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A NEW IMPLIED AVERAGE PENALTY COST METHOD FOR SOLVING TRANSPORTATION PROBLEMS

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ABSTRACT

One particular class of the linear programming problem is the transportation model. It addresses the situation where a good is transported from its source to its destination. Finding the quantities that are transported from each source to each destination in a way that minimizes overall transportation costs while meeting supply limit and demand criteria is the goal. Employment scheduling and inventory management are two more areas where the transportation model can potentially be employed. Finding a initial basic feasible solution (IBFS) is the first stage in obtaining at a minimal total cost solution for the transportation problem (TP). There are multiple ways to obtain IBFS like North West Corner Rule (NWC), Least Cost Method (LCM), Vogel's Approximation Method (VAM) etc. Every time, VAM is regarded as the most significant among them. In the present study we propose new method called "Implied Average Penalty Cost Method (IAPCM)" which provides better IBFS than VAM sometimes coincides with Optimal solution.

Keywords: Transportation Problem (TP), IBFS, VAM, IAPCM, Optimal Solution.

1. INTRODUCTION

Linear programming is a widely used mathematical technique for optimizing resource utilization. A simplified variation of the simplex methodology, known as the transportation method, can handle linear programming problems. This sort of problem, known as the Transportation Problem (TP), is widely used to solve problems that involve several product sources and destinations. The primary goal of transportation problems is to diminish the costs associated with transporting goods between several locations. This may be referred to as the cost-minimizing transportation problem. The problem of minimizing transportation costs has been debated for a long time and is popularized by T. C. Koopmans [12], G. B. Dantzig [7], A. Charnes, W. W. Cooper, and A. Henderson [6], Kirca and Satir [11], Deshmukh N.M.[8], Ahmed, Mollah Mesbahuddin, et al.[4], A. Rashid, S. S. Ahmed, and M. S. Uddin [15], A. Mhlanga et al.[13], Md. Sharif Uddin et al. [19]. The widely recognized existing methods for determining an Initial Basic Feasible Solution (IBFS) for the TPs include North West Corner Method (NWCM) [6,18], Least Cost Method (LCM) [6,18] and Vogel's Approximation Method (VAM) [6,18]. The IBFS using VAM will be the most inexpensive of the three and the closest to the optimal one. As a result, VAM is considered to be the most efficient method for identifying an IBFS for the TPs. Computational efforts are used to gauge the quality of an IBFS of the TPs. The optimal solution for locating an IBFS cannot be determined by a single approach. Because of this, rather than concentrating on finding the most suitable solution, the majority of this work is dedicated to the method of determining an IBFS. Therefore, an improved VAM method called IAPCM is proposed in this paper to obtain a better IBFS.

2. MATERIAL AND METHODS:

2.1 Mathematical Formulation of TP [1]:

Below is a linear transportation model that illustrates the transportation problem.

$$\text{Minimize } z = \sum_{i=0}^{m}\sum_{j=0}^{n} cij\, xij \qquad \text{(Objective Function)}$$

$$\text{Subject to} \quad \sum_{i=0}^{m} xij \leq si \quad i = 1,2,3,\dots\dots\dots,m \quad (capacity\ constraits)$$

$$\sum_{i=0}^{n} xij \geq dj \quad j = 1,2,3,\ldots\ldots,n \quad \text{(Demand constraints)}$$

$$xij \geq 0 \quad for\ all\ i\ and\ j \qquad \text{(Non − negative constraints)}$$

W*here,* Z=Total transportation cost to be minimized.

xij=The quantity to be shipped from source i to destination j ,

cij=Unit transportation cost from source i to destination j .

m=Total number of sources /origins.

n=Total number of destinations.

When the total supply from all sources equals the total demand from all destinations, a transportation problem is said to be balanced. If this quantity is not equal, the problem is known as an unbalanced transportation problem.

2.2 Proposed Method IAPCM to find IBFS:
This method is essentially an implied-cost method along with average penalty costs.

