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CONTROL OF FRACTIONAL ORDER NUCLEAR POWER REACTOR USING FOPID CONTROLLER

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ABSTRACT

Because nuclear power reactors are nonlinear and higher order systems, designing the controllers for such systems is always a challenge for practitioners. The novelty of this present study is to convert higher fractional order PHWR system dynamics into reduced fractional order system dynamics, which replicate the truly behaviour of the original higher fractional order system dynamics. Nowadays, FOPID controller is gaining popularity because the presence of two extra degrees of freedom, an optimized FOPID controller for the reduced ordered fractional PHWR system has been devised. The Grey Wolf optimization (GWO) algorithm is used to tune the parameters of FOPID controller. Simulation study is performed to verify the efficacy of the proposed method, and results proclaim that the proposed method has better performance.

Keywords: FOPID Controller, Grey Wolf Optimization, Model Order Reduction, Nuclear Power Reactor, Optimization

1 INTRODUCTION

Many developed countries have prioritized nuclear reactor power generation. This interest arises from the fact that power generated by nuclear reactors does not contribute to pollution. Canada Deuterium Uranium (CANDU) is one such realistic example which is Pressurized Heavy Water Reactor (PHWR). It used heavy water as a moderator and coolant while uranium as fuel. Although power generation through nuclear power reactor has the many advantages, but the other side, it requires a dependable control approach to accomplish effective power control. During the reactor operation, there may be moments when it is necessary to reduce the output power of reactor in a very short period. This is known as the step-back situation (Saha, S. et al., 2010). This reduction of power is conventionally done by putting the control rods deep inside the reactor. However, numerous physical aspects, such as gravity, the level of control rod insertion, and the experience of human operator, are crucial consideration in these conventional approaches.

A trustworthy mathematical model is most essential requirement for developing a controller for any process. Non-integer order models with dead time, often referred to as fractional-order systems with time delay, offer several benefits in mathematical modelling within the domain of control systems. These models are a more accurate representation of certain real-world processes compared to traditional integer-order models. Fractional order systems, which involve integrals and derivatives of non-integer order, have gained significant attention due to their capability to model complex dynamical systems more accurately than integer-order models (Podlubný, I., 1999), (Vinagre, B.M. et al.,2000). However, the analysis and control of such systems can be computationally expensive due to their high order. Order reduction techniques are essential to simplify these models while preserving their key dynamic characteristics. However, along with these benefits, fractional order models increase the overall mathematical complexity of the design and analysis process (Bourouba, B., Ladaci, S. and Chaabi, A.,2017).

Nuclear power reactors are extremely complex and possessing higher order system dynamics (Saha, S. et al., 2010), (Das, Saptarshi, Das, Shantanu and Gupta, A., 2011), (Liu, Cheng et al.,2009), (Vajpayee, V. et al. 2021). Hence the controlling and understanding the operations for these types of systems are very difficult, which necessitates rigorous mathematics for design and analysis. Model Order Reduction (MOR) is utilized in this context to reduce and approximate high order and complex dynamical systems (Fortuna, L., Nunnari, G. and Gallo, A.,1992). After acquiring the reduced order approximation of the high order model, the controller may be designed very easily. There are various MOR approaches in the literature. Every method has advantages and limitations. A specific MOR approach is chosen and implemented based on the task at hand to find a reduced order model of a high order system.

Here, some of the conventional methods and recent advancements in model order reduction of fractional order systems are presented. Balanced Truncation is a classical way used for the model order reduction of integer-order systems. Its extension to fractional order systems has been explored, involving the computation of Hankel singular values and truncation of state-space matrices (Verma, S.K. and Nagar, S.K., 2016). Pade approximation involves the transformation of fractional order models into a finite-order approximation using Padé series (Tavakoli-Kakhki, M. and Haeri, M.,2009). This technique retains the dominant system dynamics and can be applied to both continuous and discrete-time systems. Dominant pole retention techniques aim to identify the most significant poles of the system and truncate the state-space model while preserving the essential dynamics (Taha, M., Abu-Al-Nadi, D.I. and Hasan, O., 2016). This approach is efficient in reducing the computational complexity of fractional order systems.

