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AN EOQ MODEL FOR TIME DETERIORATING ITEMS WITH PARABOLIC DEMAND UNDER PARTIAL BACKLOGGING AND TWO-LEVEL PARTIAL CREDIT FINANCING POLICY

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ABSTRACT
An EOQ model for time deteriorating items with parabolic demand under two-level partial credit financing policy is developed. Shortages are allowed subject to partial backlogging. The backlogging rate varies inversely as the waiting time for the next replenishment. It is assumed that (1) a supplier offers a partial trade credit to the retailer ie. he pays a portion of the purchase amount at the time of placing an order and then receives a permissible delay on the rest of the outstanding amount to avoid non-payment risks (2) the retailer always obtains the partial trade credit offered by the supplier but the retailer offers the full trade credit to the customer (3) the retailer must take a loan to pay to his supplier the partial payment immediately when the order is received and then pay off the loan with the entire revenue. The main objective of this model is to minimize the total cost function with respect to optimal replenishment policy. Finally numerical examples are provided to illustrate the problem and sensitivity analysis has been carried out to depict the EOQ model.

Keywords: deteriorating items, parabolic demand, partial backlogging, two-level partial credit financing policy

1. INTRODUCTION

Inventory control is essential for companies to reduce their costs, maintaining stock, improving products quality, providing better services and managing customer demands, companies are facing greater challenges when they are working with deteriorating products. Large quantity of goods displayed in market lure the customers to buy more. If the stock is insufficient, customer may prefer some other brand, as a result it will fetch loss to the supplier. In some inventory systems such as fashionable commodities the length of waiting time for the next replenishment is the main factor in determining whether backlogging will be accepted or not, the longer the waiting time is the smaller the backlogging rate would be and vice versa. In the theory of inventory management, customers demand is a key parameter of any inventory model. The nature of demand depends on many factors viz., selling price, availability of the stock, time, quality of the product, ecofriendliness of the product, impreciseness, etc.

Ghare and Schrader [1] first established an EOQ model for an exponentially decaying item for which there is constant demand. Later, Covert and Philip [2] extended Ghare and Schrader's [1] model and developed an EOQ model for a variable deterioration rate, by assuming a two parameter Weibull distribution. Philip [3] then obtained an inventory model with a three-parameter Weibull distribution deterioration rate. Goyal and Giri [4] provided an excellent and detailed review of the literature on deteriorating inventory since the early 1990s.Moon et al. [5] formulated a model to incorporate two extreme physical characteristics of stored items into inventory model ameliorating (value or utility increase with time) and deteriorating. Shah et al. [6] estimated a coordinated decision with two level credit limit for quadratic demand, which can be seen in seasonal items, fashion goods etc. Shah et al. [7] determine an optimal shipments, ordering, and imbursement policies for integrated vendor-buyer inventory system with net credit price-sensitive trapezoidal demand. Tayal et al. [8] has investigated an inventory model for deteriorating items with seasonal products and an option of an alternative market. Freshly, Singh et al.[9] an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Shah et al. [10] evaluated an optimal down-stream credit period and cycle time for deteriorating inventory in a supply chain. Later, Shah et al. [11] studied the model on the impact of future price increase on ordering policies for perishable items under quadratic demand. Shah et al. [12]

determined a deteriorate inventory model with expiration date of items under two level trade credit and preservation technology investment for time and price sensitive demand.

In the classical economic order quantity model, it is often assumed that the shortages are either completely backlogged or completely lost. In reality, it is seen that during the shortage period either all customers wait until the arrival of next order (completely backlogged) or all customers leave the system (completely lost). However, it is more reasonable situation to consider that, some customers are able to wait for the next order to satisfy their demands during the stock out period, while others do not wish to or can wait and they have to fill their demands from other sources (partial back order case). To reflect this phenomenon, Padmanabhan and Vrat [13] considered an EOQ model for perishable items with stock-dependent demand under instantaneous replenishment with zero lead time. Abad [14] discussed a pricing and lot-sizing problem for a product with a variable rate of deterioration by allowing shortages and partial backlogging. Jalan et al. [15] assumed the deterioration as a two parameter Weibull distribution, and then they proposed an EOQ model for items with Weibull distribution deterioration shortages and trended demand. Chang and Dye [16] developed an EOQ model for deteriorating items with time varying demand and partial backlogging. Abad [17] proposed an optimal price and order-size for a reseller under partial backlogging. Goyal and Giri [18] proposed a model for recent trends in modelling of deteriorating inventory. Ouyang et al. [19] presented an inventory model for deteriorating items with exponential declining demand and partial backlogging. The rate of deterioration is assumed to be constant and the backlogging rate is inversely proportional to the waiting time for the next replenishment. Dye [20] developed two inventory models: one on determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging, and other on joint pricing and ordering for a deteriorating inventory with partial backlogging. Alamri and Balkhi [21] proposed a model for the effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates.