The proposed method can be applied to solve all types of Transportation Problem (TP). Procedure of finding an IBFS using this method is illustrated below.

- **Step-1:** Mathematical formulation of the transportation table.
- **Step 2:** Establish a dummy row or dummy column with zero transportation costs and the remaining supply or demand, respectively, to get the total supply and demand to equal.
- **Step 3:** Based on supply and demand, calculate the implied cost for each cell by multiplying the unit transportation cost by the maximum number of units of the commodity that can be produced.
- **Step 4:** The minimal implied cost from each element in each row of the TT is deducted, and the results are placed on top of the corresponding elements on the right.
- **Step 5:** Each column should undergo the same operation before being placed at the bottom right underneath the corresponding element.
- **Step 6:** Place the Average Row Penalty (ARP) and the Average Column Penalty (ACP) just after and below the supply and demand amount respectively within first brackets, which are the averages of the right-top elements of each row and the right-bottom elements of each column respectively of the TT. Do not consider dummy column or row cost 0 for calculation. This difference is called as penalty.
- **Step 7:** If there are two or more highest penalties, determine which of the ARPs and ACPs has the highest penalty; select the highest penalty along which the lowest cost element is present. Select any one of the minimum penalties, if there are any, at random.
- **Step 8:** Allocate x_{ij} = min (si , dj) on the left top of the smallest entry in the (i, j) th of the TT.
- **Step 9:** If $si < dj$, leave the i th row and readjust dj as $(dj)^* = dj - si$

If $si > dj$, leave the j th column and readjust si as $(si)^* = si - dj$

If $si = dj$ leave both i th row or j th column.

Allocate balance units in dummy column or row cost 0 in case of unbalanced TP.

- **Step 10:** Move to step 6 and repeat the steps 6–9 until an initial feasible solution has been reached.

- **Step 11:** Put these allocated values x_{ij} in original Transportation Table (TT) in corresponding cell.
- **Step 11:** Calculate, total transportation cost Z as below

$$Z = \sum_{i=0}^{m} \sum_{j=0}^{n} cij\ xij$$

3. RESULTS AND DISCUSSION:

The data used in this article are secondary data obtained from different published articles.

3.1 Mathematical Illustration 1: Balanced Transportation Problem

For illustration purpose, consider following transportation problem from Ray, G.C., and Hosain, M.E.[18] proposed in 2007.

Table No.1.1 Mathematical Model of a Transportation

	Destination					
Source	D1	D2	D3	D4	D5	Supply (S)
S1	5	7	10	50	3	5
S2	8	6	9	12	14	10
S3	10	9	8	10	15	10
Demand(D)	3	3	10	5	4	25/25

Total demand = Total supply = 25. Hence TP is balanced type.

Iteration 1: Based on supply and demand, calculate the implied cost for each cell by multiplying the unit transportation cost by the maximum number of units of the commodity that can be produced.

Table No.1.2: Iteration 1- Determination of implied cost

	Destination					
Source	D1	D2	D3	D4	D5	Supply (S)
S1	5 **15**	7 **21**	10 **50**	50 **25**	3 **12**	5
S2	8 **24**	6 **18**	9 **90**	12 **60**	14 **56**	10
S3	10 **30**	9 **27**	8 **80**	10 **50**	15 **60**	10
Demand(D)	3	3	10	5	4	25/25

Iteration 2:

Follow step 4 to 10 of proposed algorithm until we obtain initial basic feasible solution.