Recently, many researchers have utilized optimization algorithms for obtaining reduced order models from a higher order system. Due to the advancements in technology, many metaheuristic optimization algorithms are proposed which are built on the behaviour of various classes living in their natural environments. These algorithms have been extensively used to obtain the optimal parameters in different applications. Some of the main optimization algorithms are presented here. Genetic Algorithms (GAs) are widely applied to optimize the model order reduction of fractional order systems. They explore the search space of possible reduced-order models and evolve solutions that minimize the error between the reduced and original systems. GAs can handle both continuous and discrete orders (Tominaga, D., Koga, N. and Okamoto, M.,2000). Particle Swarm Optimization (PSO) is another optimization technique employed for order reduction (Wang, D., Tan, D. and Liu, L., 2017). It uses a population-based approach to search for optimal reduced-order models. PSO is computationally efficient and can be applied to both continuous and discrete-time systems. Simulated annealing is a stochastic optimization method used for model order reduction of fractional order systems. It explores the search space by simulating a cooling process to determine the global minimum error between the reduced and original systems. Ant colony optimization algorithm (Al Salami, Nada MA, 2009) is a nature-inspired optimization technique that has been applied to order reduction. It uses the concept of ant colonies to hunt for optimal reduced-order models by considering the trade-off between accuracy and complexity.

Optimization techniques aim to minimize the error between the reduced-order and original high-order system. This leads to a more accurate approximation of the system's dynamics compared to traditional techniques like Balanced Truncation or Pade approximation (Jain, S., Hote, Y.V. and Saxena, S., 2019)). Optimization techniques can efficiently explore the solution space to find the best reduced-order model, often in a faster and more automated manner than manual or heuristic methods. Optimization techniques can handle various types of fractional order systems, both continuo us and discrete-time, and can adapt to different objectives and constraints, allowing for more flexibility in model reduction, order reduction of fractional order systems using optimization techniques is a better option compared to traditional methods due to its ability to preserve accuracy, computational efficiency, flexibility, and the potential to find global optima. These techniques offer a promising avenue for simplifying complex fractional order systems while retaining their essential dynamic behaviour. therefore, the present work aims to find the reduced order model of a fractional order PHWR system using Grey Wolf Optimization (GWO) algorithm (Mirjalili, S., Mirjalili, S.M. and Lewis, A., (2014).

The next step is to develop a FOPID controller for the reduced fractional order of model of the system. Variety of characteristics, including disturbance rejection capability, handling of parametric uncertainty, robustness etc (Podlubný, I., 1999), (Das, S., 2011) have been considered. Numerous control strategies are reported in the earlier reported literature. Out of the variety of controllers Proportional Integral derivative (PID) controller is widely used. Despite their widespread acceptance, PID controllers have certain limitations, such as poor disturbance rejection, inadequate handling of parametric uncertainty (Hamamcı, S.E., 2007), Lamba, R., Singla, S.K. and Sondhi, S. , 2017), (Shah, P. and Agashe, S., 2016). In the present study FOPID controllers have been used.

FOPID controllers offer more design flexibility because they have five degrees of freedom to be tuned, whereas conventional PID controller has only three degrees of freedom (Hamamcı, S.E., 2007), Lamba, R., Singla, S.K. and Sondhi, S. , 2017), (Shah, P. and Agashe, S., 2016). Many researchers have developed FOPID controllers for use in a wide range of industrial applications, demonstrating the adaptability and widespread acceptance of this sort of control technique. Shah, P. and Agashe, S. (2016) discusses the merits of FOPID controller over other alternative control techniques. For power control of the PHWR system, a variety of control schemes have been created, tested, and applied. In (Sondhi, S. and Hote, Y.V, (2014) a fractional FOPID controller was constructed using frequency domain tuning methods. However, undershoots were found in this design during operation. Here, to determine the unknown controller settings, the stability boundary locus approach was utilized. The overall design, however, suffers from a protracted settling time in this technique.

