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MHD FLOW AND HEAT CONVECTION WITH JOULE HEATING IN A CHANNEL FILLED WITH OPEN-CELL METAL FOAM POROUS MEDIA OF VARIABLE PERMEABILITY

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ABSTRACT
A channel region of semi-infinite length between two nonporous parallel plates packed with isotropic homogeneous open-celled porous metallic foam is considered for the steady flow convective heat transfer of electrically conducting viscous incompressible fluid in the presence of static transverse magnetic field. The effect of magnetic field in the form of Joule dissipation is taken into account in the local thermal non equilibrium (LNTE) model for the study of heat convection. The modeled problem is governed by a set of coupled non-linear differential equations. The solutions are obtained by the quasi-numerical method, the Differential Transform Method (DTM). The velocity profiles and temperature profiles for fluid phase and solid phase are discussed through graphs. The heat convection property for both the fluid phase and solid phase is discussed by Nusselt number. With the increase in Hartmann number, Reynolds number, Stenton number and Biot type number, the value of heat transfer coefficient for both fluid and solid increases.

Keywords: Parallel-plate channel, LNTE model DTM, Open-cell metallic foam, Hartmann number etc.

INTRODUCTION
Metal foam is a cellular structure consisting of a solid metal (e.g. Aluminum, Copper) and containing a large volume of gas filled in pores. The metal foams are made of non-flammable metal and have a natural characteristic of high porosity of 75-95% of the volume. These types of porous materials have a wide range of applications in medical, automobile, defense and various industrial purposes. Qualitatively metallic foam is comparatively good to polymer foams as these are stiffer, stronger, fire resistant, better weather properties and high energy absorbent. The flow through channel type configurations are encountered in many industries in one or another ways for cooling purposes or for heat convection processes. In industry, metallic foams are primarily used as heat exchangers. During the review of literature, it was found that for numerous application purposes the flow through a channel filled with porous medium are studied. Chamkha et al.[4] discussed unsteady laminar hydromagnetic flow and heat transfer in porous channels with temperature dependent properties. Zhao et al. [10] studied the heat dissipation capability of highly porous cellular metal foams with open cells subject to forced air convection with combined experimental and analytical approach. Baoku, et al. [2] investigated effects of thermal radiation and magnetic field on hydromagnetic Couette flow of a highly viscous fluid with temperature-dependent viscosity and thermal conductivity at constant pressure through a porous channel. Xu, et al. [8] investigated fully developed forced convective heat transfer in a parallel-plate channel partially filled with highly porous, open-celled metallic foam. Xu, et al. [9] worked out for fully developed flow and forced convective heat transfer through highly porous, open-celled metallic foams bounded between two infinite parallel plates. Piller, et al. [7] numerically investigated the natural convection in inclined parallel-plate channel partly filled with metal foams. Attia, et al. [1] examined the effect of variation of physical variables on a steady flow through a porous medium with heat transfer between parallel plates with temperature dependent viscosity. In view of enhancement of convection of heat due to metallic foam porous media and significance of magnetic field in engineering applications, the present study is an attempt to investigate the effect of transverse magnetic field applied on Darcy flow through parallel

plates filled with high porous metallic foam with exponentially decreasing permeability along the width of the axisymmetric channel.

PROBLEM FORMULATION

A horizontal channel of two parallel plates filled with open cell metallic foam porous media is taken to study flow and heat convection for viscous incompressible electrically conducting fluid. The plates are at 2R distance apart and symmetrical about the x-axis while the axis of y is normal to the channel. A constant transverse magnetic field $(0, B_0, 0)$ is applied. The permeability of the metallic foam is taken as exponentially decreasing along the width of the channel, i.e. with respect to y and given by $K(y) = K_0 e^{-\frac{y}{R}}$. It is also assumed that the flow is fully developed and no thermal radiation. Under the above assumptions and geometrical configurations, the modeled problem is defined by equation of continuity, equations of motion and heat equation.

The equation of continuity for incompressible fluid with velocity field $\vec{q}$ is defined by

$$\nabla.\vec{q} = 0$$

(1)

The equation of motion for the present problem is given by

$$\frac{\rho}{\varphi^2}(\vec{q}\cdot\nabla\vec{q})\vec{q} = -\nabla p + \frac{\mu}{\varphi}\nabla^2\vec{q} - \frac{\mu}{K}\vec{q} + \vec{J}\times\vec{B}$$

(2)

For the local thermal non-equilibrium model the two heat equations for flow of fluid and solid is given by Energy equation for the fluid in porous medium (LTNE)

$$(\rho C_p)_f\left(\frac{1}{\varphi}\right)\vec{q}.\nabla T_f = \nabla\left[\kappa_{fe}\nabla T_f\right] + h_{sf}a_{sf}\left(T_s - T_f\right) + \frac{\vec{J}^2}{\rho\varphi^2} \quad \textbf{(3)}$$

