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TWO WAREHOUSE EOQ MODEL FOR IMPERFECT QUALITY ITEMS WITH STOCK AND TIME DEPENDENT DEMAND & LEARNING EFFECT UNDER TRADE CREDIT FINANCING POLICY SUBJECT TO PARTIAL BACKLOGGING

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ABSTRACT
A two warehouse (Own warehouse and Rented warehouse) EOQ model for imperfect quality items with stock and time dependent demand has been developed. It is assumed that the supplier has perfect and imperfect quality items. When the supplier provides lots for sale to his retailer, the retailer separates the whole lot by inspection process into perfect and imperfect quality items. The percentage of defective items present in the lot follows S-shape learning curve. For the imperfect quality items, retailer will avail a price discount from the supplier and customers also will get a price discount. Shortage at Own warehouse is allowed subject to partial backlogging. With this assumption, the model has been framed with learning effect and trade credit financing policy. Different cases based on the trade credit period have been considered. The objective of this work is to minimize the total inventory cost and to find the optimal length of replenishment and the optimal order quantity. Computational algorithms are designed to find the optimal order quantity and the optimal cycle time. To elucidate our model, hypothetical numerical examples and sensitivity analysis are carried out to decide the feasibility of renting a warehouse by availing a trade credit period for imperfect quality items.

Keywords: Two warehouses, Imperfect quality items, Stock and time dependent demand, Learning Effect, Trade credit financing policy, Partial backlogging

1.INTRODUCION
Demand plays a vital role while developing an inventory model. In real-life inventory systems, the demand rate of an item usually influenced by available stock and time. Large quantity of goods displayed in market attracts the customers to buy more. If the stock is insufficient, customer may prefer some other, it will go for loss to the supplier. Chung-yuan Dye [4] developed an Inventory model for deteriorating items with stock dependent demand and partial backlogging under the conditions of permissible delay in payments. Kun-Shan Wu, Liang-YuhOuyang, Chih-Te Yang [20] developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Geetha and Udayakumar [10] developed an Optimal replenishment policy for deteriorating items with time sensitive demand under trade credit financing. Khurana and Chaudhary [8] proposed an inventory model using stock and price-dependent demand for deteriorating items under shortage backordering. Krishna Prasad and Bani Mukherjee [19] developed an inventory model for stock and time dependent demand for time varying deterioration rate with shortage. Khurana and Chaudhary [7] developed a deteriorating inventory model for stock and time-dependent with partial backlogging.

In the real life, it is not always seen that all the products are of perfect nature. The sellers' inspection policies can uphold their reputation as well as fulfill customer demand and minimize their total cost. Jaggi et al. [3, 16], a mathematical model for the inventory was proposed for the imperfect items with the policy of financing period under shortages and permissible delay on payment. In Jaber and Salameh [15], a mathematical model was derived with shortages and backorder under leaning effect. Further, in Jaber et al. [13], the idea was stretched with the help of learning concepts for the imperfect items. A mathematical idea under impact of learning for the imperfect items was established in Khan et al. [17]. In Jaber and Khan [14], a model was discussed about the order of lots and the number of shipments for the imperfect items using the concept of learning. A model for defective items under learning effect and shortages was offered by Konstantaras et al. [18]. Wahab and Jaber [23] developed a note on Economic order quantity model for items with imperfect quality, different holding costs, and learning effects. Goyal [33] developed Economic order quantity model for imperfect lot with partial backordering under the effect of learning and advertisement dependent imprecise demand. After that, an optimal quantity model of Sangal et al. [28] was proposed with learning impact and shortages where deterioration is a function of time. In De and Mahata [6], an inventory model was investigated for defective items under cloudy fuzzy atmosphere. The commendable work in De and Mahata [6] has been improved by this present paper with the help of a learning effect and the financing period, where defective items follow the S-shape learning curve. Kuppulakshmi et al. [21] considered a fuzzy-based inventory model for defective items under penalty cost. The effects of learning operate as a significant function for reducing the inventory cost and also optimizing the total profit of the inventory system. Osama et al. [25] developed a Supply Chain Model with Learning Effect and Credit Financing Policy for Imperfect Quality Items under Fuzzy Environment.

Another important aspect in inventory management is, the price discount, low cost storage, huge demand etc. and under such a situation one may decide to procure large quantity of the items which would arise the problem of storing. As the capacity of own warehouse is limited, therefore one has to decide where to stock the goods. There are many such situations requiring additional storage facility. This additional storage capacity may be a Rented Warehouse (RW). This is to be hired to store the excess quantity. A model considering the effect of two warehouses was considered by Hartley [12] in which he assumed that the holding cost in RW is greater than that in OW; therefore, items in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero, and then items in OW are released.

In this direction, researchers have developed their inventory model for a single warehouse which has unlimited capacity. This assumption is not applicable in real-life situation. When an attractive price discount for bulk purchase is available, the management decides to purchase a huge quantity of items at a time. These goods cannot be stored in the existing storage (the owned warehouse with limited capacity). Another equally important aspect associated with inventory management is to decide where to stock the goods. There are many such situations requiring additional storage facility, for instance when one has to procure a larger stock that can't be accommodated in one's Own Warehouse (OW) because of its limited capacity. This additional storage capacity may be a Rented Warehouse (RW). A model considering the effect of two warehouses was considered by Hartley [12] in which he assumed that the holding cost in RW is greater than that in OW, due to the non-availability of better preserving facility which results in higher deterioration rate; therefore, items in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero, and then items in OW are released. Hence to reduce the holding cost, it is more economical to consume the goods of rented warehouse at the earliest.

Inventory model with double storage facility OW and RW was first developed by Hartley [12]. Palanivel M, Sundararajan R, Uthayakumar R [27] Two warehouse inventory model with non-instantaneously deteriorating items, stock dependent demand, shortages and inflation, Sahu and Bishi [32] extended the inventory deteriorating Items under permissible delay in payments. After his pioneering contribution, several other researchers have attempted to extend his work to various other realistic situations. In this connection, mention may be made of the studies undertaken by Sarma [29,30], Murdeshwar and Sathe [24], Pakkala and Achary [26], Dave [5], Bhunia and Maity [2] Yang [37,38], Singh and Sahu [31], Lee [22], Dey et al.[9] to name only a few.