The following are the primary goals of this present work:

- Determine the reduced fractional order model for higher fractional order PHWR using GWO algorithm while minimizing an Integral square error (ISE) based cost function.
- Designing and tune a FOPID controller for the determined reduced fractional order model and tuning its parameters using the GWO algorithm.
- Comparison of the performance of designed controller based on performance indices e.g., ISE, IAE, and ITAE.

The rest paper is organized as follows: Section 2 describes the MOR of the higher fractional order PHWR system using GWO algorithm by minimizing the ISE in detail. Section 3 discusses the design and implementation of the optimization based FOPID controller, as well as the procedures involved in the GWO algorithm. The simulation studies and performance analyses are shown in Section 4. The final portion contains the conclusions.

2. Model Order Reduction of PHWR System

Four different high order NIOPTD transfer functions of PHWR system for corresponding operating conditions are tabulated in Table 1. These operating conditions depicts the different levels of control rods insertion into the reactor of PHWR system. For example, P_{100}^{30} shows the transfer function for the operating condition in which the level of control rod insertion is up to 30%, and the corresponding output power of the reactor is 100%. As for P_{100}^{30} operating condition, the insertion level of rods is the least among all the other operating conditions, the power control is hard to achieve on this case. Therefore, in the present work the controller design is performed for this operating condition only.

Table 1: Transfer functions for PHWR system.

Label	Transfer function (NIOPTD) for PHWR system
P_{100}^{30}	$\frac{1522.8947}{s^{2.0971}+8.1944s^{1.0036}+7.7684}e^{-2.0043*10^{\wedge}-12}$
P_{90}^{30}	$\frac{1339.2345}{s^{2.0972}+8.1906s^{1.0036}+7.7075}e^{-1.5968*10^{\wedge}-9}$
P_{80}^{30}	$\frac{1027.3027}{s^{2.0163}+6.7859s^{0.99388}+6.5268}e^{-2.5346*10^{\wedge}-5}$
P_{70}^{30}	$\frac{1074.396}{s^{2.0961}+8.2663s^{1.0037}+7.8641}e^{-3.143*10^{\wedge}-10}$

The reduced fractional order model for PHWR system ais obtained by minimizing a performance index using GWO algorithm. ISE, Integral absolute error (IAE), Integral time square error (ITAE), etc. are some of the commonly used performance indices. Here, ISE is used as a performance index as it is simple and provides better approximation.

$$f_i = \int_0^T \left[w(i\Delta t) - w_r(i\Delta t) \right]^2 dt, \qquad (1)$$

ωηερε $w(i\Delta t)$ δενοτεσ τηε ηιγηερ–ορδερ σψστεμ϶σ υνιτ στεπ ρεσπονσε ανδ $w_r(i\Delta t)$ ρεπρεσεντσ τηε υνιτ στεπ ρεσπονσε οφ τηε δεσιρεδ ρεδυχεδ ορδερ μοδελ ατ $t = \Delta t$. Φλοωχηαρτ οφ τηε ΓΩΟ αλγοριτημ ισ πρ οϖιδεδ ιν Φιγυρε 4. Τηε ρεδυχε ορδερ μοδελ οβταινεδ υσινγ τηε ΓΩΟ αλγοριτημ ισ γιϖεν βελοω:

$$G_{\mathrm{Re}\,duced}(s) = \frac{195.3808}{1.077s^{1.0459} + 1.0013} \qquad (2)$$