Permissible delay in payment is one of the most important and popular strategies in the business world. With this strategy, the suppliers attract more customers, and this leads to more sales of the product. In practice, the supplier usually allows the wholesaler a fixed credit period for settling the account, and the wholesaler in turn provides a similar credit period to its customers. Teng [22] introduced an inventory model with trade credit financing by distinguishing the difference between unit cost and unit price. Huang [23] established an EOQ model under two levels of trade credit policy. Huang [24] established an EOQ model under conditionally permissible delay in payments. Liao [25] extended Huang's model [23] to analyze the impact of the two-level trade credit on EPQ model for an exponentially deterioration items. Huang and Hsu [26] and Chung [27] considered an EOQ model under the wholesaler, which provides the partial trade credit polity to the retailer in a supply chain. Teng and Chang [28] explored the optimal replenishment decisions in the EPQ model under two-level trade credit policy from the viewpoint of the retailer and manufacturer. Kreng and Tan [29] considered the supplier to allow the wholesaler a fixed credit period linked to the order quantity for settling the account. Liao et al. [30] explored the optimal strategy of deterioration items with capacity constraints under two-levels of trade credit policy. Many additional related investigations can be found in the articles by Ouyang et al. [31, 32], Teng and Goyal [33], Liao [34, 35], Huang and Liao [36], Chung et al. [37–40], Lin and Srivastava [41] and Liao et al. [42], and in the references cited in each of these earlier works.

Another realistic situation obtaining momentum is partial trade credit financing in the supply chain model, that is, paying partial payment for goods immediately after receiving those goods and covering the rest when the trade credit period uses out. Huang and Hsu [43] investigated retailers' inventory policy to show that the retailer acquires full trade credit from its supplier yet merely furnishes partial trade credit to consumers. On this basis, Wu and Chan [44] took deteriorating items into account, and related articles can be found in Mahata and Mahata [45], Thangam [46] , Soni and Joshi [47], Chen et al. [48], Wu and Chan [49], Wu et al. [50], Liao et al. [51], Mahata [52], Mahata [53] and Mahata and De [54] and their references. Liao & Huang [55] established lot-sizing policies for deterioration items under two-level trade credit with partial credit to credit-risk retailer and limited storage capacity, Most of the existing inventory models with trade credit assumed that the wholesaler's capital is unrestricted, and the stocks ordered is fully paid for upon receipt. However, from the viewpoint of practice, the wholesaler may take a loan to pay off the amount owed to the supplier immediately when the order is received and then pay off the loan with entire revenue.

In this work, we have studied an EOQ model for time deteriorating items with parabolic demand under partial backlogging and two level partial credit financing policy. The shortages are allowed and this is partially backlogged with a constant backlogging rate. The supplier offers a partial trade credit to a retailer ie. he pays a portion of the purchase amount at the time of placing an order and then receives a permissible delay on the rest of the outstanding amount to avoid non-payment risks. The retailer always obtains the partial trade credit offered by the supplier but the retailer offers the full trade credit to the customer. Also the retailer must take a loan to pay to his supplier the partial payment immediately when the order is received and then pay off the loan with the entire revenue. The proposed model is formulated mathematically by using ordinary differential equations and the corresponding optimization is obtained as a cost minimization problem.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in the entire paper.

2.1 ASSUMPTIONS:

1. The inventory system involves one item.
2. The replenishment occurs instantaneously at an infinite rate
3. Lead time is zero.
4. There is no replacement or repair of deteriorated units.
5. Shortages are allowed and the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant.
6. The demand rate $D(t)$ at time t is assumed to be

$$D(t)=\begin{cases} a+bt+ct^2, 0\le t\le t_1 \\ B, t_1\le t\le T \end{cases}$$

Where a is positive constant, b,c are the time-dependent consumption rate parameter, $0\le b,c\le 1$

7. The fixed credit period offered by the supplier to the retailer is no less than the credit period permitted by the retailer to the customer (ie) $M\ge N$
8. As the order is filled, the retailer must take a loan (with interest charged of I_p) to pay the supplier the partial payment of $(1-\alpha)PQ$ immediately and then pay off the loan with entire revenue.
9. The retailer offers a credit period N to every customer. The retailer does not receive money from its customers until the time N.
10. During the credit period $M(>N)$, sales revenue is deposited in an interest bearing account with rate I_e. At the end, if the permissible delay M, the supplier pays off all units sold, keeps the profit for use in other activities, and starts paying for the interest charges with the rate I_r on the loan.

2.2 NOTATIONS:

1. $I(t)$ - Inventory level at any time t where $0 \le t \le T$
2. $D(t)$ - Demand rate function.
3. θ - Deterioration rate, $0 < \theta < 1$.
4. P **-** Purchase cost, $ per unit
5. K **-** The Replenishment cost per order ($/order)
6. h - Holding cost per unit per unit time
7. s - Shortage cost, $ per unit per unit time
8. p - Selling price, $ per unit per unit time
9. π - Opportunity cost due to lost sale, $ per time per unit time
10. δ - The backlogging parameter (a positive constant) $0 < \delta < 1$.
11. Q - Lot size of total order
12. t_1 - Time at which shortages starts, $0 \le t_1 \le T$.
13. T - The length of the Replenishment cycle
14. α - the fraction of the delay payments permitted by the supplier $(0 \le \alpha \le 1)$
15. I_m - Maximum product amount at the very beginning of the cycle
16. I_b - Maximum storage amount at the end of the cycle
17. M - The retailer's trade-credit period offered by supplier in years
18. N - The customer's trade-credit period offered by retailer in years
19. I_e - Interest which can be earned per $ per year by retailer
20. I_p **-** Interest charges per $ in stocks per year by the supplier
21. $TC(t_1,T)$ - The total cost per unit time, $ per unit time
22. $T_0 = \frac{1}{\theta}\log\left(1+\frac{\theta p(M-N)}{(1-\alpha)P}\right)$