Energy equation for solid matrix (LTNE)

$$\nabla\left[\kappa_{se}\nabla T_s\right] - h_{sf}a_{sf}\left(T_s - T_f\right) = 0 \quad \textbf{(4)}$$

Where κ_{fe}, κ_{se} are the effective thermal conductivities of fluid and solid respectively, T_f and T_s are volume-averaged Temperature of fluid and solid respectively, h_{sf} is the heat transfer coefficient between the solid and fluid, ρ is the density of the fluid, C_p is the specific heat of the fluid, $\vec{q}$ is the volume-averaged velocity, a_{sf} specific surface area, $\vec{J}$ current density, $\vec{B}$ applied magnetic field. In view of the physical configuration of the problem, the equation of continuity (1) reduces to

$$\frac{\partial u}{\partial x} = 0 \quad (5)$$

$\Rightarrow$ u is only function of y

Under the prescribed geometry the equations from (2) to (4) reduces into The equation of motion

$$\mu_f \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} + \frac{\mu_f}{K} u + \sigma B_0{}^2 u$$

(6)

$$\kappa\left(\frac{\partial^2 T_f}{\partial y^2}\right) + a_{sf} h_{fs}\left[T_s - T_f\right] + \sigma B_0{}^2 u^2 = 0 \qquad (7)$$

$$\kappa_s \frac{\partial^2 T_s}{\partial y^2} + a_{sf} h_{fs}\left[T_f - T_s\right] = 0$$

(8)

The corresponding boundary conditions are:

$$y = 0: \qquad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial T_f}{\partial y} = 0 = \frac{\partial T_s}{\partial y}$$

(9)

$$y = R: \qquad u = 0, \quad T_f = T_s = T_w$$

where T_w is the temperature of the plate.

METHOD OF SOLUTION

To make the equations (5) to (9) dimensionless introducing following non-dimensional quantities

$$x^* = \frac{x}{R}, \quad y^* = \frac{y}{R}, \quad u^* = \frac{\mu_f u}{GR^2}, \quad G = -\frac{\partial p}{\partial x}, \quad \theta_f = \frac{T_f}{T_w}, \quad \theta_s = \frac{T_s}{T_w},$$

$$Da = \frac{K_0}{R^2}, \quad M = \sqrt{\frac{\sigma B_0{}^2 R^2}{\mu_f}}, \quad \Pr = \frac{\mu_f C_p}{\kappa_f}, \quad \mathrm{Re} = \frac{\rho R\left(\frac{GR^2}{\mu_f}\right)}{\mu_f},$$

$$S = \frac{a_{sf} h_{sf} u_f}{\rho C_p GR}, \quad Ec = \frac{\nu_f{}^2}{C_p T_w R^2}, \quad B_i = \frac{a_{sf} h_{sf} R^2}{\kappa_{se}}$$

(10)

where G the constant pressure gradient, Da the Darcy number, M the Hartmann number, Pr the Prandtl number, Re the Reynolds number, S the Stanton number, Ec the Eckert number and B_i the Biot type number.

For the sake of convenience dropping the asterisk the dimensionless form of the equations (5) to (9) respectively are given as under

$$\frac{d^2 u}{dy^2} = -1 + \left(\frac{1}{Da}\right) u e^y + M^2 u$$

(11)

$$\left(\frac{1}{\Pr \mathrm{Re}}\right)\frac{d^2\theta_f}{dy^2}+S\left(\theta_s-\theta_f\right)+M^2\,\mathrm{Re}\,Ecu^2=0$$

(12)

$$\frac{d^2\theta_s}{dy^2}+B_i\left(\theta_s-\theta_f\right)=0$$

(13)

The corresponding boundary conditions are

$$\mathrm{y}=0: \qquad \frac{\partial u}{\partial y}=0; \qquad \frac{\partial\theta_f}{\partial y}=0=\frac{\partial\theta_s}{\partial y}$$

(14)

$$y=1: \qquad u=0; \qquad \theta_f=\theta_s=1$$

The momentum equation and coupled energy equations, are nonlinear differential equations, whose solution can be obtained by the Differential Transform Method (DTM), The DTM is a quasi-numerical method able to solve equation easily. The efficiency of the method can be seen in literature. Biazar et al. [3] used DTM for solving quadratic Ricatti differential equation. Farshid [5] had solved linear and non linear systems of ordinary differential equations using DTM. Javed [6] used one dimensional DTM for solving higher order boundary value problems in finite domain.

