

$\gamma_{ss}(G) \leq p - \beta$,

where $\beta$ is the independence number of a graph.

**Proof**: Let M be a maximum independent set of vertices in G. Then M has at least two vertices and every in M is adjacent to some vertex in V-M. This implies that V-M is a strong split dominating set of G. Thus,

$\gamma_{ss}(G) \leq |V - M| = p - \beta$.

**Corollary 6.** For any graph G,

$\gamma_{ss}(G) \leq \alpha$,

where $\alpha$ is the vertex covering number of a graph G.

We state without proof a straight forward result that characterizes strong dominating set of G that are strong split dominating sets.

**Theorem 7.** A strong dominating set D of G is a split strong dominating set if and only if there exist two vertices $x,y \in V\text{-}D$ such that every x-y path contains a vertex of D.

**Theorem 8.** For any graph G,

(i) $\gamma_s(G) \leq \gamma_{ss}(G)$

(ii) $k(G) \leq \gamma_{ss}(G)$,

where k(G) is connectivity of G.

**Proof**: Follows by the definitions of $\gamma_s(G)$, $\gamma_{ss}(G)$ and k(G).

**Theorem 9.** For any graph G with an end-vertex,

$\gamma_s(G) = \gamma_{ss}(G)$.

Furthermore, there exists a $\gamma_{ss}$-set of G containing all vertices adjacent to end-vertices.

**Proof:** Let $v$ be an end-vertex of G. Then there exists a cut-vertex x adjacent to $v$. Let D be a $\gamma_s$-set of G. Suppose $x \in D$, then D is a $\gamma_{ss}$-set of G. Suppose $x \in V\text{-}D$, then $v \in D$ and hence $D - \{v\} \cup \{x\}$ is a $\gamma_{ss}$-set of G. Repeating this process for all such cut-vertices adjacent to end -vertices, we obtain a $\gamma_{ss}$-set of G containing all cut-vertices adjacent to end –vertices.

**Theorem 10.** If diam (G) =2, then

$$\gamma_{ss}(G) \le \delta(G),$$

Where $\delta(G)$ is minimum degree of G.

**Proof:** Let v be a vertex of minimum degree in G. Since diam (G)=2 there exists a vertex u not adjacent to *v*. Hence u must be adjacent to some vertex in N(*v*). Hence it follows that N(*v*) is a strong split dominating set of G. Thus $\gamma_{ss}(G) \le \delta(G)$.

**Results on strong nonsplit domination in graphs:**
We start with some elementary results, since their proofs are trivial, we omit the same.

**Theorem 11.**

$$\gamma_s(G) \le \gamma_{sns}(G).$$

**Theorem 12.** For any graph G,

$$\gamma_s(G) = \min\{\gamma_{ss}(G), \gamma_{sns}(G)\}$$

In [26], Sampathkumar and Pushpa Latha gave necessary and sufficient conditions for a minimal strong dominating set.

**Theorem 13 [26].** Let D be a minimal strong dominating set. Then, for each $v \in D$, one of the following holds:

(i) No vertex in D strongly dominates v.

(ii) There exists a vertex $u \in V$-D such that v is the only vertex in D which strongly dominates u.

**Theorem 14.** A strong nonsplit dominating set D of G is minimal if and only if for each vertex $v \in D$ one of the following conditions is satisfied.

(i) No vertex in D strongly dominates v.

(ii) There exists a vertex $u \in V$-D such that v is the only vertex in D which strongly dominates u.

(iii) $N(v) \cap (V-D) = \phi$.

**Proof**: Suppose D is minimal. On the contrary, if there exists a vertex $v \in D$ such that *v* does not satisfy any of the given conditions, then by Theorem, D'=D-{*v*} is a strong dominating of set G and by (iii) <V-D'> is connected. This implies that D' is a strong split dominating set of G, a contradiction.

Sufficiency is straight forward.

Next we obtain a relation between $\gamma_{sns}$(G) and $\gamma_{sns}$(H), where H is any spanning sub-graph of G. We omit the proof.

**Theorem 15.** For any spanning sub-graph H of G,

$$\gamma_{sns}(G) \le \gamma_{sns}(H).$$

**Theorem 16.** For any graph G,

$$\gamma_{sns}(G) \ge \frac{(2p-q-1)}{2}$$

Further the equality holds if G=$K_2$.

**Proof:** Let D be a $\gamma_{sns}$-set of G. Since <V-D> is connected.

$q \geq |V-D|+|V-D|-1$.

Thus, $\gamma_{sns}(G) \geq \frac{(2p-q-1)}{2}$ .

further, the equality holds if G=$K_2$.