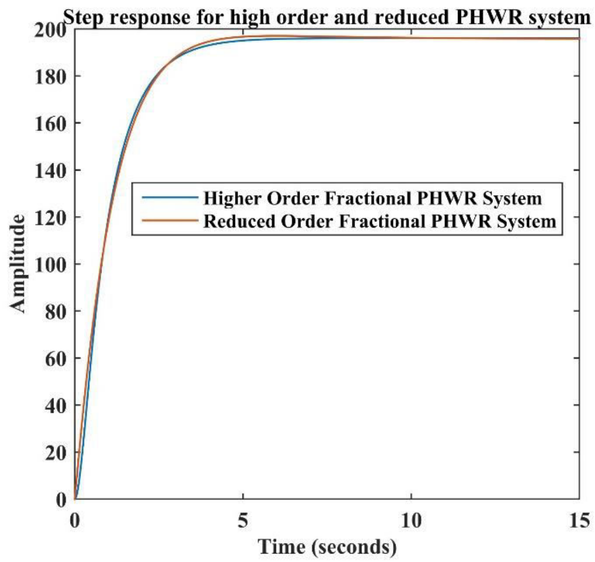

Figure 1: Open-loop response for the reduced and original system.

Figure 1 shows the fair comparison of the open-loop step responses for the original higher and reduced fractional model to show the good approximation of the high and reduced order model of PHWR. It is clear from the Figure 1 that both the responses are very close to each other which validates that the reduced order model, in fact, holds almost all the characteristics of the higher order fractional system.

3. Proposed Fractional Order Controller

The mathematical modelling of the FOPID controller is given in (3).

$$u(t) = k_P e(t) + k_I D_t^{-\lambda} e(t) + k_D D_t^{\mu} e(t) \qquad (3)$$

In Laplace domain the above equation becomes as given below,

$$G_{Controller}(s) = k_P + k_I s^{-\lambda} + k_D s^{\mu} \qquad (4)$$

here k_P, k_I, k_D, λ and μ are the controller parameters.

As it can be seen from the above equation, five parameters of controller are required to be tuned in order to get the complete controller design Integral Squared Error (ISE) is used to formulate the objective function. Figure 2 depicts a pictorial representation of the structure of the designed FOPID controller. The GWO Algorithm is used to find the best values. The obtained optimization based FOPID controller is given in (5) below:

$$G_C(s) = \frac{0.000584s^{1.0116} + 0.005s^{1.115} + 0.00489}{s^{1.0116}} \tag{5}$$

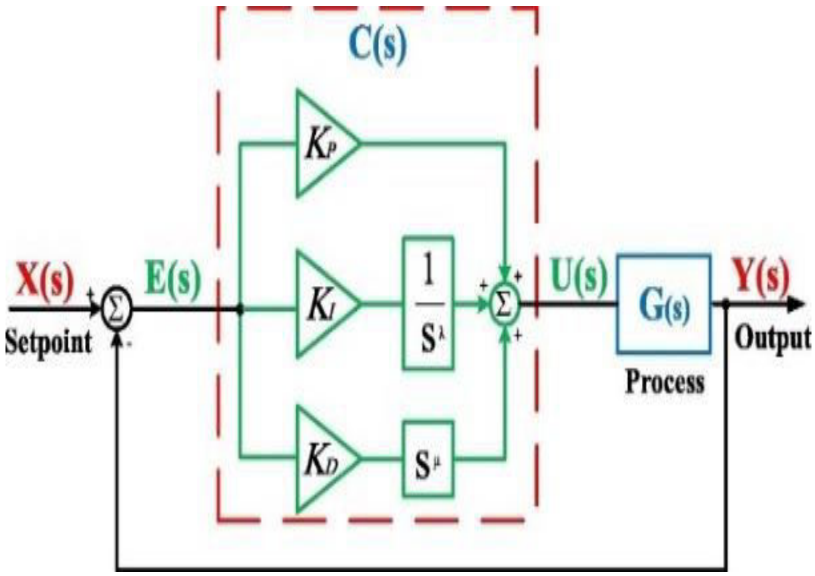

Figure 2: Structure of control scheme.

3.1 ***Grey Wolf Optimization (GWO) Algorithm***

The GWO algorithm is a meta-heuristic optimisation method based on grey wolf hunting and leadership behaviour.