In the literature we find that differential transform U(k) of the derivative $\frac{d^k u(y)}{dy^k}$ is defined by

$$U(k)=\frac{1}{k!}\left[\frac{d^k u(y)}{dy^k}\right]_{y=y_0}$$

(15)

The inverse differential transform of U(k) is defined by

$$u(y)=\sum_{k=0}^{\infty}U(k)\left(y-y_0\right)^k$$

(16)

The Basic formulae of DTM on a function are given in the Appendix-A.

I. Solution for Velocity Profile

Applying DTM on the momentum equation (11), the equation in transformed space with parameter k gives the following recurrence relation.

$$(k+2)(k+1)U(k+2) = -\delta(k) + \frac{1}{Da}\sum_{t=0}^{k}\frac{U(k-t)}{t!} + M^2U(k) \qquad \textbf{(17)}$$

The initial condition on derivative of u reduces to $U(1)=0$

And the boundary condition gives

$$\sum_{k=1}^{N} U(k) = 0 \qquad \text{at y=1}$$

(18)

where U(k) is Differential Transform of *the derivative* u(y)

Since the value of U(k) at k=0 is not known, therefore considering $U(0)=\alpha$ (constant).

The value of arbitrary constant α is to be determined by the boundary condition. Corresponding to the values for k=0, 1, 2, 3 and 4 respectively, the recurrence relation (17) gives

$$U(2) = \left(\frac{1}{2}\right)\left(-1+\left(\frac{1}{Da}+M^2\right)\right)\alpha$$

$$U(3) = \frac{\alpha}{6Da}$$

$$U(4) = \frac{\alpha}{24Da} + \left(\frac{1}{24}\right)\left(\frac{1}{Da}+M^2\right)\left(-1+\left(\frac{1}{Da}+M^2\right)\alpha\right)$$

$$U(5) = \left(\frac{\alpha}{120Da}\right)\left(1+\frac{1}{Da}+M^2\right)+\frac{1}{40Da}\left(-1+\left(\frac{1}{Da}+M^2\right)\alpha\right)$$

$$U(6) = \frac{1}{30}\left[\begin{array}{l}\left(\frac{1}{Da}+M^2\right)\left(\frac{\alpha}{24Da}+\left(\frac{1}{24}\right)\left(\frac{1}{Da}+M^2\right)\right)\left(-1+\left(\frac{1}{Da}+M^2\right)\right)+ \\ \frac{\alpha}{6Da^2}+\frac{1}{4Da}\left(-1+\left(\frac{1}{Da}+M^2\right)\alpha\right)+\frac{\alpha}{24Da}\end{array}\right]$$

Using these values in the equation (18) the solution of the problem up to 6th order is given by

$$u(y) = U(0)+U(1)y+U(2)y^2+U(3)y^3+U(4)y^4+U(5)y^5+U(6)y^6 \qquad \textbf{(19)}$$

To find value of α, invoking the boundary condition, u(1) =0 in the equation (19) we have

$$U(0)+U(1)+U(2)+U(3)+U(4)+U(5)+U(6)=0$$

(20)

$$\alpha = \frac{\frac{1}{2}+\frac{1}{24}\left(\frac{1}{Da}+M^2\right)+\frac{1}{30Da}+\frac{1}{720}\left(\frac{1}{Da}+M^2\right)^2}{\left(\begin{array}{l} 1+\frac{1}{2}\left(\frac{1}{Da}+M^2\right)+\frac{151}{720Da}+\frac{1}{24}\left(\frac{1}{Da}+M^2\right)^2+\frac{1}{120Da}\left(1+\frac{1}{Da}+M^2\right)+\frac{1}{30Da}\left(\frac{1}{Da}+M^2\right) \\ +\frac{1}{30}\left(\frac{1}{Da}+M^2\right)\left(\frac{1}{24Da}+\frac{1}{24}\left(\frac{1}{Da}+M^2\right)^2\right)+\frac{1}{180Da^2} \end{array}\right)}$$

which on simplification gives

(21)

Invoking value of α in the equation (19) and using MATLAB Programming the velocity profiles are computed and presented through graphs.

II. Skin-Friction

The non-dimensional shearing stress on the upper plate in terms of local skin-friction coefficient is derived and computed values are given in the Table-1.