3. MATHEMATICAL FORMULATION:

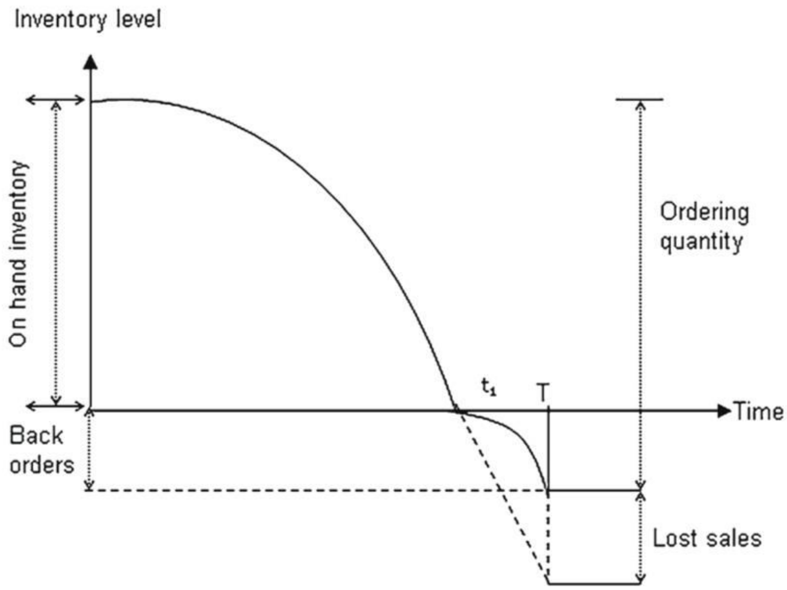


Fig. 1 Graphical representation of the inventory system

Based on the assumptions mentioned earlier, this section presents the following inventory model formulation. Initially an retailer purchase Q units of goods. The reduction of inventory is due to the combined effect of the demand as well as the deterioration in the interval $[0,t_1)$ a variable and ultimately falls to zero at $t=t_1$. Therefore shortages are allowed to occur and the demand during the period $[t_1,T]$ is partially backlogged rate. During the stock-out period, the backlogged rate is variable and is dependent on the length of the waiting time for the next replenishment. So the backlogged rate for negative inventory is denoted as $\frac{1}{1+\delta(T-t)}$, where δ is the backlogging parameter $0<\delta<1$.

Hence the inventory level $I(t)$ is given by

$$\frac{dI_1(t)}{dt}=-(a+bt+ct^2)-\theta I(t), 0\le t\le t_1$$

$$\frac{dI_2(t)}{dt}=-\frac{B}{1+\delta(t-t)}, t_1\le t\le T \qquad (1)$$

With boundary conditions $I_1(t_1)=I_m, I_2(t_1)=0$ and $I(t)$ is continuous at $t=t_1$.

The solutions of the above differential equation are

$$I_1(t)=\frac{1}{\theta^3}\left[-\theta^2 a+\theta b-2c-bt\theta^2+2tc\theta-ct^2\theta^2+\frac{e^{-\theta t}\left(\theta^2 a-\theta b-2c+bt_1\theta^2-2t_1c\theta+ct_1^2\theta^2+I_m\theta^3\right)}{e^{-\theta t_1}}\right] \qquad (2)$$

$$I_2(t) = \frac{B\log\left(1+\delta(T-t)\right) - B\log\left(1+\delta T - \delta t_1\right)}{\delta} \qquad (3)$$

Again exploiting the boundary condition $I_1(0) = I_m$, we get

$$I_m = \frac{1}{\theta^3}\left(\theta^2 a e^{-\theta t_1} - \theta b e^{-\theta t_1} + 2ce^{-\theta t_1} - \theta^2 a + \theta b - 2c - bt_1\theta^2 + 2t_1 c\theta - ct_1^2\theta^2\right) \qquad (4)$$

Again exploiting the boundary condition $I_2(T) = -I_b$, we get

$$I_b = \frac{B\log\left(1+\delta T - \delta t_1\right)}{\delta} \qquad (5)$$

The total number of ordered amount is

$$Q = I_m + I_b$$

$$= \frac{1}{\theta^3}\left(\theta^2 a e^{-\theta t_1} - \theta b e^{-\theta t_1} + 2ce^{-\theta t_1} - \theta^2 a + \theta b - 2c - bt_1\theta^2 + 2t_1 c\theta - ct_1^2\theta^2\right) + \frac{B\log\left(1+\delta T - \delta t_1\right)}{\delta}$$

(6)

The total inventory profit consists of the following components.

a) Ordering cost per cycle is K. (7)

b) Inventory holding cost HC per cycle is given by

$$HC = h\int_0^{t_1} I_1(t)dt$$

$$= -\frac{1}{6}\frac{1}{\theta^4}\Big[h\Big(-6e^{\theta t_1}\theta^2 a + 6e^{\theta t_1}\theta b - 12e^{\theta t_1}c + 3\theta^3 bt_1^2 e^{\theta t_1} + 6\theta^3 at_1 e^{\theta t_1} + 2\theta^3 ct_1^3 e^{\theta t_1} - 6\theta^2 bt_1 e^{\theta t_1}$$

$$+ 6\theta^2 a + 12t_1 c\theta e^{\theta t_1} - 6\theta b + 12c - 6\theta^2 ct_1^2 e^{\theta t_1}\Big)e^{-\theta t_1}\Big]$$