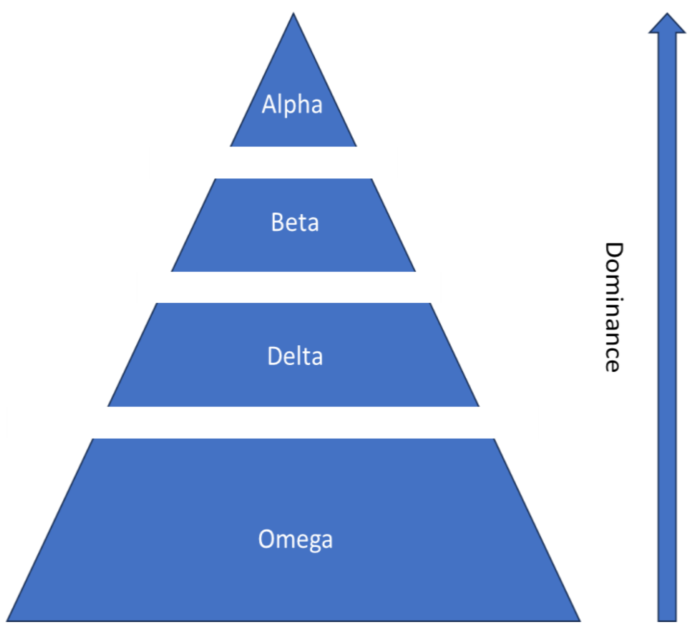

Figure 3: Dominance of grey wolves.

It is one of the most recent and advanced optimization methods, and it has been widely used in a variety of domains and applications, including control systems. Grey wolves are assumed to be at the top of the food chain in this method. Wolves are classified into four groups: Alpha, Beta, Delta, and Omega. Figure 3 depicts this.