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{y=1}$$
$$= U(1)+2U(2)+3U(3)+4U(4)+5U(5)+6U(6)$$

TABLE 1: VARIATION OF SKIN FRICTION AT THE PLATE WITH RESPECT TO THE PHYSICAL PARAMETERS

M	Da	C_f
0.5	0.1	-0.29315
1	0.1	-0.28512
1.5	0.1	-0.27369
2	0.1	-0.26079
0.5	1	-0.6832
0.5	0.01	-0.57732
0.5	0.001	-7.84733

III. Solution for Temperature Profiles

Now to find temperature profiles θ_f and θ_s, we have to solve the coupled differential equations (12) and (13) . Letting P(k) and L(k) are differential transform of $\theta_f(y)$ and $\theta_s(y)$ respectively. Applying DTM on the equations (12) and (13), the recurrence equations respectively are

$$\frac{1}{\Pr\mathrm{Re}}(k+2)(k+1)P(k+2)+S(L(k)-P(k))+M^2\,\mathrm{Re}\,Ec\sum_{t=0}^{k}U(t)U(k-t)=0$$

(22)

$$(k+2)(k+1)L(k+2)+B_i(P(k)-L(k))=0$$

(23)

The DTM of derivative at initial values for θ_f and θ_s gives

$$P(1) = L(1) = 0$$

(24)

During the calculation of the values of Differential Transform coefficients from the recurrence relations (22) and (23), the values of $P(0)$, $L(0)$ are required. Since $P(0)$, $L(0)$ are not known,

so let $P(0) = a$ and $L(0) = b$, where a and b are arbitrary constants to be determined with the help of prescribed boundary conditions on θ_f and θ_s . From the recurrence relations (22) and (23), for k=0, 1, 2, 3 … respectively we get

$$P(2) = -\frac{\Pr \mathrm{Re}}{2}\left(S(b-a) + M^2 \mathrm{Re}\, Ec\alpha\right)$$

$$L(2) = -\frac{B_i}{2}(a-b)$$

$$L(3) = 0$$

$$P(4) = \frac{\Pr \mathrm{Re}}{12}\left[S\left[(b-a)(B_i + S\Pr \mathrm{Re}) + M^2 \mathrm{Re}^2 Ec\alpha\right] + M^2 \mathrm{Re}\, Ec\alpha\left(-1 + \left(\frac{1}{Da} + M^2\right)\alpha\right)\right]$$

$$P(3) = 0$$

$$L(4) = \frac{B_i}{24}(\Pr \mathrm{Re}\, S + B_i)(b-a) + \frac{M^2 \mathrm{Re}^2 Ec\alpha B_i}{24}$$

$$P(5) = -\frac{M^2 \mathrm{Re}^2 \Pr Ec\alpha^2}{60Da}$$

$$L(5) = 0$$

Using these values the inverse transform of *P(k) and L(k) gives the* solution of the problem up to 5^{th} order. The expression of θ_f *and* θ_s *are given by the following equations respectively.*

$$\theta_f(y) = a - \frac{\Pr \mathrm{Re}}{2}\left(S(b-a) + M^2 \mathrm{Re}\, Ec\alpha\right)y^2 - \frac{\Pr \mathrm{Re}}{24}\begin{bmatrix} S\left[(b-a)(B_i + S\Pr \mathrm{Re}) + M^2 \mathrm{Re}^2 \Pr Ec\alpha\right] \\ + M^2 \mathrm{Re}\, Ec\alpha\left(-1 + \left(\frac{1}{Da} + M^2\right)\alpha\right) \end{bmatrix} y^4$$
$$- \frac{M^2 \mathrm{Re}^2 \Pr Ec\alpha^2}{60Da} y^5$$

(25)

For the obtaining the values of a and b imposing the boundary conditions $\theta_f(1)=1, \theta_s(1)=1$

(26)

It gives the following linear system of equations

$$A_1 a - B_1 b = C_1$$

$$\theta_s(y) = b - \frac{B_i}{2}(a-b)y^2 + \left[\frac{B_i}{24}(\Pr \operatorname{Re} S + B_i)(b-a) + \frac{M^2 \operatorname{Re}^2 Ec\alpha B_i}{24}\right]y^4$$

(27)

$$-A_2 a + B_2 b = C_2 \qquad (28)$$

where, the constants A_1, A_2, B_1, B_2, C_1 and C_2 are defined below

$$A_1 = 1 + \frac{\Pr \operatorname{Re} S}{2} + \frac{S \Pr \operatorname{Re}(B_i + S \Pr \operatorname{Re})}{24}$$

$$B_1 = -\frac{\Pr \operatorname{Re} S}{2} + \frac{\Pr \operatorname{Re} S(B_i + S \Pr \operatorname{Re})}{24}$$

$$C_1 = \frac{M^2 \operatorname{Re}^2 \Pr Ec\alpha}{2} + \frac{M^2 \operatorname{Re}^3 \Pr^2 EcS\alpha}{24} + \frac{M^2 \operatorname{Re}^2 \Pr Ec\alpha}{24}\left(-1 + \left(\frac{1}{Da} + M^2\right)\right) + \frac{M^2 \operatorname{Re}^2 \Pr Ec\alpha^2}{60Da} + 1$$

$$A_2 = \frac{B_i}{2} + \frac{B_i(\Pr \operatorname{Re} S + B_i)}{24}$$

$$B_2 = 1 + \frac{B_i}{2} + \frac{B_i}{24}(\Pr \operatorname{Re} S + B_i)$$

$$C_2 = 1 - \frac{M^2 \operatorname{Re}^2 EcB_i\alpha}{24}$$

solving (27) and (28), the value of arbitrary constants a and b are known and given by

$$a = \frac{B_1 C_2 + B_2 C_1}{A_1 B_2 - A_2 B_1} \qquad b = \frac{A\ C_2 + A_2 C_1}{A_1 B_2 - A_2 B_1}$$

Hence, the solution θ_f and θ_s are known. Further the numerical values of θ_f and θ_s for various set of physical parameters are computed with MATLAB Programming and discussed graphically.