In the classical time, the payment of the items was done exactly at the time of delivery or before it. But in the modern era, as the business is getting huge and complex, this practice is not possible. Nowadays, the retailer need not clear his dues at the time of delivery. Now Trade Credit is also known as permissible delay in payment, the practice followed by every business. In this, a grace period is provided by the supplier to his retailers to complete the payment. Trade credit is an essential tool for financing growth for many businesses. The number of days for which a credit is given is determined by the company allowing the credit and is agreed on by both the company allowing the credit and the company receiving it. By payment extension date, the company receiving the credit essentially could sell the goods and use the credited amount to pay back the debt. To encourage sales, such a credit is given. During this credit period, the retailer can accumulate and earn interest on the encouraged sales revenue. In case of an extension period, the supplier charges interest on the unpaid balance. Hence, the permissible delay period indirectly reduces the cost of holding cost. In addition, trade credit offered by the supplier encourages the retailer to buy more products. Hence, the trade credit plays a major role in inventory control for both the supplier as well as the retailer. Goyal [11] developed an EOQ model under the condition of a permissible delay in payments. Aggarwal and Jaggi [1] then extended Goyal's model to allow for deteriorating items under permissible delay in payments. Uthayakumar and Geetha [34,35] developed a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging and non-instantaneous deteriorating items with two levels of storage under trade credit policy.

The effects of learning play a vital role for reducing the inventory cost and optimizing the total profit of the inventory model. Some authors discussed the results of the learning shape in the same direction, such as Wright [36] and Jaber et al. [13]. From Figure 1, it can be observed that the curve rises slowly as one becomes familiar with the basics of a skill. The steep part occurs when one has enough experience to start "putting it all together." Then, a second phase of fast development is entered, known as Learning Phase 2. Skills are added along with the progress. At a certain moment, development achieves a speed of steady development, followed by a period of slower development. The final phase of top of the progress is known as maturity phase.

In Figure 2, the S-shape learning curve is graphically represented with the help of the available data which is provided below in the form of a formula: $P(n) = \frac{d}{f+e^{gn}}$ where d, f and $g > 0$ are the model parameters, n is the cumulative number of cycle and with $P(n)$ is the percentage of defective items per cycle n.

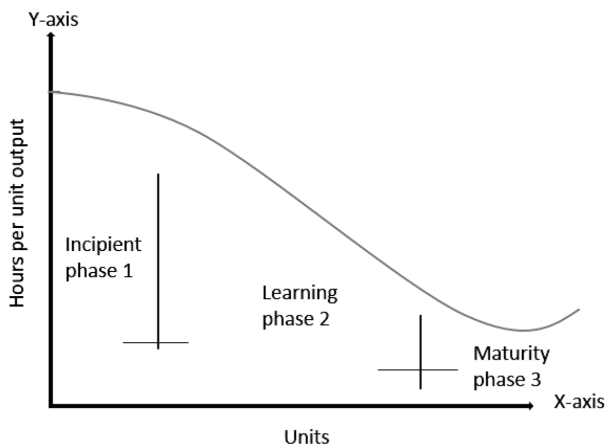

Fig 1: Learning Stage

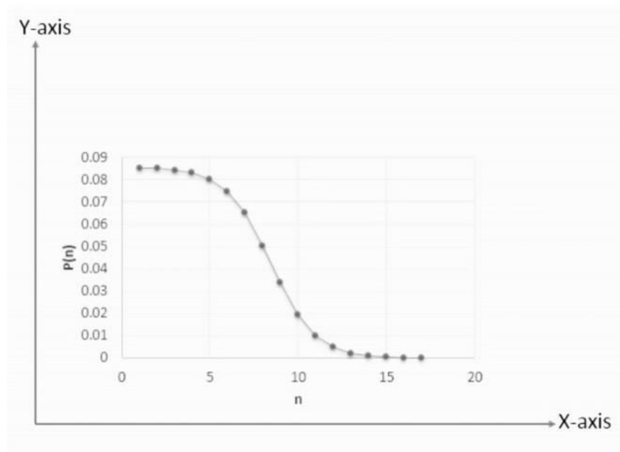

Fig 2: S-Shape Learning Curve

This paper aims to develop a two-warehousing inventory model for imperfect quality items with stock and time dependent demand under learning effect and the supplier offers the retailer a trade credit period to settle the amount. It is also assumed that the inventory holding cost in RW is higher than that in OW but the deterioration rate in RW is less than that in OW because RW offers better preserving facilities. In addition, shortages are allowed in OW and are partially backlogged. The optimal replenishment schedule has also been proposed. Finally, the numerical examples and managerial insights elucidate the performance of the model.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in the entire paper.

2.1 ASSUMPTIONS

1. Demand rate is known and which is stock and time dependent. The Consumption rate $D(t)$ at time t is assumed to be

$D(t) = \begin{cases} a + b\, I(t) + ct, & I(t) > 0, \\ a + ct, & I(t) \le 0 \end{cases}$ where a, b and c are positive constants.

2. The owned warehouse OW has limited capacity of W units and the rented warehouse RW has unlimited capacity. For economic reasons, the items of RW are consumed first and continues with those in OW once inventory stored at RW is exhausted. This implies that $t_r < T$.
3. The replenishment rate is infinite and the lead time is zero. The time horizon is infinite.
4. Both screening as well as demand proceeds simultaneously, but the screening rate is assumed to be greater than demand rate.
5. Imperfect quality items follow the S-shape learning curve with $P(n) = \frac{d}{f+e^{gn}}$ where d, f and $g > 0$ are the model parameters, n is the cumulative number of cycle and with $P(n)$ is the percentage of defective items per cycle n.
6. The supplier offers the retailer a permissible delay to settle the account on credit. During the trade credit period the account is not settled, the revenue is deposited in an interest bearing account. At the end of the permissible delay, the retailer pays off the items ordered, and starts to pay the interest charged on the items in stock.
7. Shortages are allowed and they are partially backlogged, the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant where $0 < \delta < 1$ and $(T - t)$ is the waiting time $(t_1 \le t \le T)$.