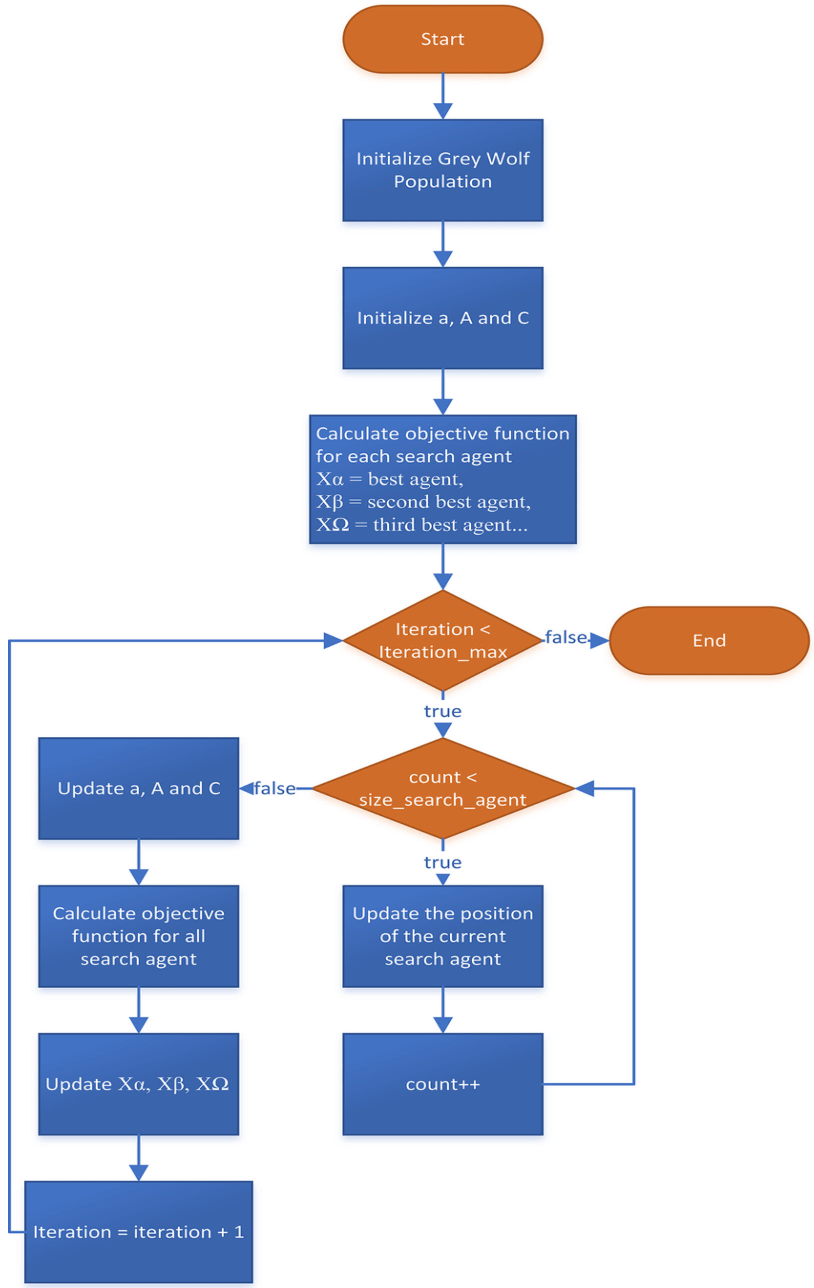

Figure 4: Flowchart of GWO

The Alpha wolf is the most dominant in Figure 3. The grey wolf algorithm consists of three major phases, which are as follows:

1. Tracking and approaching the prey.
2. Encircling and tormenting the prey until it stops moving.

3. Attacking on the prey.

The flowchart in Figure 5 provides a more detailed overview of the steps involved in the GWO. The GWO Pseudo code is as follows:

START

Initialization of algorithm parameters

Position generation wolves (P_w), agent (P_A)

Fitness of agent is determined.

Estimate X_α, X_β and X_Ω

While ($t<N_{max}$)

{

For

{

update P_{curr}

}

End for

Search agents are updated

Fitness value is calculated

Update X_α, X_β and X_Ω

Count++

}

End while

Return best value (X_α)

END

Where P_w is position of wolves, P_A is position of agent, N_{max} is maximum number of iteration and P_{curr} is current position of search agent.

4. Results and Discussion

All simulations studies have been run in the MATLAB/SIMULINK platform. To illustrate the resilience of the developed controller, its performance is analysed in the presence of with and without disturbance. To test the robustness of the controller, parametric uncertainty of +25 % have been considered in the coefficients of the transfer function of PHWR system. For each of these scenarios, the simulation results for step response are shown from Figure 5 to Figure 8.

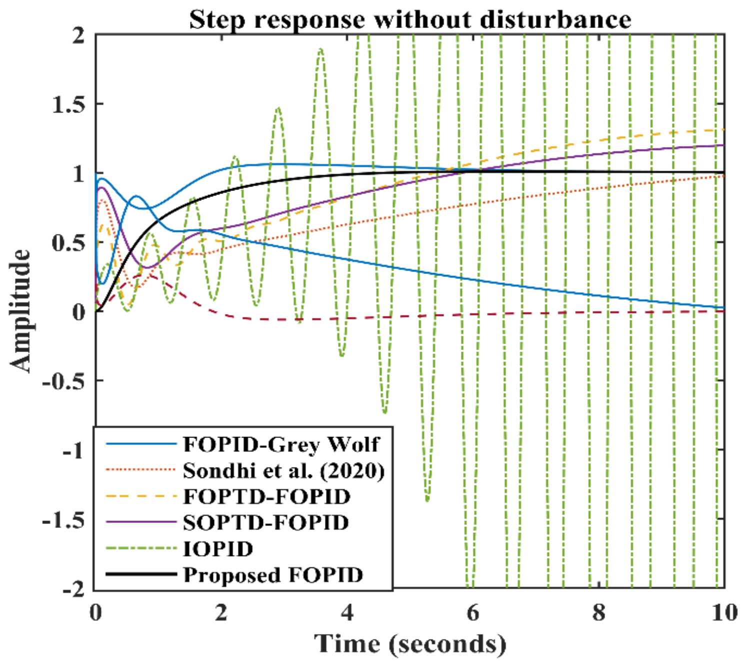

Figure 5: Step response without disturbance.