IV. Nusselt Number

The dimensionless coefficient of heat transfer at the upper plate is given by

$$Nu_f = \left(\frac{\partial \theta_f}{\partial y} \right)_{y=1} \quad (for\ fluid)$$

$$= \Pr \mathrm{Re}\left(S(b-a) + M^2 \mathrm{Re}\, Ec\alpha\right) - \frac{\Pr \mathrm{Re}}{6} \left[\begin{array}{l} S\left[(b-a)(B_i + S \Pr \mathrm{Re}) + M^2 \mathrm{Re}^2 \Pr Ec\alpha\right] \\ + M^2 \mathrm{Re}\, Ec\alpha \left(-1 + \left(\frac{1}{Da} + M^2 \right) \alpha \right) \end{array} \right]$$

$$- \frac{M^2 \mathrm{Re}^2 \Pr Ec\alpha^2}{12 Da}$$

Table. 2: Variation in Nusselt Number for Fluid and Solid Phase Due to Physical Parameters

M	Da	S	B_i	Re	Ec	Pr	Nu(Fluid)	Nu(Solid)
0.5	0.1	0.2	0.5	10	0.1	5	0.31812	0.09212
1.0	0.1	0.2	0.5	10	0.1	5	1.21079	0.35109
1.5	0.1	0.2	0.5	10	0.1	5	2.51984	0.73213
0.5	0.01	0.2	0.5	10	0.1	5	0.04213	0.01174
0.5	0.001	0.2	0.5	10	0.1	5	0.00459	0.00126
0.5	0.1	0.5	0.5	10	0.1	5	0.19551	0.07602
0.5	0.1	1.5	0.5	10	0.1	5	0.11914	0.06666
0.5	0.1	0.2	1.5	10	0.1	5	0.433	0.25914
0.5	0.1	0.2	2.5	10	0.1	5	0.534	0.40664
0.5	0.1	0.2	0.5	20	0.1	5	0.87933	0.31649
0.5	0.1	0.2	0.5	40	0.1	5	2.47865	1.13561
0.5	0.1	0.2	0.5	10	0.05	5	0.15906	0.04606
0.5	0.1	0.2	0.5	10	0.01	5	0.03181	0.00921
0.5	0.1	0.2	0.5	10	0.1	7	0.37683	0.12517
0.5	0.1	0.2	0.5	10	0.1	10	0.45142	0.17325

RESULTS AND DISCUSSION

The effect of magnetic field on the fluid flow profile is shown in the figures 1 to 4 for various values of Darcy's number. The parabolic nature of the fluid flow diminished when permeability of the medium i.e. the value of Darcy's number is reduced. The fluid velocity in the vicinity of the plate have higher gradient for value of Da=0.01 and 0.001 as observed in the figure 3 and 4, while for the value of Da=1.0 and 0.1 the flow profiles are parabolic as observed in the figures 1 and 2. It is also observed in figure 1 and 2 that when Da=1.0, 0.1 the flow velocity decelerate in the complete channel region with the increase of magnetic field intensity, while for Da=0.01 the effect of magnetic field on the flow velocity in vicinity of the plate is overturned. The figure 4 shows that Da=0.001 then the flow velocity is unaffected by the mild transverse magnetic field. In figure 5 to 8, the flow behaviour with various values of Darcy's number are presented. It is observed that in the central region of the channel the flow velocity decreases with the decrease of Darcy's number But for comparatively very small value of Darcy's number the flow velocity augmented near the plate and a high velocity gradient is appeared.