2.2 NOTATIONS

In addition, the following notations are used throughout this paper:

OW	-	The owned warehouse
RW	-	The rented warehouse
D	-	The demand per unit time
K	-	The replenishment cost per order ($/order)
p_1	-	The purchasing cost per perfect quality item ($/unit)
p_2	-	The purchasing cost per defective item ($/unit)
c_1	-	The selling price per perfect quality item ($/unit)
c_2	-	The selling price per defective item ($/unit)
c_s	-	The screening cost per unit item ($/unit)
S	-	The shortage cost per unit item ($/unit)
h_o	-	The holding cost per unit per unit time in RW
h_r	-	The holding cost per unit per unit time in OW
π	-	The opportunity cost per unit item ($/unit)
M	-	Permissible delay in settling the accounts

$P(n)$	-	Percentage of defective items
μ	-	Screening rate per unit item
I_p	-	The interest charged per dollar in stocks per year
I_e	-	The interest earned per dollar per year
$I_o(t)$	-	The inventory level in OW at time t
$I_r(t)$	-	The inventory level in RW at time t
I_m	-	Maximum Inventory level.
I_b	-	Maximum amount of shortage demand to be backlogged
W	-	The storage capacity of OW
Q	-	The retailer's order quantity (a decision variable)
TC_i	-	The total relevant costs
t_r	-	The time at which the inventory level reaches zero in RW
t_1	-	The time at which the inventory level reaches zero in OW
T	-	The length of replenishment cycle (a decision variable

3. MODEL FORMULATION

In the present study a two warehouse inventory model has been developed. There are certain circumstances, where the owned warehouse of the retailer is insufficient to store the goods. In that situation, the retailer may go for rented warehouse. To suit to this case, we develop the replenishment problem of a two-warehousing inventory model for a single imperfect quality item with stock and time dependent demand under trade credit financing policy and partial backlogging has been considered. Initially, the items are stored in Owned Warehouse (OW), after satisfying the OW; remaining items are stored in Rented Warehouse (RW) but uses the items of RW prior to the items of OW to satisfy the demand in order to reduce the inventory carrying charge (holding cost). A lot size of Q units enters instantaneously to the system. After meeting the backorders, I_m units enter the inventory system, out of which W units are kept in OW and the remaining $(I_m - W)$ units are kept in the RW. It is assumed that each lot received contains percentage defectives $\alpha = P(n)$ which follows S-Shape learning curve. Screening (inspection) process of the whole received lot is undertaken at a rate of μ units per unit time; During this period the demand and screening process occurs parallel and demand is fulfilled from the items which are found to be of perfect quality by the screening process and the defective items are used to eliminate backorders and sold immediately after the screening process at time t_s as a single batch with a price discount. Shortages are allowed and partially backlogged which are eliminated during the screening process as it has been assumed that screening rate is greater than the demand rate. The behavior of the inventory level is illustrated in Fig. 1, where T is the cycle length αQ is the number of defectives withdrawn from inventory, $t_s = \frac{Q}{\mu}$ is the total screening time of Q units ordered per cycle which is less than cycle time T and t_1, t_r are the inventory level reaches zero in OW and RW respectively.

Let $I_r(t)$ be the inventory level in RW at any time t, in the interval $(0, t_r)$. The change in the inventory level in RW is given by the following differential equation

$$\frac{dI_r(t)}{dt} = -(a + b\, I_{r1}(t) + ct), \qquad 0 \le t \le t_r$$

The solution of this differential equation with the initial condition $I_r(0) = (I_m - W)$ is given by

$$I_{r1}(t) = \frac{-a}{b} + \frac{c}{b^2} - \frac{ct}{b} + e^{-bt}\left\{(I_m - W) + \frac{a}{b} - \frac{c}{b^2}\right\} \qquad 0 \le t \le t_s$$

After the screening process, the number of defective items in RW at time t_s is $\alpha(I_m - W)$

Hence, the inventory level in RW during $t_s \le t \le t_r$ is given by

$$I_{r2}(t) = \frac{-a}{b} + \frac{c}{b^2} - \frac{ct}{b} + e^{-bt}\left\{(I_m - W) + \frac{a}{b} - \frac{c}{b^2}\right\} - \alpha(I_m - W)$$

From the boundary condition $I_r(t_r) = 0$, the maximum inventory level as

$$I_M = W - \frac{e^{-bt_r}(ab-c) - ab + c(1-bt_r)}{b^2\{e^{-bt_r} - \alpha)\}}$$

Let $I_o(t)$ be the inventory level in OW at any time t, in the interval $(0, t_1)$. The change in the inventory level in OW is given by the following differential equation

$$\frac{dI_o(t)}{dt} = -(a + b\, I_{r1}(t) + ct), \qquad 0 \le t \le t_1$$

Since there is no change in the inventory level in OW at the interval $(0, t_r)$ due to demand, the inventory level in OW is

$$I_{o1}(t) = W \qquad 0 \le t \le t_s$$

After the screening process, the number of defective items in OW at time t_s is αW

Therefore, the inventory level in OW at the interval (t_s, t_r) is given by

$$I_{o2}(t) = (1 - \alpha)W \qquad t_s \le t \le t_r$$

During the interval (t_r, t_1) the inventory level decreases to zero due to demand and from the boundary condition $I_{o3}(t_1) = 0$, the inventory level in OW for this period is given by

$$I_{o3}(t) = \frac{-a}{b} + \frac{c}{b^2} - \frac{ct}{b} + e^{-b(t-t_1)}\left\{\frac{a}{b} - \frac{c}{b^2} + \frac{ct_1}{b}\right\} - \alpha W$$

During the interval (t_1, T), the inventory level is given by the differential equation

$$\frac{dI_{o4}(t)}{dt} = \frac{-(a + ct)}{1 + \delta(T - t)}, \qquad t_1 \le t \le T$$

Solving the above differential equation with the condition $I_{o4}(t_1) = 0$ is given by

$$I_{o4}(t) = \frac{c}{\delta}(t - t_1) + \frac{(c + \delta(a + cT))}{\delta^2}\{\log(1 + \delta(T - t)) - \log(1 + \delta(T - t_1))\}$$

Furthermore, the continuity of $I_{o2}(t_r) = I_{o3}(t_r)$ we get

$$W = \frac{1}{b^2}(ab - c(1 - bt_1))(e^{-b(t_r - t_1)} - 1)$$

The maximum backlogging quantity is given by

$I_{o4}(T) = I_b$

$I_b = \frac{c}{\delta}(T - t_1) - \frac{(c+\delta(a+cT))}{\delta^2}\{\log(1 + \delta(T - t_1))\}$