(8)

c) The shortage cost in the interval $[t_1, T)$ denoted by SC is given by,

$$SC = s\int_{t_1}^{T} -I_2(t)dt$$

$$= -\frac{sB\left(\log\left(1+\delta T - \delta t_1\right) - \delta T + \delta t_1\right)}{\delta^2} \qquad (9)$$

d) The opportunity cost due to lost sales denoted by OC is given by,

$$OC = \pi\int_{t_1}^{T} B\left[1 - \frac{1}{1+\delta(T-t)}\right]dt$$

$$= \pi BT - \pi\left[Bt_1 + \frac{B\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{10}$$

e) The Purchasing cost for the first replenishment cycle by PC is given by

$$PC = P(I_m + I_b)$$

$$= P\left[\frac{1}{\theta^3}\left(\theta^2 ae^{-\theta t_1} - \theta be^{-\theta t_1} + 2ce^{-\theta t_1} - \theta^2 a + \theta b - 2c - bt_1\theta^2 + 2t_1c\theta - ct_1^2\theta^2\right)\right.$$

$$\left. + \frac{B\log(1+\delta T-\delta t_1)}{\delta}\right] \tag{11}$$

Therefore the total inventory cost (X)= Ordering cost + Holding cost + Shortage cost + Opportunity cost + Purchasing cost

i.e. $X = K + HC + SC + OC + PC$

$$= \left\{\begin{array}{l} K + -\frac{1}{6}\frac{1}{\theta^4}\left[h\left(-6e^{\theta t_1}\theta^2 a + 6e^{\theta t_1}\theta b - 12e^{\theta t_1}c + 3\theta^3 bt_1^2 e^{\theta t_1} + 6\theta^3 at_1 e^{\theta t_1} + 2\theta^3 ct_1^3 e^{\theta t_1} - 6\theta^2 bt_1 e^{\theta t_1}\right.\right. \\ \left.\left. + 6\theta^2 a + 12t_1c\theta e^{\theta t_1} - 6\theta b + 12c - 6\theta^2 ct_1^2 e^{\theta t_1}\right)e^{-\theta t_1}\right] - \frac{sB(\log(1+\delta T-\delta t_1) - \delta T + \delta t_1)}{\delta^2} \\ + \pi BT - \pi\left[Bt_1 + \frac{B\log(1+\delta T-\delta t_1)}{\delta}\right] + P\left[\frac{1}{\theta^3}\left(\theta^2 ae^{-\theta t_1} - \theta be^{-\theta t_1} + 2ce^{-\theta t_1} - \theta^2 a + \theta b - 2c\right.\right. \\ \left.\left. - bt_1\theta^2 + 2t_1c\theta - ct_1^2\theta^2\right) + \frac{B\log(1+\delta T-\delta t_1)}{\delta}\right] \end{array}\right\}$$

(12)

According to the assumptions, there are two cases that should be considered in annual interest payable.

Case(i): $t_1 + N \le M < T_0$

In this case, the retailer takes a loan to pay the retailer the amount $(1-\alpha)PQ$ immediately when the order is filled at time 0. Additionally, the retailer pays off the loan from sales revenue at the time t given by

$$t = N + \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1} - 1}{\theta}\right)$$

which is less than time M. Further, the annual interest payable is

$$IP_1 = \frac{(1-\alpha)PI_r\left(a + bt + ct^2\right)}{2\theta^2}\left(e^{\theta t_1} - 1\right)\left[2\theta N + \frac{(1-\alpha)P}{p}\left(e^{\theta t_1} - 1\right)\right] \tag{13}$$

In this case the retailer pays off the loan from sales revenue before the permissible delay M. The interest earned starts from the time t given by

$$t = N + \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1} - 1}{\theta}\right) \quad \text{to} \quad t = t_1 + N$$

is $\frac{pI_e(a+bt+ct^2)}{2}\left[t_1 - \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1}-1}{\theta}\right)\right]^2$

And the interest earned starts from the time $t = t_1 + N$ to $t = M$ is given by

$$pI_e(a+bt+ct^2)\left\{(M - t_1 - N)\left[t_1 - \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1}-1}{\theta}\right)\right]\right\}$$

The annual interest earned is given by

$$IE_1 = \frac{pI_e(a+bt+ct^2)}{2\theta^2}\left[\theta t_1 - \frac{(1-\alpha)P}{p}\left(e^{\theta t_1} - 1\right)\right]\left[2\theta(M-N) - \theta t_1 - \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right] \quad (14)$$

Case (ii): $M < t_1 + N \le T_0$

In this case the interest earned is same as previous case

$$IP_2 = \frac{(1-\alpha)PI_r(a+bt+ct^2)}{2\theta^2}\left(e^{\theta t_1} - 1\right)\left[2\theta N + \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right] \quad (15)$$

In this case the interest earned starts from the time

$$t = N + \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1} - 1}{\theta}\right) \quad \text{to} \quad t = M$$

The annual interest earned in this case is

$$IE_2 = \frac{pI_e(a+bt+ct^2)}{2\theta^2}\left[\theta(M-N) - \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]^2 \quad (16)$$