Table No.1.3: Iteration 2-Solution by IAPCM

	Destination									
Source	D1	D2	D3	D4	D5	Supply (S)				
S1	3 5 0	9 7 3	38 10 0	**1*** 13 5 0	**4*** 0 3 0	~~5~~ ~~1~~ 0	12.6	15.75	18	**25.5***
S2	**3*** 6 8 9	**3*** 0 6 0	**4*** 72 9 40	42 12 35	38 14 44	~~10~~ ~~7~~ ~~4~~ 0	31.6	**30***	-	-

S3	3 10 15	0 9 9	**6*** 53 8 30	**4*** 23 10 25	33 15 48	~~10~~ ~~4~~0	22.5	19.75	**26.3***	-
Demand(D)	~~3~~0	~~3~~0	~~10~~ ~~6~~0	~~5~~ ~~1~~ 0	~~4~~0	25/25				
	8	4	23.33	23.33	**30.67***					
	8	4	23.33	23.33	-					
	-	-	23.33	23.33	-					
	-	-	23.33	23.33	-					

Note: Value with * and bold appearing at left top corner of cell is allocation of units.

Total transportation cost =

$z = \sum_{i=0}^{m} \sum_{j=0}^{n} c_{ij} x_{ij}$= 5x1 + 3x 4 + 3 x 8 + 3 x 6 + 4 x 9 + 6 x 8 + 4 x 10 = **183**

In the above solution obtained by IAPCM, No. of occupied cells = 7, and m+ n -1= 3 +5 -1= 7 indicates rim requirements are satisfied. Hence this initial basic solution obtained by IAPCM is feasible with total transportation cost **183**. For comparison purpose, Initial solution obtained by VAM is 187 and Optimum solution obtained by MODI method is also **183**. This indicates that Initial solution obtained by IAPCM is even better than VAM and equal to MODI method.

3.2 Mathematical Illustration 2: Unbalanced Transportation Problem

For illustration purpose, consider following transportation problem from Ray, G.C., and Hosain, M.E.[43] proposed in 2007.

Table No.2.1: Initial table

	D1	D2	D3	D4	D	Supply (S)
S1	5	4	8	6	5	600
S2	4	5	4	3	2	400
S3	3	6	5	8	4	1000
Demand(D)	450	400	200	250	300	1600/2000

Here, total demand is 1600 and total supply is 2000. Hence given transportation problem is unbalanced type. We add dummy column Dm with all costs zero and demand 400. We implied cost as below.

Iteration 1: Determination of **implied cost** same as in case of illustration 1

Table No.2.2: Iteration 1- Determination of implied cost

	D1	D2	D3	D4	D	Dm	Supply (S)
S1	5 2250	4 1600	8 1600	6 1500	5 1500	0 0	600
S2	4 1600	5 2000	4 800	3 750	2 600	0 0	400
S3	3 1350	6 2400	5 1000	8 2000	4 1200	0 0	1000
Demand(D)	450	400	200	250	300	400	2000/2000

Iteration 2:

Follow step 4 to 10 of proposed algorithm until we obtain initial basic feasible solution.

Table No.2.3: Iteration 2-Solution by IAPCM

	Destination										
Source	D1	D2	D3	D4	D5	Dm	Supply (S)				
S1	750 5 900	**400*** 100 4 0	 100 8 800	0 6 750	0 5 900	**200*** 0 0 0	~~600 200~~ 0	190	237 .5	237 .5	316. 6
S2	100 4 250	1400 5 400	200 4 0	**250*** 150 3 0	**150*** 0 2 0	 0 0 0	~~400 150~~ 0	370	425	-	-
S3	**450*** 350 3 0	1400 6 800	**200*** 0 5 200	1000 8 1250	**150*** 200 4 600	**200*** 0 0 0	~~1000 850~~ ~~400 200~~ 0	590	487 .5	487 .5	**583. 3***
Demand (D)	~~450~~ 0	~~400~~ 0	~~200~~ 0	~~250~~ 0	~~300~~ ~~150~~ 0	~~400~~ ~~200~~ 0	2000/200 0				
	350	400	333. 3	**666.6***	500	0					
	350	400	500	-	**500***	0					
	450	400	**-**	--	-	0					
	-	400		-	-	0					

Note: Value with * and bold appearing at left top corner of cell is allocation of units.