Hence the maximum order quantity is $Q = I_m + I_b$

The total inventory cost per cycle consists of the following elements

a) Cost of placing order is K

b) Sales Revenue is the sum of revenue generated by the demand meet during the time period $(0, T)$ and by the sale of imperfect quality items is $c_1(1-\alpha)Q + c_2\alpha Q$

c) Purchase Cost is $p_1(1-\alpha)Q + p_2\alpha Q$

d) Screening Cost $c_s Q$

e) Inventory holding cost HC per cycle is given by

$$HC = h_r\left\{\int_0^{t_s} I_{r1}(t)dt + \int_{t_s}^{t_r} I_{r2}(t)dt\right\} + h_o\left\{\int_0^{t_s} I_{o1}(t)dt + \int_{t_s}^{t_r} I_{o2}(t)dt + \int_{t_r}^{t_1} I_{o3}(t)dt\right\}$$

$$HC = \frac{h_r}{b^3}\left\{(-(I_m - W)b^2 - ba + c)e^{-bt_r} + \left(I_m - W - at_r - \frac{ct_r^2}{2}\right)b^2 + (a + ct_r)b - c\right\} + \frac{h_o}{b^3}\left\{(ab + bct_1 - c)e^{b(t_1 - t_r)} + Wb^3(t_r + \alpha(t_s - t_1)) - (t_1 - t_r)b^2\left(c\frac{(t_1+t_r)}{2} + a\right) - (ab + bct_r - c)\right\}$$

f) Shortage cost per cycle SC is given by

$$SC = s\int_{t_1}^{T} -I_{o4}(t)dt$$

$$SC = -\frac{s}{2\delta^3}\left\{2(a\delta + cT\delta + c)\log(1 + \delta(T - t_1)) - \delta(T - t_1)\{\delta(c(T + t_1) + 2a) + 2c\}\right\}$$

g) Opportunity cost per cycle due to lost sales OC is given by

$$OC = \pi\int_{t_1}^{T}\left(a + ct - \frac{a+ct}{1+\delta(T-t)}\right)dt$$

$$OC = \frac{-\pi}{2\delta^2}\{2(a\delta + cT\delta + c)\log(1 + \delta(T - t_1)) - \delta(T - t_1)\{\delta(c(T + t_1) + 2a) + 2c\}\}$$

Based on the assumptions and description of the model, the total annual cost which is a function of t_r, t_1, and T is given by

$$TC(t_1, T) = \begin{cases} TC_1(t_1, T), & t_s < M \le t_r \\ TC_2(t_1, T), & t_r < M \le t_1 \\ TC_3(t_1, T), & M > t_1 \end{cases}$$

Figure 3 depicts the following 3 cases.

3.1 CASE 1: $0 < t_s \le M \le t_r$

When the credit period is shorter than or equal to the length of period with positive inventory stock of items ($M \leq t_1$), payment for goods is settled and retailer starts paying the interest for the goods still in stocks with annual rate I_p. Thus the interest payable denoted by IP_1 and it is given by

$$IP_1 = p_1 I_p \left\{ \int_M^{t_r} I_{r2}(t)dt + \int_M^{t_r} I_{o2}(t)dt + \int_{t_r}^{t_1} I_{o3}(t)dt \right\}$$

$$IP_1 = p_1 I_p \{((2ct_1 + 2a)b - 2c)e^{b(t_1 - t_r)} + (2(I_m - W)b^2 + 2ab - 2c)(e^{-bM} - e^{-bt_r}) + b^3\{(2W - 2(I_m - W)\alpha)t_r + (2(MI_m - Wt_1)\alpha - 2MW)\} + (cM + ct_1 + 2a)(M + t_1)b^2 - 2(cM - a)b + 2c\}$$

We assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with a rate I_e. Thus the interest earned per cycle is given by IE_1

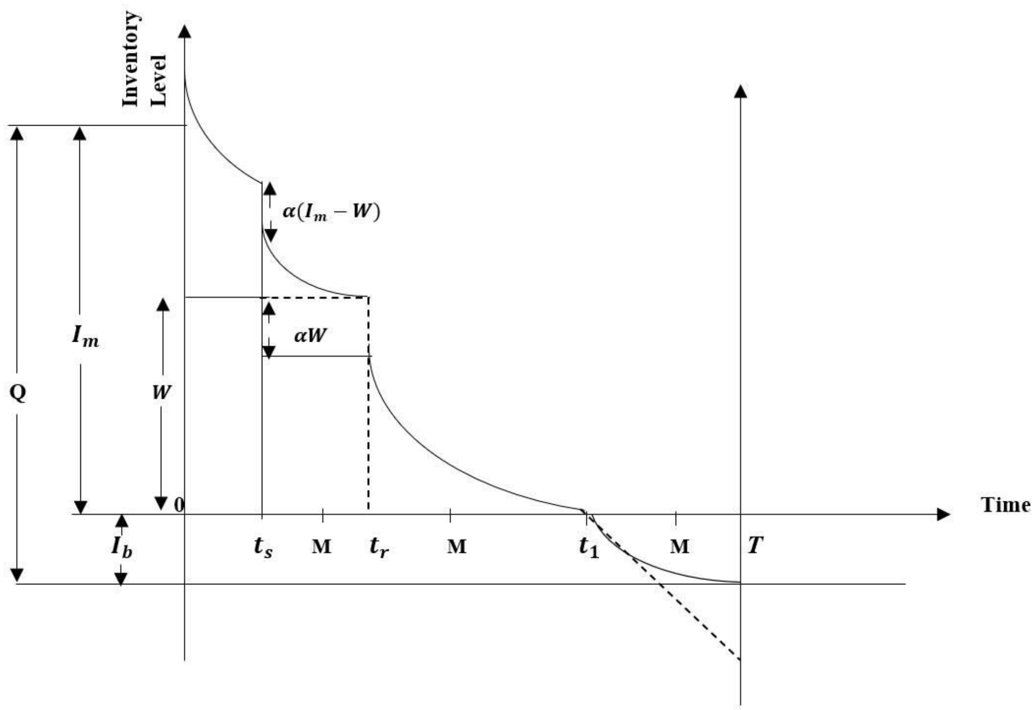

Fig: 3

$$IE_1 = c_1 I_e \left\{ \int_0^{t_s} (a + bI_{r1}(t) + ct)t\,dt + \int_{t_s}^{M} (a + bI_{r2}(t) + ct)t\,dt \right\} + c_2 I_e \{(I_m - W)\alpha + \alpha W\}$$

$$IE_1 = c_1 I_e \left\{ \frac{1}{2b^3} \{2(bM + 1)((-I_m + W)b^2 - ab + c)e^{-bM} + \alpha(M^2 - t_s{}^2)(-I_m + W)b^4 + (cM^2 - 2W + 2I_m)b^2 + 2ab - 2c\} \right\} + c_2 I_e \{(I_m - W)\alpha + \alpha W\}$$