Case (iii): $t_1 + N \ge T_0$

In this case, the retailer takes a loan to pay the supplier the amount $(1-\alpha)PQ$ at the initial time as well and the loan can be paid off from the revenue received at the time t given by

$$t = N + \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1} - 1}{\theta}\right),$$

which is greater than the time M. Hence, clearly the retailer takes the second loan to pay the rest of αPQ at time M but starts paying off from the sales revenue after the time t given by

$$t = N + \frac{(1-\alpha)P}{p}\left(\frac{e^{\theta t_1} - 1}{\theta}\right).$$

The annual interest payable is given by

$$IP_3 = \left\{ \frac{(1-\alpha)PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta N + \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]\right\} +$$
$$\left\{ \frac{\alpha PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta(N-M) + \frac{(2-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]\right\} \quad (17)$$

In this case, because $T_0 < t_1 + N$, the payoff time of the partial payment is longer than the permissible delay M. Futhermore, there is no interest earned

$$IE_3 = 0 \quad (18)$$

Total Cost Function:

$$TC_i(T,t_1) = \begin{cases} TC_1, t_1 + N \le M < T_0 \\ TC_2, M < t_1 + N \le T_0 \\ TC_3, t_1 + N \ge T_0 \end{cases} \quad (19)$$

$$TC_1(T,t_1) = \frac{1}{T}\left[X - IE_{11} + IP_{11}\right]$$

$$= \left\{ \begin{array}{l} K + -\frac{1}{6}\frac{1}{\theta^4}\left[h\left(-6e^{\theta t_1}\theta^2 a + 6e^{\theta t_1}\theta b - 12e^{\theta t_1}c + 3\theta^3 bt_1^2 e^{\theta t_1} + 6\theta^3 at_1 e^{\theta t_1} + 2\theta^3 ct_1^3 e^{\theta t_1} - 6\theta^2 bt_1 e^{\theta t_1}\right.\right. \\ \left.\left. + 6\theta^2 a + 12t_1 c\theta e^{\theta t_1} - 6\theta b + 12c - 6\theta^2 ct_1^2 e^{\theta t_1}\right)e^{-\theta t_1}\right] - \frac{sB\left(\log\left(1+\delta T - \delta t_1\right) - \delta T + \delta t_1\right)}{\delta^2} \\ + \pi BT - \pi\left[Bt_1 + \frac{B\log\left(1+\delta T-\delta t_1\right)}{\delta}\right] + P\left[\frac{1}{\theta^3}\left(\theta^2 ae^{-\theta t_1} - \theta be^{-\theta t_1} + 2ce^{-\theta t_1} - \theta^2 a + \theta b - 2c\right.\right. \\ \left.\left. - bt_1\theta^2 + 2t_1 c\theta - ct_1^2\theta^2\right) + \frac{B\log\left(1+\delta T - \delta t_1\right)}{\delta}\right] \\ - \frac{pI_e\left(a+bt+ct^2\right)}{2\theta^2}\left[\theta t_1 - \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]\left[2\theta(M-N) - \theta t_1 - \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right] \\ + \frac{(1-\alpha)PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta N + \frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right] \end{array} \right\} \quad (20)$$

$$TC_2(T,t_1) = \frac{1}{T}\left[X - IE_{21} + IP_{21}\right]$$

$$=\left\{\begin{array}{l} K+-\frac{1}{6}\frac{1}{\theta^4}\left[h\left(-6e^{\theta t_1}\theta^2 a+6e^{\theta t_1}\theta b-12e^{\theta t_1}c+3\theta^3 bt_1^2 e^{\theta t_1}+6\theta^3 at_1 e^{\theta t_1}+2\theta^3 ct_1^3 e^{\theta t_1}-6\theta^2 bt_1 e^{\theta t_1}\right.\right. \\ \left.\left.+6\theta^2 a+12t_1 c\theta e^{\theta t_1}-6\theta b+12c-6\theta^2 ct_1^2 e^{\theta t_1}\right)e^{-\theta t_1}\right]-\frac{sB\left(\log\left(1+\delta T-\delta t_1\right)-\delta T+\delta t_1\right)}{\delta^2} \\ +\pi BT-\pi\left[Bt_1+\frac{B\log\left(1+\delta T-\delta t_1\right)}{\delta}\right]+P\left[\frac{1}{\theta^3}\left(\theta^2 ae^{-\theta t_1}-\theta be^{-\theta t_1}+2ce^{-\theta t_1}-\theta^2 a+\theta b-2c\right.\right. \\ \left.\left.-bt_1\theta^2+2t_1 c\theta-ct_1^2\theta^2\right)+\frac{B\log\left(1+\delta T-\delta t_1\right)}{\delta}\right]-\frac{pI_e\left(a+bt+ct^2\right)}{2\theta^2}\left[\theta(M-N)-\frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]^2 \\ +\frac{(1-\alpha)PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta N+\frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right] \end{array}\right\} \tag{21}$$