**Theorem 17.** For any graph G,

$\gamma_{sns}(G) \leq p-\omega(G)+1$

where $\omega$(G) is the clique number of G.

**Proof**: Let U be the set of vertices of G such that <U> is complete with $|U|=\omega$ (G).

Then for any $u \in U$, (V-U) $\cup \{u\}$ is a strong nonsplit dominating set of G.

Thus $\gamma_{sns}(G) \leq |(V-U) \cup \{u\}|$=p-$\omega$(G)+1.

Now we list the exact values of $\gamma_{sns}(G)$ for some standard graphs:

(i) For any complete graph $K_p$ with $p \geq 2$ vertices,

$\gamma_{sns}(G)=1$.

(ii)For any complete bipartite graph $K_{r,t}$

$$\gamma_{sns}(K_{r,t}) = \begin{cases} 2 \ \ if \ r,t \geq 2, r=t \\ r+t-1 \quad \text{otherwise} \end{cases}$$

(iii) For any cycle $C_p$,

$\gamma_{sns}(C_p)=p-2$

(iv) For any wheel $W_p$,

$\gamma_{sns}(W_p)=1$

(v) For any path with $p \geq 6$ vertices,

$\gamma_{sns}(P_p)=p-2$

Recently, Kulli and Janakiram introduced the concept of block nonsplit domination as follows:

A dominating set of D of a connected graph G= (V,E) is a block nonsplit dominating set if the induced sub-graph <V-D> is a block in G. The block non-split domination number $\gamma_{bns}(G)$ is the minimum cardinality of a block nonsplit dominating set of G [20]. In this, we extend the concept of strong non-spilt domination to block strong nonsplit domination as follows:

A strong dominating set D of a connected graph G=(V,E) is said to be a block strong nonsplit dominating set if the induced sub graph <V-D> is a block in G. The block strong nonsplit domination number $\gamma_{bsns}(G)$ of G is the minimum cardinality of a block strong nonsplit dominating set of G.

All graphs considered here are assumed to be connected.

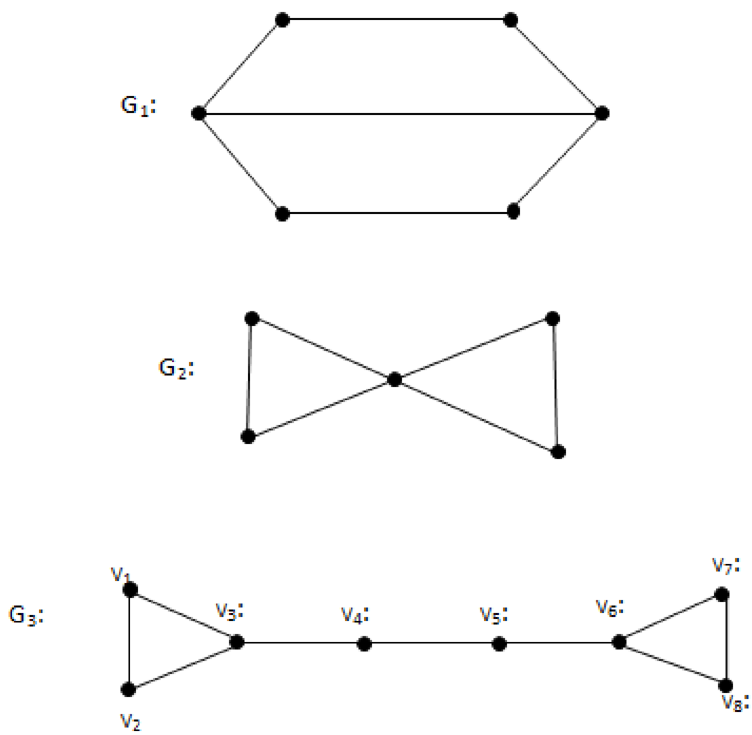


**Figure.2**

**In figure 2** , $G_1$, $G_2$ have no block strong nonsplit dominating sets, whereas for $G_3$, $\{v_1, v_2, v_3, v_6, v_7, v_8\}$ is a block strong nonsplit dominating set.

**Theorem 18.** A graph G has a block strong nonsplit dominating set if and only if there exists a block in G containing only cut vertices of G.

**Proof** : Let D be a block strong nonsplit dominating set G, then <V-D> is a block in G. Since D is a strong dominating set, each vertex in V-D is a cut-vertex of G.

Converse is obvious.

## CONCLUSION

In this paper, we define the notions of accurate split and non-split dominations in graphs. We got many bounds on accurate split and non-domination numbers. Exact values of these new parameters are obtained for some standard graphs. As a future work, the readers extend the results and study the applications of the parameters in a wider sense.

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