Thus, the total annual cost which is a function of t_1 and T is given by

$$TC_1(t_1, T) = \frac{1}{T}\{K - \text{Sales Revenue} + \text{Purchase Cost} + \text{Screening Cost} + HC + DC + SC + OC + IP_1 - IE_1\}$$

$TC_1(t_1,T) = \frac{1}{T}\Big\{K - \{c_1(1-\alpha)Q + c_2\alpha Q\} + p_1(1-\alpha)Q + p_2\alpha Q + c_s Q + \frac{h_r}{b^3}\Big\{(-(I_m - W)b^2 - ba + c)e^{-bt_r} + \left(I_m - W - at_r - \frac{ct_r^2}{2}\right)b^2 + (a + ct_r)b - c\Big\} + \frac{h_o}{b^3}\Big\{(ab + bct_1 - c)e^{b(t_1-t_r)} + Wb^3(t_r + \alpha(t_s - t_1)) - (t_1 - t_r)b^2\left(c\frac{(t_1+t_r)}{2} + a\right) - (ab + bct_r - c)\Big\} - \frac{s}{2\delta^3}\Big\{2(a\delta + cT\delta + c)\log\big(1 + \delta(T - t_1)\big) - \delta(T - t_1)\{\delta(c(T + t_1) + 2a) + 2c\}\Big\} - \frac{\pi}{2\delta^2}\{2(a\delta + cT\delta + c)\log\big(1 + \delta(T - t_1)\big) - \delta(T - t_1)\{\delta(c(T + t_1) + 2a) + 2c\}\} + p_1 I_p\{\big((2ct_1 + 2a)b - 2c\big)e^{b(t_1-t_r)} + (2(I_m - W)b^2 + 2ab - 2c)(e^{-bM} - e^{-bt_r}) + b^3\{(2W - 2(I_m - W)\alpha)t_r + (2(MI_m - Wt_1)P(n) - 2MW)\} + (cM + ct_1 + 2a)(M + t_1)b^2 - 2(cM - a)b + 2c\} - c_1 I_e\Big\{\frac{1}{2b^3}\{2(bM + 1)\big((-I_m + W)b^2 - ab + c\big)e^{-bM} + \alpha(M^2 - t_s^2)(-I_m + W)b^4 + (cM^2 - 2W + 2I_m)b^2 + 2ab - 2c\}\Big\} + c_2 I_e\{(I_m - W)\alpha + \alpha W\}\Big\}$

3.2 CASE 2: $t_r \leq M \leq t_1$

Interest payable for this period is denoted by IP_2 is given by

$IP_2 = p_1 I_p\left\{\int_M^{t_1} I_{o3}(t)dt\right\}$

$IP_2 = p_1 I_p\Big\{\frac{1}{2b^3}\{(2b(a + ct_1) - 2c)e^{-b(M-t_1)} + 2WP(n)(M - t_1)b^3 + (M - t_1)\{(M + t_1)c + 2a\}b^2 - 2b(a + cM) + 2c\}\Big\}$

The interest earned from the accumulated sales during this period is

$IE_2 = c_1 I_e\left\{\int_0^{t_s}(a + bI_{r1}(t) + ct)tdt + \int_{t_s}^{t_r}(a + bI_{r2}(t) + ct)tdt + \int_{t_r}^{M}(a + bI_{o3}(t) + ct)tdt\right\} + c_2 I_e\{(I_m - W)\alpha + \alpha W\}$

$IE_2 = c_1 I_e\Big\{\frac{1}{2b^3}\{(bM + 1)(2b(a + ct_1) - 2c)e^{-b(M-t_1)} + (2b(a + ct_1) - 2c)(bt_r + 1)e^{b(t_1-t_r)} + 2\big((-I_m + W)b^2 - ab + c\big)(bt_r + 1)e^{-bt_r} - \alpha\big((-I_m + W)t_s^2 + (I_m - 2W)t_r^2 + M^2W\big)b^4 + (cM^2 + 2I_m - 2W)b^2 + 2ab - 2c\}\Big\} - 2c_2 I_e\{(I_m - W)\alpha + \alpha W\}$

Thus, the total annual cost which is a function of t_1 and T is given by

$TC_2(t_1,T) = \frac{1}{T}\{K + \text{Sales Revenue} + \text{Purchase Cost} + \text{Screening Cost} + HC + DC + SC + OC + IP_2 - IE_2\}$

$$TC_2(t_1,T)=\frac{1}{T}\Big\{K-\{c_1(1-\alpha)Q+c_2\alpha Q\}+p_1(1-\alpha)Q+p_2\alpha Q+c_sQ+\frac{h_r}{b^3}\Big\{(-(I_m-W)b^2-ba+c)e^{-bt_r}+\left(I_m-W-at_r-\frac{ct_r^{\,2}}{2}\right)b^2+(a+ct_r)b-c\Big\}+\frac{h_0}{b^3}\Big\{(ab+bct_1-c)e^{b(t_1-t_r)}+Wb^3(t_r+\alpha(t_s-t_1))-(t_1-t_r)b^2\left(c\frac{(t_1+t_r)}{2}+a\right)-(ab+bct_r-c)\Big\}-\frac{s}{2\delta^3}\Big\{2(a\delta+cT\delta+c)\log\big(1+\delta(T-t_1)\big)-\delta(T-t_1)\{\delta(c(T+t_1)+2a)+2c\Big\}-\frac{\pi}{2\delta^2}\{2(a\delta+cT\delta+c)\log\big(1+\delta(T-t_1)\big)-\delta(T-t_1)\{\delta(c(T+t_1)+2a)+2c\}\}+p_1I_p\Big\{\frac{1}{2b^3}\{(2b(a+ct_1)-2c)e^{-b(M-t_1)}+2W\alpha(M-t_1)b^3+(M-t_1)\{(M+t_1)c+2a\}b^2-2b(a+cM)+2c\Big\}-c_1I_e\Big\{\frac{1}{2b^3}\{(bM+1)(2b(a+ct_1)-2c)e^{-b(M-t_1)}+(2b(a+ct_1)-2c)(bt_r+1)e^{b(t_1-t_r)}+2\big((-I_m+W)b^2-ab+c\big)(bt_r+1)e^{-bt_r}-\alpha\big((-I_m+W)t_s^{\,2}+(I_m-2W)t_r^{\,2}+M^2W\big)b^4+(cM^2+2I_m-2W)b^2+2ab-2c\Big\}-2c_2I_e\{(I_m-W)\alpha+\alpha W\}\Big\}$$