Figure 9 shows that the fluid temperature is more than the porous matrix for each value of Hartman number. It is plausible that on increasing magnetic strength i.e. the Hartman number the fluid temperature as well as temperature of the solid i.e. porous matrix increases significantly as a consequence of Joule heating. The result support and validated mathematical model by the physical concept. It is observed in figure 10 that the rise in temperature at small value of Da is very small as compared to the rise at large value of Da, The effect of Stenton number on temperature is shown through figure 11. The increase in Stanton number imparts cooling effect in both the fluid and porous matrix temperature. Figure 12 demonstrate that the increase in Biot number B_i supports in the enhancement of temperature of the fluid and porous matrix foam. Figure 13 demonstrate that close to the plate the temperature of the fluid and porous matrix foam increases with the increase of Prandtl number. But a peculiar behavior of rise in the temperature for the value of Pr=5.0 is observed for both fluid and solid phase temperature increases with the increase in Reynolds number and Eckert number. From Table 1, it is observed that From Table 2 it is observed that coefficient of heat transfer at the plate for both the fluid phase and solid phase increases with the increase in Hartmann number, Darcy number, Biot type number, Reynolds number, Eckert number and Prandtl number while decreases with the increase in Stanton number.

CONCLUSION

As the permeability of the medium decreases, the parabolic nature of fluid flow diminishes. When Darcy's number is very small, the flow velocity increases near the plate, leading to a steep velocity gradient. The temperature of both the fluid and the porous matrix rises significantly with an increase in magnetic field intensity due to Joule heating. Conversely, an increase in the Stanton number induces a cooling effect on the temperatures of both the fluid and the porous matrix, while a higher Biot number enhances these temperatures.

- The parabolic nature of the fluid flow diminished when permeability of the medium is reduced
- For comparatively very small value of Darcy’s number the flow velocity augmented near the plate and a high velocity gradient is appeared
- The fluid temperature as well as temperature of the porous matrix increases significantly with the increase in intensity of the magnetic field as a consequence of Joule’s heating effect.
- The increase in Stanton number imparts cooling effect in both the fluid and porous matrix temperature.
- The increase in Biot type number B_i supports in enhancement of temperature of the fluid and porous matrix.

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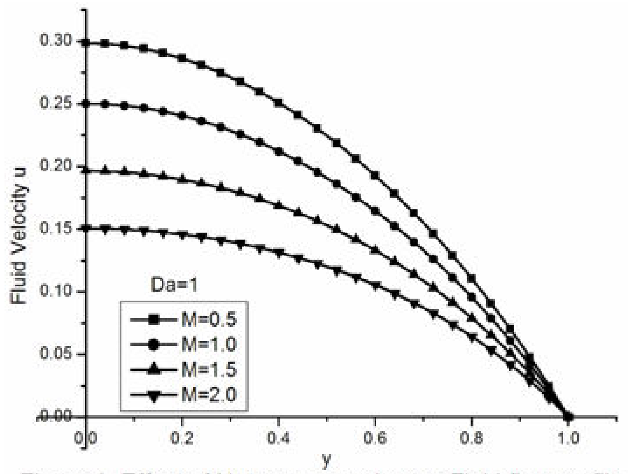

Figure 1. Effect of Hatrmann number on Fluid flow profile.

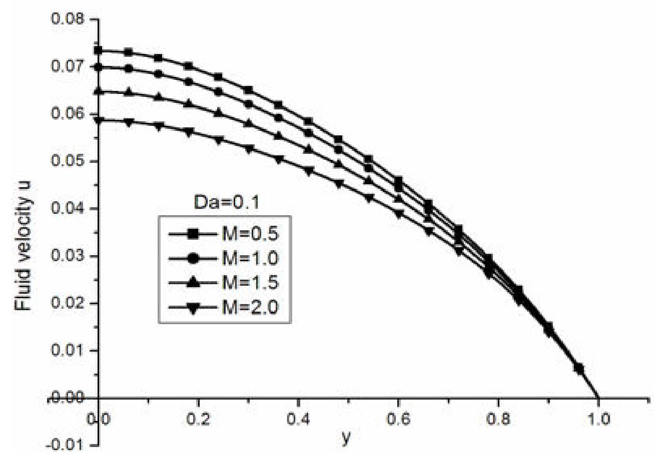

Figure 2. Effect of Hartmann number on Fluid flow profile.

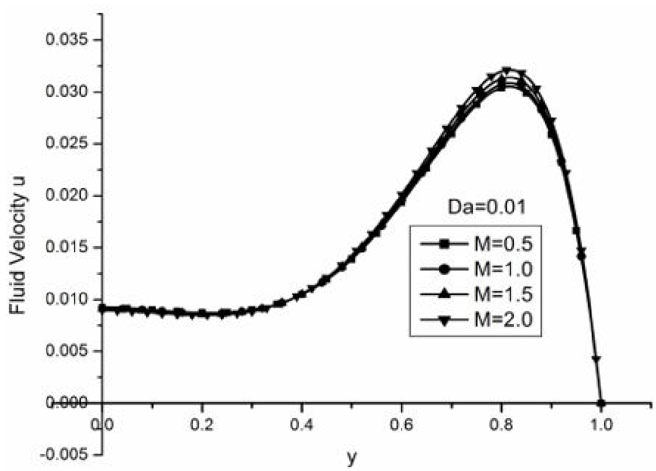

Figure 3. Effect of Hartmann number on Fluid flow profile.