Total transportation cost = $z = \sum_{i=0}^{m} \sum_{j=0}^{n} c_{ij}\, x_{ij}$ = 400 x 4 + 200 x 0 + 250 x 3 + 150x 2 + 450 x 3 + 200 x 5 + 150 x4 + 200 x 0 = **5600**

In the above solution obtained by IAPCM, No. of occupied cells = 8, and m+ n -1= 3 +6 -1= 8 indicates rim requirements are satisfied. Hence this initial basic solution obtained by IAPCM is feasible with total transportation cost **5600**. For comparison purpose, Initial solution obtained by VAM is 6000 and Optimum solution obtained by MODI method is also **5600**. This also indicates that Initial solution obtained by IAPCM is even better than VAM and equal to MODI method.

To support this claim, we now consider different secondary data sets obtained from available literature and find IBFS using differ methods like NWCM, LCM, VAM, IAPCM and finally optimum solution using Modified Distribution (MODI) Method.

3.3 Numerical Examples of Transportation Problems:

Table No. 3.1: Data sets

Numerical Example No.	Dimension of cost matrix	Source (Reference)	Data
1	3x3	10	[cij]=[8 6 10; 7 11 11; 4 5 12] ; [si]=[150,175,275] ;[d*j*]=[200,100,300] (Balanced TP)
2	3x4	04	[cij]=[10 8 4 3; 12 14 20 2; 6 9 23 25] ; [si]=[500, 400, 300] ;[d*j*]=[250, 350, 600, 150] (Unbalanced TP)

3	4x4	05	[cij]=[7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3] ; [si]=[30, 25, 20, 15] ;[d*j*]=[30, 30, 20, 10] (Balanced TP)
4	3x3	16	[cij]=[4 5 6; 3 1 5; 2 4 4]; [si]=[12, 11, 7] ; [d*j*]=[6, 5, 8] (Unbalanced TP)
5	3x5	17	[cij]=[5 7 10 5 3; 8 6 9 12 14; 10 9 8 10 15] ; [si]=[5, 10 10]; [d*j*]=[3, 3, 10, 5, 4] (Balanced TP)
6	3x5	16	[[cij]=[5 4 8 6 5; 4 5 4 3 2; 3 6 5 8 4]; [si]=[600, 400, 1000]; [d*j*]=[450, 400, 200, 250, 300] (Unbalanced TP)
7	3x3	4	[cij]=[15 7 25; 8 12 14; 17 19 21] ; [si]=[12,17,7] ;[d*j*]=[12,10,14] (Balanced TP)
8	3x3	9	[cij]=[4 8 8; 16 24 16; 8 16 24] ; [si]=[76, 82, 77] ; [d*j*]=[72, 102, 41] (Unbalanced TP)
9	3x4	13	cij]=[1 2 1 4; 4 2 5 9; 20 40 30 10] ; [si]=[30, 50, 20]; [d*j*]=[20, 40, 30, 10] (Balanced TP)
10	3x4	2	[cij]=[3 1 7 4; 2 6 5 9; 8 3 3 2] ; [si]=[300,400,500] ;[d*j*]=[250,350,400,200]