3.3 CASE 3: $t_1 \le M \le T$

For this period there is no interest payable, but the interest earned from the accumulated sales during this period is

$$IE_3=c_1I_e\left\{\int_0^{t_s}(a+bI_{r1}(t)+ct)tdt+\int_{t_s}^{t_r}(a+bI_{r2}(t)+ct)tdt+\int_{t_r}^{t_1}(a+bI_{o3}(t)+ct)tdt\right\}+c_2I_e\{(I_m-W)\alpha+\alpha W\}-(M-t_1)\left\{c_1I_e\left\{\int_0^{t_s}(a+bI_{r1}(t)+ct)tdt+\int_{t_s}^{t_r}(a+bI_{r2}(t)+ct)tdt+\int_{t_r}^{t_1}(a+bI_{o3}(t)+ct)tdt\right\}+c_2I_e\{(I_m-W)\alpha+\alpha W\}\right\}$$

$$IE_3=c_1I_e(1+M+t_1)\Big\{\frac{1}{2b^3}\{(bt_r+1)\big(2(b(a+ct_1)-c)\big)e^{b(t_1-t_r)}+2\big((-I_m+W)b^2-ab+c\big)(bt_r+1)e^{-bt_r}-b^4\big(\alpha(-I_m+W)t_s^{\,2}+(I_m-2W)t_r^{\,2}+t_1^{\,2}W\big)-(ct_1^{\,2}-2I_m+2W+2at_1)b^2\}-c_2I_e\alpha(2W-I_m)\Big\}$$

Thus, the total annual cost which is a function of t_1 and T is given by

$$TC_3(t_1,T)=\frac{1}{T}\{K+\text{Sales Revenue}+\text{Purchase Cost}+\text{Screening Cost}+HC+DC+SC+OC-IE_3\}$$

$$TC_3(t_1,T)=\frac{1}{T}\Big\{K-\{c_1(1-\alpha)Q+c_2\alpha Q\}+p_1(1-\alpha)Q+p_2\alpha Q+c_sQ+\frac{h_r}{b^3}\Big\{(-(I_m-W)b^2-ba+c)e^{-bt_r}+\left(I_m-W-at_r-\frac{ct_r^{\,2}}{2}\right)b^2+(a+ct_r)b-c\Big\}+\frac{h_0}{b^3}\Big\{(ab+bct_1-c)e^{b(t_1-t_r)}+Wb^3(t_r+\alpha(t_s-t_1))-(t_1-t_r)b^2\left(c\frac{(t_1+t_r)}{2}+a\right)-(ab+bct_r-c)\Big\}-\frac{s}{2\delta^3}\Big\{2(a\delta+cT\delta+c)\log\big(1+\delta(T-t_1)\big)-\delta(T-t_1)\{\delta(c(T+t_1)+2a)+2c\Big\}-\frac{\pi}{2\delta^2}\{2(a\delta+cT\delta+c)\log\big(1+\delta(T-t_1)\big)-\delta(T-t_1)\{\delta(c(T+t_1)+2a)+2c\}\}-c_1I_e(1+M+t_1)\Big\{\frac{1}{2b^3}\{(bt_r+1)\big(2(b(a+ct_1)-c)\big)e^{b(t_1-t_r)}+2\big((-I_m+W)b^2-ab+c\big)(bt_r+1)e^{-bt_r}-b^4\big(\alpha(-I_m+W)t_s^{\,2}+(I_m-2W)t_r^{\,2}+t_1^{\,2}W\big)-(ct_1^{\,2}-2I_m+2W+2at_1)b^2\}-c_2I_e\alpha(2W-I_m)\Big\}\Big\}$$

The necessary conditions for the total annual cost $\partial TC_i(t_1,T)$ is convex with respect to t_1 and T are $\frac{\partial TC_i(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_i(t_1,T)}{\partial T}=0$ where $i=1,2,3$ (1)

Provided they satisfy the sufficient conditions

$$\left.\frac{\partial^2 TC_i(t_1,T))}{\partial {t_1}^2}\right|_{(t_1^*,T^*)}>0, \left.\frac{\partial^2 TC_i(t_1,T))}{\partial T^2}\right|_{(t_1^*,T^*)}>0 \text{ and}$$

$$\left.\left\{\left(\frac{\partial^2 TC_i(t_1,T)}{\partial {t_1}^2}\right)\left(\frac{\partial^2 TC_i(t_1,T))}{\partial T^2}\right)-\left(\frac{\partial^2 TC_i(t_1,T)}{\partial t_1 \partial T}\right)^2\right\}\right|_{(t_1^*,T^*)}>0 \quad (2)$$

To acquire the optimal values of t_1 and T that minimize $TC_i(t_1,T)$, we develop the following algorithm to find the optimal values of t_1 and T (say, ${t_1}^*$ and T^*).

ALGORITHM

Step 1: Start

Step 2: Evaluate $\frac{\partial TC_i(t_1,T)}{\partial t_1}$ and $\frac{\partial TC_i(t_1,T)}{\partial T}$ where $i=1,2,3$.

Step 3: Solve the simultaneous equation $\frac{\partial TC_i(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_i(t_1,T)}{\partial T}=0$ by fixing M, t_r and initializing the values of $K, a, b, c, \alpha, \delta, s, \pi, p_1, p_2, c_1, c_2, h_r, h_o, t_s, I_p, I_e$

Step 4: Choose one set of solution from step 3.

Step 5: If the values in equation (2) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step 3.

Step 6: Evaluate $TC_i({t_1}^*,T^*)$

Step 7: End

Our aim is to find the optimal values of t_1 and T which minimize $TC({t_1}^*,T^*)$

$TC({t_1}^*,T^*)=Min\{TC_1({t_1}^*,T^*), TC_2({t_1}^*,T^*), TC_3({t_1}^*,T^*)\}$

4. NUMERICAL EXAMPLES

The following examples illustrate our solution procedure:

Example 1: Consider an inventory system with the following data: $K=1000, a=500, b=2, c=0.2, t_s=0.0137, t_r=0.0411, h_r=2, h_o=1, p_1=20, p_2=15, c_1=35, c_2=25, c_s=0.5, s=18, \pi=10, M=0.039, \delta=0.56, I_p=.13, I_e=0.12, \alpha=0.05, W=250$

in appropriate units. In this case, we see that $t_s<M<t_r$. Therefore, applying algorithm for Case 1, we get the optimal solutions, $t_1=0.6274, T=0.7169$ the corresponding total cost $TC=1804.30$ and the ordering quantity $Q=275.65$ units

Example 2: The data are the same as in Example 1 except $M=0.0822$ in appropriate units. In this case, we see that $t_r<M<t_1$. Therefore, applying algorithm for Case 2, we get the optimal solutions, $t_1=0.3512, T=0.6354$ the corresponding total cost $TC=1748.88$ and the ordering quantity $Q=281.38$ units.