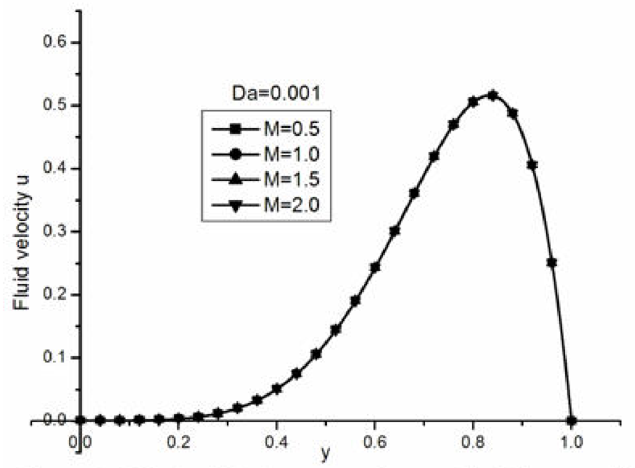

Figure 4. Effect of Hartmann number on Fluid flow profile.

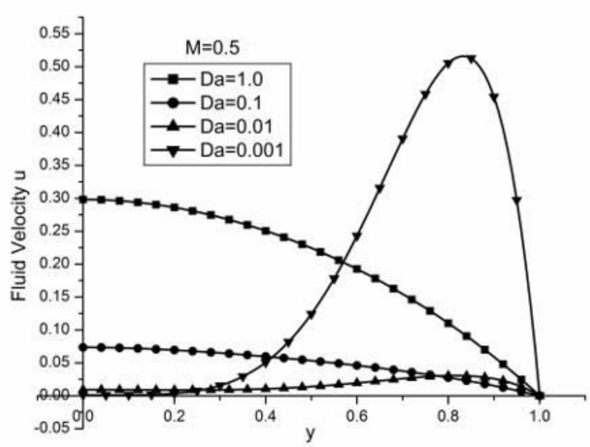

Figure 5. Effect of Darcy number on Fluid flow profile.

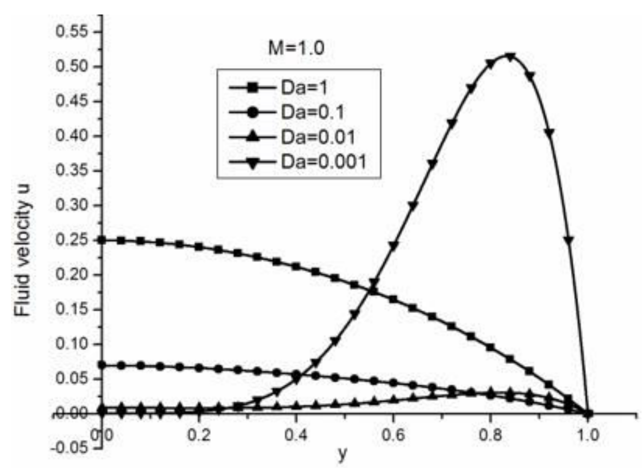

Figure 6. Effect of Darcy number on Fluid flow profile.

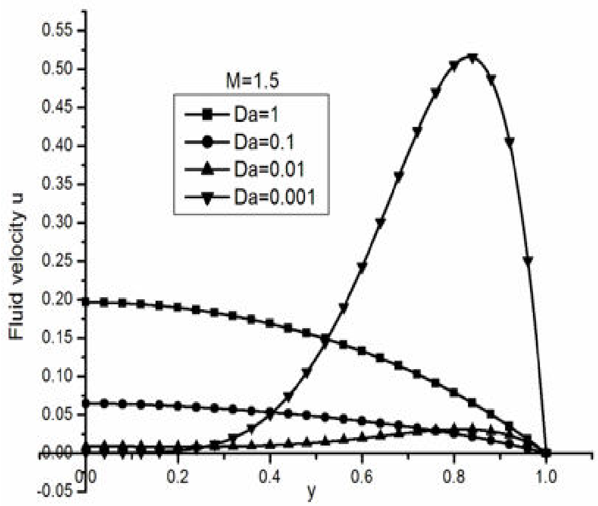

Figure 7. Effect of Darcy number on Fluid flow profile.