3.4 Comparison of IBFS with Optimum Solution (OS):

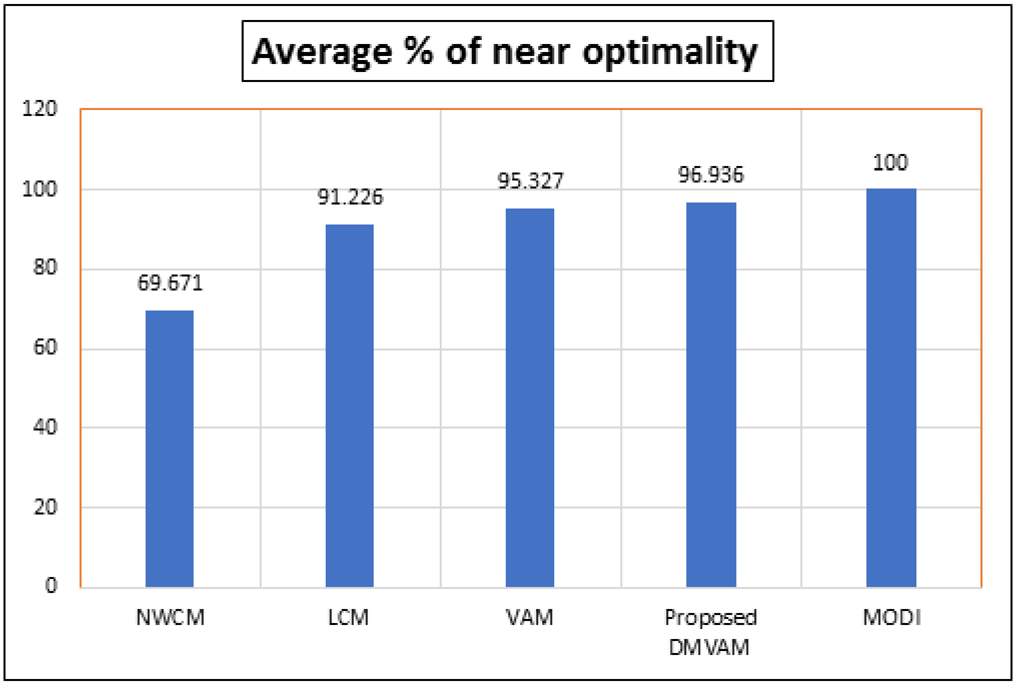
Initially we obtain IBFS from differ method and OS from MODI Method. Further we find % of deviation from OS.

Table No. 3.2: Comparison of IBFS with Optimum Solution (OS)

	IBFS (I_R)				**OS(F_R)**	**% of near optimality**			
Example No.	NWCM	LCM	VAM	Proposed IAPCM	MODI	NWCM	LCM	VAM	Proposed IAPCM
1	5925	4550	5127	4550	4520	76.28	99.34	88.16	99.34
2	18800	8800	8350	8000	7750	41.22	88.07	92.81	96.88
3	540	435	470	455	410	75.93	94.25	87.23	90.11
4	90	57	57	57	57	63.33	100	100	100
5	234	191	187	183	183	78.21	95.81	97.86	100
6	8150	6450	6000	5600	5600	68.71	86.82	93.33	100
7	545	433	425	425	425	77.98	98.15	100	100
8	2968	2968	2424	2424	2424	81.67	81.67	100	100
9	600	560	450	450	450	75.00	80.36	100	100
10	19700	13100	12250	13850	11500	58.38	87.79	93.88	83.03
Average % of near optimality						**69.67**	**91.227**	**95.33**	**96.94**

If the percentage is 100, indicates that the obtained result is numerically equal to the optimal solution. From above table no. --- it is seen that average % near to optimality of Proposed IAPCM is 96.936%. It is better than other IBFS method and ensures the better performance of the proposed method. In 60% cases, proposed technique is directly yielding optimal result for transportation problems.

Chart 1: Comparison of average % of near optimality of different IBFS with MODI



4. CONCLUSION

Transportation models are the most cost-effective way for transporting products from numerous manufacturing facilities or manufacturers to different warehouses. This is a crucial aspect in logistics management. The first step toward obtaining a minimal total cost solution to a transportation problem is identifying an initial feasible solution. There are several methods to obtain IBFS. Among them, VAM is always considered as best.

In contrast to the methods currently found in the literature, we have developed an efficient method (IAPCM) for the cost reduction of transportation problems that is relatively simple to comprehend and yields better results. To develop this method for the scenario of several plants and destinations, future study may be conducted. The study's limitation is that just ten randomly selected data sets are examined in order to evaluate performance.

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