$$TC_3(T,t_1)=\frac{1}{T}\left[X-IE_{22}+IP_{22}\right]$$

$$=\left\{\begin{array}{l} K+-\frac{1}{6}\frac{1}{\theta^4}\left[h\left(-6e^{\theta t_1}\theta^2 a+6e^{\theta t_1}\theta b-12e^{\theta t_1}c+3\theta^3 bt_1^2 e^{\theta t_1}+6\theta^3 at_1 e^{\theta t_1}+2\theta^3 ct_1^3 e^{\theta t_1}-6\theta^2 bt_1 e^{\theta t_1}\right.\right. \\ \left.\left.+6\theta^2 a+12t_1 c\theta e^{\theta t_1}-6\theta b+12c-6\theta^2 ct_1^2 e^{\theta t_1}\right)e^{-\theta t_1}\right]-\frac{sB\left(\log\left(1+\delta T-\delta t_1\right)-\delta T+\delta t_1\right)}{\delta^2} \\ +\pi BT-\pi\left[Bt_1+\frac{B\log\left(1+\delta T-\delta t_1\right)}{\delta}\right]+P\left[\frac{1}{\theta^3}\left(\theta^2 ae^{-\theta t_1}-\theta be^{-\theta t_1}+2ce^{-\theta t_1}-\theta^2 a+\theta b-2c\right.\right. \\ \left.\left.-bt_1\theta^2+2t_1 c\theta-ct_1^2\theta^2\right)+\frac{B\log\left(1+\delta T-\delta t_1\right)}{\delta}\right]-0 \\ +\left\{\frac{(1-\alpha)PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta N+\frac{(1-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]\right\} \\ +\left\{\frac{\alpha PI_r\left(a+bt+ct^2\right)}{2\theta^2}\left(e^{\theta t_1}-1\right)\left[2\theta(N-M)+\frac{(2-\alpha)P}{p}\left(e^{\theta t_1}-1\right)\right]\right\} \end{array}\right\} \tag{22}$$

The necessary conditions for the total profit $\partial TC_i(T,t_1)$ is concave with respect to T and t_1 are $\frac{\partial TC_i(T,t_1)}{\partial T}=0$ and $\frac{\partial TC_i(T,t_1)}{\partial t_1}=0$

Provided they satisfy the sufficient conditions $\left.\frac{\partial^2 TC_i(T,t_1)}{\partial T^2}\right|_{(T^*,t_1^*)}>0, \left.\frac{\partial^2 TC_i(T,t_1)}{\partial {t_1}^2}\right|_{(T^*,t_1^*)}>0$

and $\begin{bmatrix} \dfrac{\partial^2 TC_i}{\partial T^2} & \dfrac{\partial^2 TC_i}{\partial T \partial t_1} \\ \dfrac{\partial^2 TC_i}{\partial t_1 \partial T} & \dfrac{\partial^2 TC_i}{\partial {t_1}^2} \end{bmatrix} > 0$

We develop the following algorithm to find the optimal values of T and t_1 (say T^*, t_1^*) that maximize $TC_i(T, t_1)$

SOLUTION PROCEDURE:

The problem mentioned above is solved by using the following algorithm:

Step1: Start

Step 2: Plug all the value of the required parameters of the proposed model in the equation (19)

Step 3: Put $\dfrac{\partial TC_i}{\partial t_1} = \dfrac{\partial TC_i}{\partial T} = 0$, $i = 1,2,3$

Step 4: Solve the optimization problem TC_i for $i = 1,2,3$ and store the optimal value of t_1^*, T^*, TC^*, S^* and R^*

Step 5: Compare the value of TC_1, TC_2 and TC_3.

Step 6: Choose the minimum value among TC_1, TC_2 and TC_3.

Step 7: Stop

NUMERICAL EXAMPLES:

Example 1: Consider the inventory system with the following data $K = 1000, a = 300, b = 2, p = 34, c = 0.2,, \alpha = 0.3, \theta = 0.12,, \delta = 0.596, P = 12, h = 1.6$
$s = 10, \pi = 8, B = 350, t = 2, I_e = 0.11, I_r = 0.13, M = 0.5, N = 0.065$

in appropriate units. In this case $t_1 + N \le M < T_0$. Using the algorithm we obtain the optimal solution as $T = 0.81299$, $t_1 = 0.30778$. Hence the Total cost per unit time is $TC_1 = 3497.086$, $Q = 101.605$ units.

Example 2: Taking all the parameters same except $M = 0.12, N = 0.058$ in appropriate units. In this case $M < t_1 + N \le T_0$. Using the algorithm we obtain the optimal solution as $T = 0.90426$, $t_1 = 0.61546$. Hence the Total cost per unit time is $TC_2 = 3642.406$ \$, $Q = 104.121$ units.

Example 3: Taking all the parameters same except $M = 0.09, N = 0.064$ in this case $t_1 + N \ge T_0$. Using the algorithm we obtain the optimal solution as $T = 0.81584$, $t_1 = 0.38628$. Hence the Total cost per unit time is $TC_3 = 3231.433$, $Q = 80.668$ units.