Example 3: The data are the same as in Example 1 except $M = 0.3288$ in appropriate units. In this case, we see that $M > t_1$. Therefore, applying algorithm for Case 3, we get the optimal solutions, $t_1 = 0.3248, T = 0.6152$ the corresponding total cost $TC = 1553.89$ and the ordering quantity $Q = 297.21$ units.

Effect of change in various parameters of the inventory in example 3

Changing parameter	% Change in parameter	Change in parameter	t_1	T	TC	Q
a	-20%	400	0.3285	0.5552	1811.29	292.69
	-10%	450	0.3267	0.5850	1676.25	294.95
	0%	500	0.3248	0.6152	1553.89	297.21
	+10%	550	0.3229	0.6456	1443.21	299.47
	+20%	600	0.3209	0.6763	1342.81	301.72
b	-20%	1.6	0.3377	0.6261	1533.16	297.00
	-10%	1.8	0.3311	0.6204	1543.94	297.10
	0%	2.0	0.3248	0.6152	1553.89	297.21
	+10%	2.2	0.3189	0.6104	1563.05	297.31
	+20%	2.4	0.3134	0.6059	1571.47	297.42
c	-20%	0.16	0.3248	0.6152	1553.88	297.20
	-10%	0.18	0.3248	0.6152	1553.88	297.21
	0%	0.20	0.3248	0.6152	1553.89	297.21
	+10%	0.22	0.3248	0.6152	1553.89	297.21
	+20%	0.24	0.3248	0.6152	1553.90	297.21
δ	-20%	0.448	0.4028	0.8451	1206.58	298.21
	-10%	0.504	0.3660	0.7317	1347.43	297.69
	0%	0.560	0.3248	0.6152	1553.89	297.21
	+10%	0.616	0.2783	0.4957	1876.66	296.74
	+20%	0.672	0.2253	0.3733	2434.95	296.30
α	-20%	0.040	0.3314	0.6374	1483.11	296.95
	-10%	0.045	0.3281	0.6263	1517.87	297.08
	0%	0.050	0.3248	0.6152	1553.89	297.21
	+10%	0.055	0.3215	0.6041	1591.23	297.34
	+20%	0.060	0.3181	0.5931	1629.96	297.47
t_r	-20%	0.0329	0.2711	0.4928	2032.33	292.58
	-10%	0.0370	0.2988	0.5545	1763.62	294.91
	0%	0.0411	0.3248	0.6152	1553.89	297.21
	+10%	0.0452	0.3512	0.6793	1374.53	299.64
	+20%	0.0493	0.3760	0.7422	1229.93	302.04
M	-20%	0.2630	0.3246	0.6147	1555.41	297.21
	-10%	0.2959	0.3247	0.6150	1554.65	297.21
	0%	0.3288	0.3248	0.6152	1553.89	297.21
	+10%	0.3617	0.3249	0.6154	1553.13	297.21
	+20%	0.3946	0.3250	0.6157	1552.38	297.21

Graphical Representation of the Effect of Change in Various Parameters of the Inventory