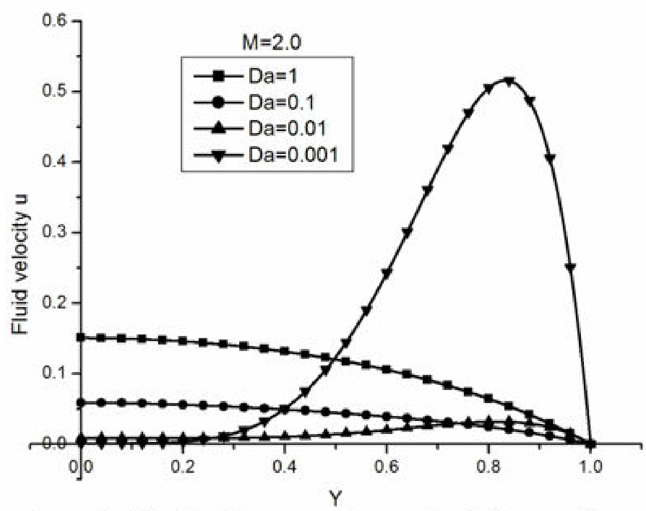

Figure 8. Effect of Darcy number on Fluid flow profile.

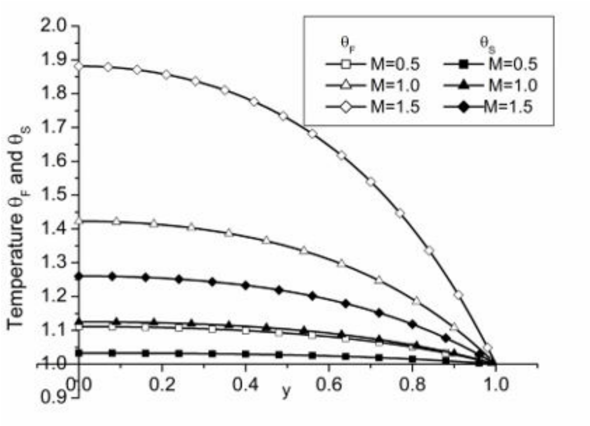

Figure 9. Effect of Hartmann number on temperature profile at Da=0.1, S=0.2, D=0.5, Re=10, Ec=0.1, Pr=5

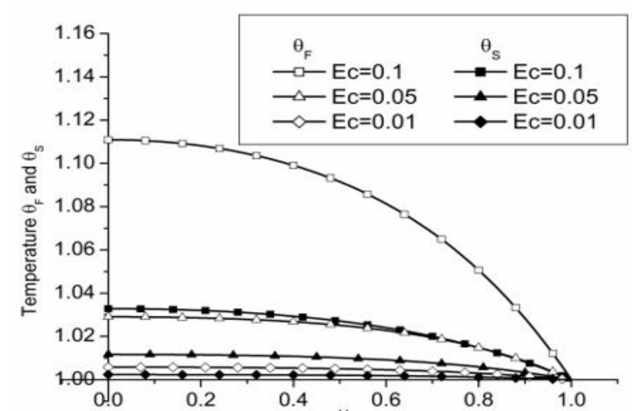

Figure 15. Effect of Eckert number on temperature profile at M=0.5,Da=0.1,S=0.2,D=0.5,Re=10,Pr=5

Appendix-A	
The fundamental mathematical operations under DTM	
Function	***Differential transform***
$u(y)=f(y)\pm g(y)$	$U(k)=F(k)\pm G(k)$
$u(y)=\lambda g(y)$	$U(k)=\lambda G(k)$
$u(y)=\dfrac{\partial g(y)}{\partial y}$	$U(k)=(k+1)G(k+1)$
$u(y)=\dfrac{\partial^m g(y)}{\partial y^m}$	$U(k)=(k+1)...(k+m)G(k+m)$
$u(y)=y^m$	$U(k)=\delta(k-m)=\begin{cases}1 & at \quad k=m \\ 0 & otherwise\end{cases}$

$u(y)=f(y)g(y)$	$U(k)=\sum_{r=0}^{k}F(r)G(k-r)$
$u(y)=f_1(y)f_2(y)...f_m(y)$	$U(k)=\sum_{k_1}^{k}...\sum_{k_{m-1}=0}^{k_2}F_1(k_1)F_2(k_2-k_1)....F_m(k-k_{m-1})$

Appendix-B

a_{sf} Specific surface area

K Permeability

$\vec{q}$ Velocity field

$\vec{J}$ Current density

$\vec{B}$ Applied Magnetic Field

P Pressure

G Constant pressure

Da Darcy's number

M Hartmann number

Pr Pandtl number

Re Reynolds number

S Stenton number

Ec Eckert number

R Half of the distance between parallel plates

u,v,w velocity components along (x,y,z) coordinate axis

h_{sf} Local heat transfer Coefficient

T_f Fluid Temperature

T_s Solid material Temperature

T_w Wall Temperature

Greek Symbols:

κ_e Effective Thermal Conductivity

ρ Fluid density

φ Viscous dissipation term

μ_f Fluid viscosity

θ_f Non Dimensional fluid temperature

θ_s Non Dimensional solid material temperatur