Effect of change in various parameter of the inventory is presented in the following table

Changing parameter	Change in parameter	t_1	T	TP	Q
K	996	0.38598	0.81552	3230.389	80.6713
	998	0.38601	0.81612	3233.945	80.8111
	1002	0.38606	0.81732	3241.045	81.0910
	1004	0.38609	0.81793	3244.590	81.2311

p	28	0.38549	0.81816	3295.704	81.4522
	30	0.38570	0.81762	3273.829	81.2629
	32	0.38588	0.81714	3254.575	81.0972
	34	0.38604	0.81672	3237.497	80.9510
θ	0.116	0.38647	0.81535	3229.396	80.4733
	0.118	0.38636	0.81564	3230.718	80.5852
	0.122	0.38614	0.81622	3233.361	80.8087
	0.124	0.38603	0.81651	3234.681	80.9205
δ	0.592	0.38688	0.81924	3236.117	81.3861
	0.594	0.38673	0.81715	3232.091	80.8729
	0.596	0.38658	0.81506	3228.072	80.3615
	0.598	0.38643	0.81298	3224.059	79.8520
α	0.26	0.38582	0.81752	3242.938	81.2051
	0.28	0.38593	0.81712	3240.219	81.0781
	0.32	0.38614	0.81632	3234.770	80.824
	0.34	0.38625	0.81593	3232.040	80.696
P	11.6	0.38889	0.80428	3144.758	77.1044
	11.8	0.38756	0.81008	3188.265	78.9025
	12.2	0.38496	0.82181	3276.074	82.4880
	12.4	0.38369	0.82774	3320.357	84.2161
M	0.086	038596	0.81699	3239.312	81.0357
	0.088	0.38600	0.81686	3238.405	80.9934
	0.092	0.38607	0.81659	3236.588	80.9081
	0.094	0.38611	0.81646	3235.679	80.8665
N	0.058	0.38661	0.81460	3222.913	80.2737
	0.062	0.38637	0.81549	3229.002	80.5558
	0.066	0.38613	0.81637	3235.073	80.8381
	0.068	0.38601	0.81681	3238.102	80.9792