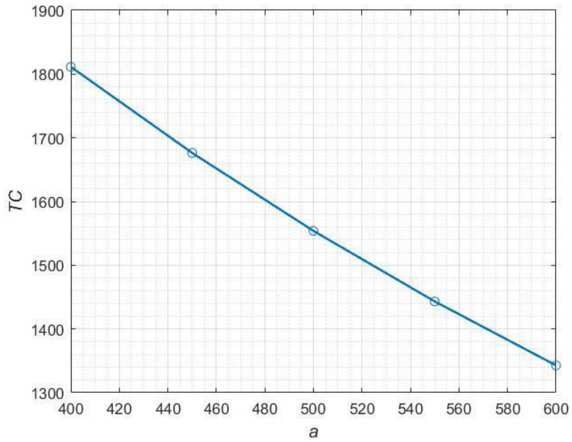

Fig 4. Effect of change of a on Total cost

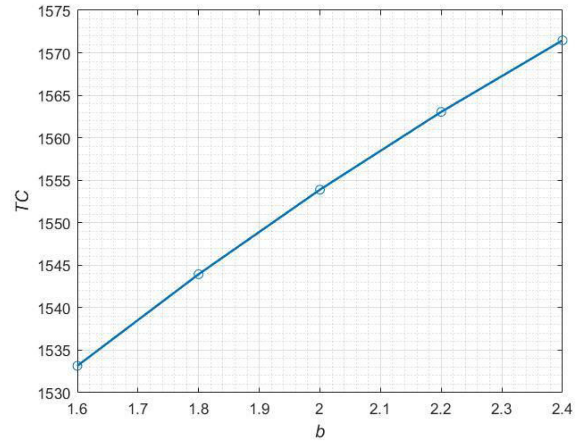

Fig 5. Effect of change of b on Total cost

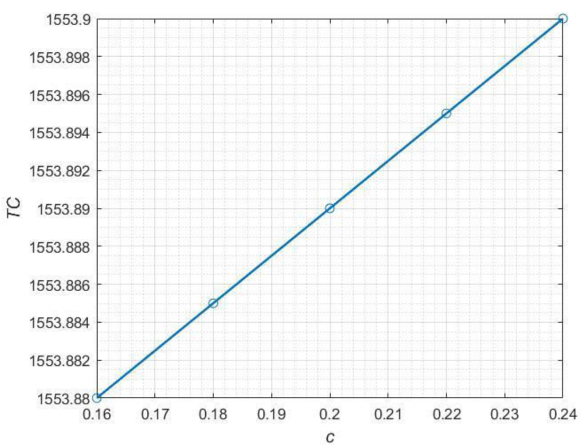

Fig 6. Effect of change of c on Total cost

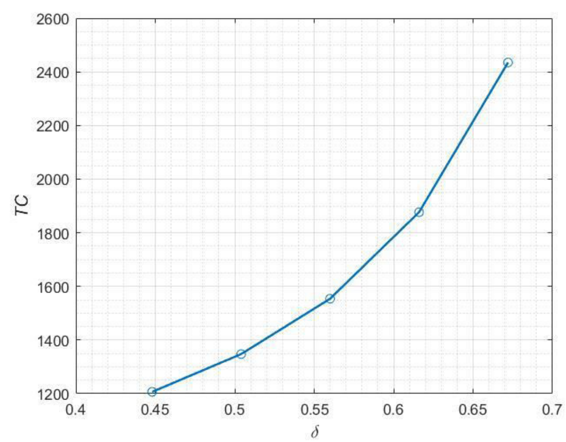

Fig 7. Effect of change of δ on Total cost

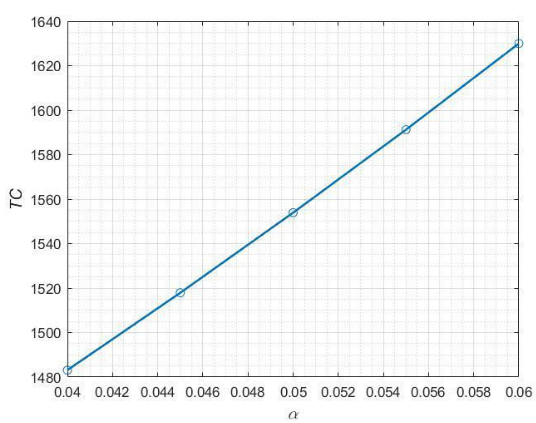

Fig 8. Effect of change of α on Total cost

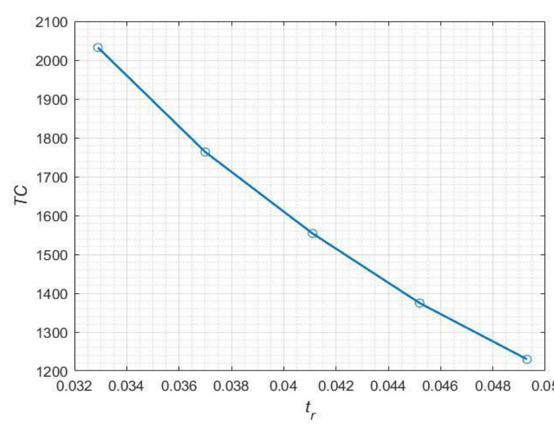

Fig 9. Effect of change of t_r on Total cost

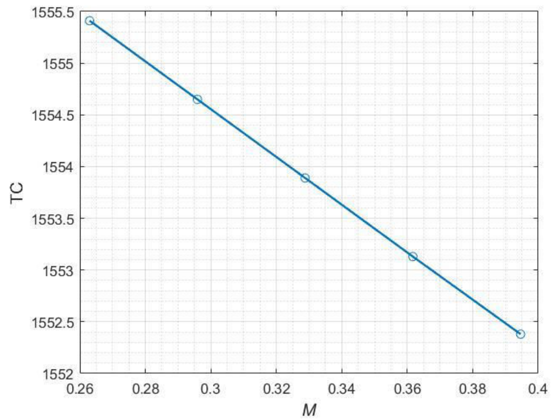

Fig 10. Effect of change of M on Total cost

5. SENSITIVITY ANALYSIS

A change in the values of parameters may happen due to uncertainties. In order to examine these changes, a sensitivity analysis will help in decision-making. We now study the effects of changes in the values of the system parameters $a, b, c, \delta, \alpha, t_r$ and M on the optimal replenishment policy of Example 3. We change one parameter at a time, keeping the other parameters unchanged. The results are summarized in Table 1. Based on our numerical results, we obtain the following managerial implications:

(1) When the parameter a increases, the total optimal cost (TC), the time at which the inventory level becomes zero in OW (t_1) decreases. But the cycle length (T) and order quantity (Q) increases. That is, increasing of the parameter a will decrease the total cost of the retailer.

(2) When the parameter b increases, the total optimal cost (TC) and the order quantity (Q) increases. But the time at which the inventory level becomes zero in OW (t_1), the cycle length (T) increases. That is, increasing of the parameter b will increase the total cost of the retailer.

(3) When the deterioration rate c increases, the total optimal cost (TC) and the order quantity (Q) increases. The time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) are decreasing by small quantity. That is, increasing of the parameter c will increase the total cost of the retailer.

(4) If the backlogging parameter δ increases, the total optimal cost (TC) increases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) and the order quantity (Q) decreases. That is, to minimize the cost, the retailers should decrease the backlogging parameter.

(5) If percentage of defective items (α) increases, the total optimal cost (TC) and the order quantity (Q) increases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) are decreasing. That is, increasing percentage of defective items will increase the total cost of the retailer.

(6) If the time at which the inventory level becomes zero in RW (t_r) increases, the total optimal cost (TC) decreases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) and the order quantity (Q) increases. That is, increasing of the parameter t_r will decrease the total cost of the retailer.

(7) If the Credit period M increases, the total optimal cost (TC) decreases. But the time at which the inventory level becomes zero in OW (t_1), the cycle length (T) and the order quantity (Q) increases. That is, in order to minimize the cost, the retailers should increase the Credit period M.

From the sensitivity analysis we could see that increasing the parameters a, t_r, M will decrease the total annual inventory cost and decreasing the parameters b, c, δ, α will decrease the total annual inventory cost.

6. CONCLUSION

In this model, a two-warehouse EOQ model for imperfect quality items with stock and time dependent demand under trade credit period and partial backlogging has been developed. Our model suits well for the retailer in situations involving unlimited storage space. The aim of this paper is to obtain the optimal solution of cycle length, time intervals, order quantity and minimizing the total cost of the retailer. We presented a computational algorithm to find the optimal solution. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions are also provided. Numerical examples and a sensitivity analysis are carried out. From the managerial insights we could see that the rate of change of the parameters $a, b, c, \alpha, \delta, t_r$ and M affects the total annual inventory cost and ordering quantity. From the results obtained, we see that the retailer can reduce total annual inventory cost by ordering lower quantity when the supplier provides a permissible delay in payments by improving storage conditions for stock and time dependent imperfect quality items and learning effect to reduce the percentage of defective items in each replenishment. Therefore, this model provides a new managerial insight which helps the industry to reduce the total inventory cost by renting a warehouse and availing a trade credit period with learning effect of defective items.

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