Fig.2: Effect of K on Total Profit

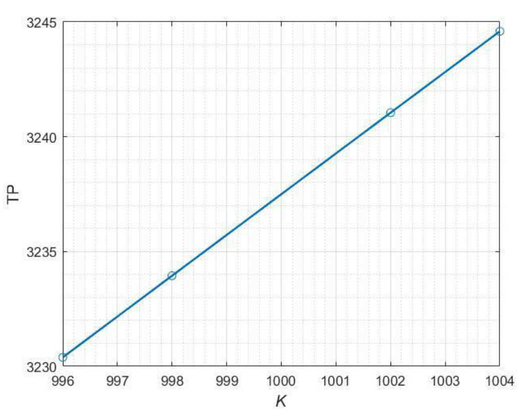


Fig.3: Effect of p on Total Profit

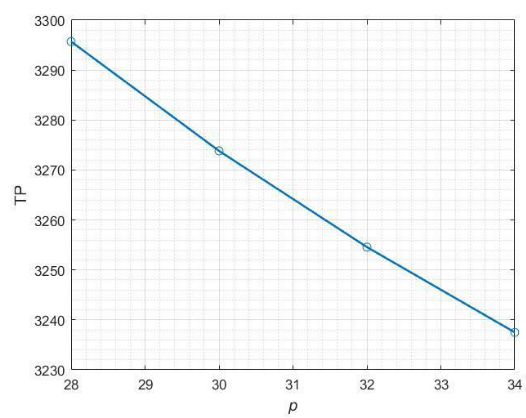


Fig.4: Effect of θ on Total Profit

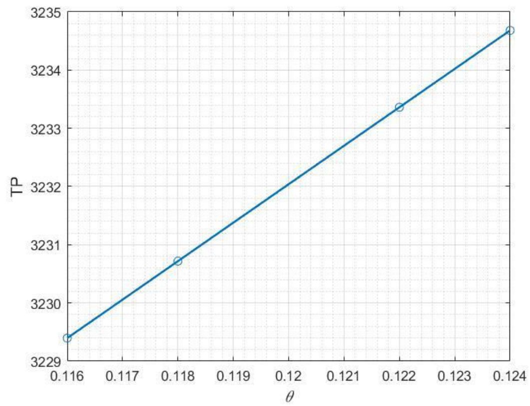


Fig.5: Effect of δ on Total Profit

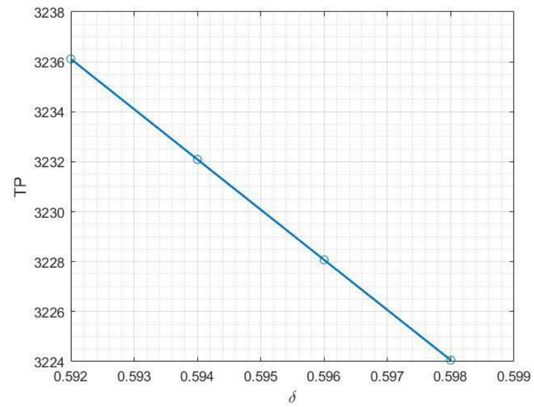


Fig.6: Effect of α on Total Profit

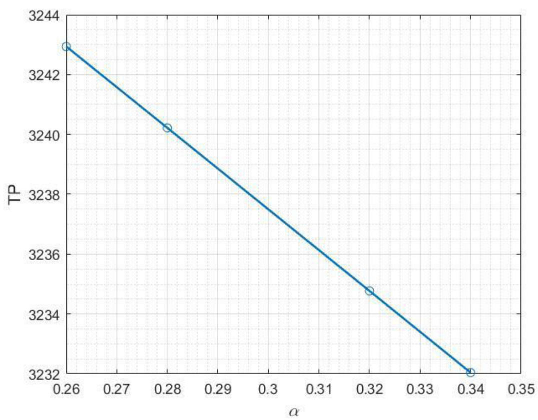


Fig.7: Effect of P on Total Profit

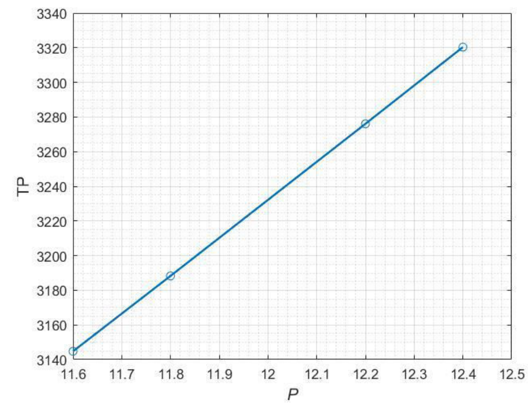


Fig.7: Effect of M on Total Profit

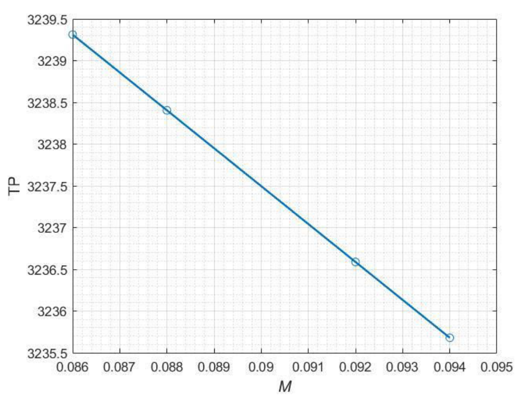
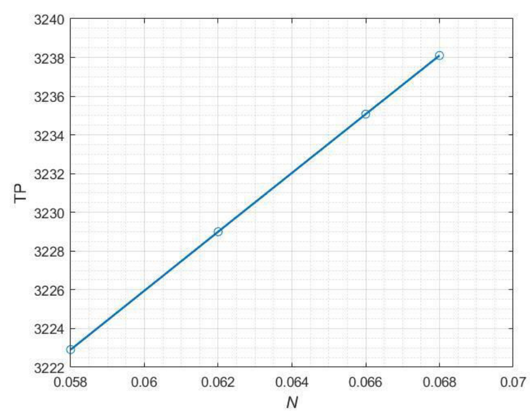


Fig.8: Effect of N on Total Profit



4. SENSITIVITY ANALYSIS:

Now, let us study on changes in the values of the system parameters based on the optimal replenishment policy of Example 3. One parameter is changed at a time, keeping the other parameters unchanged. The results are summarized in Table 1. Based on the numerical results, we obtain the following managerial implications

1. The total profit (TP) is highly sensitive with the change of the ordering cost K and significantly sensitive with respect to parameters P and p. The retailer should give more concentration in selecting these parameters for taking the optimal decision. It is moderately sensitive with change in the parameters K, δ, α and N and is less sensitive with change in the parameters θ and M.
2. The order quantity (Q) is reasonably sensitive with the change of the parameters δ, P and it is less sensitive for the parameters $K, \theta, \alpha, M, N, p$
3. From Table, we see that the increases in rate of deterioration θ leads to an increase in the total cost. Hence when the deterioration rate of products is more, the retailer should order less.
4. The backlogging rate decreases with increase in the backlogging parameter δ. Hence, when the backlogging rate increases, the total cost decreases. To achieve minimum total cost , the retailer should increases the backlogging rate by ordering more quantity.
5. When the unit purchase cost increases the retailer's total cost per unit time increases. Thus the retailers must reduce the unit purchase cost at lower cost but negotiating with the supplier ensuring them that they are going to make a higher order size.
6. When the fraction of delay payments α increase, the optimal order cycle T decreases, and the optimal total cost decrease. Increasing the delay payment α decrease default risks with credit-risk customers, demand rate and the optimal order cycle T. So, results in an decrease in the optimal total cost.
7. As the length of credit period M increases, the total optimal cost per unit time decreases. This clearly suggests that if the permissible delay period increases, then it helps the retailer to prolong the payment to the supplier without penalty and earn more from the interest earned and eventually results in minimum cost. To acquire minimum cost, industrial managers have to concentrate on their credit period tactics and cycle length.
8. If the length of the period N increases, the total cost increases slightly. Due to the increase in the trade credit period of the customer, total cost for the retailer is increasing.

5. CONCLUSION

In this work, described an appropriate EOQ model for deteriorating items with parabolic demand, in which the retailer receives an up-stream partial trade credit from its supplier, while the retailer offers a down-stream full trade credit to its customer under some realistic features. Shortages are allowed and can be partially backlogged, where the backlogging rate is dependent on the time of waiting for the next replenishment. The retailer always obtains the partial trade credit, which is independent of the order quantity offered by the supplier, but the wholesaler offers the full trade credit to the customer. Secondly, the retailer must take a loan to pay its supplier the partial payment immediately when the order is received and then pay off the loan with entire revenue. Behaviour of different parameters have been discussed through the numerical example and sensitivity analysis. Sensitivity analysis are presented to show the relationship between the changing parameter and the decision variables. From sensitivity analysis carried out the rate of change in the parameters $\varphi, K, p, h, \theta, \delta, \lambda, P, N$ and M is analyzed which helps to the business organization to make better managerial decision.

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