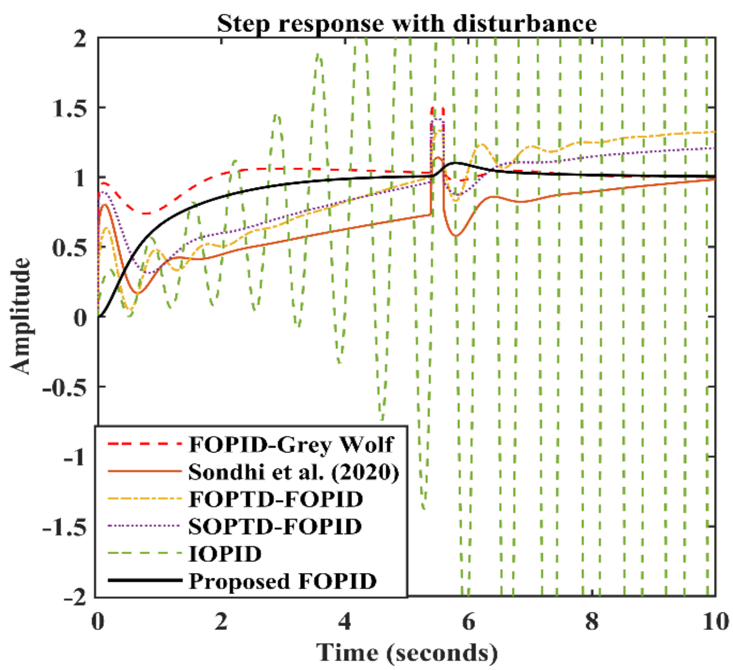

Figure 6: Step response with the disturbance

Figures 5 and 6 depict the step responses without and with disturbance. At 5.5 seconds, a disturbance signal is introduced. It is observed from Figure 6 that the developed controller can handle and reject the disturbance efficiently. Figure 7 and 8 show the step responses for +25% and -25% parametric uncertainty, respectively. The simulated response shows that the controller can easily handle parameter uncertainty, demonstrating the design's robustness.

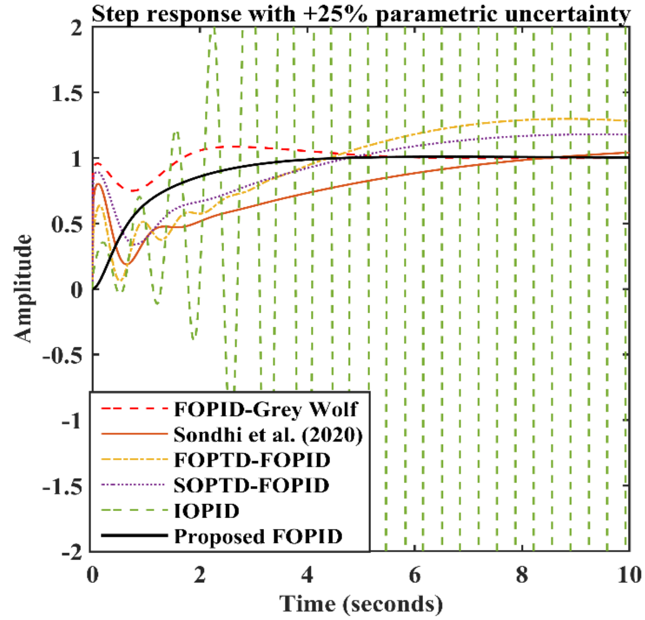

Figure 7: Step response under +25% uncertainty

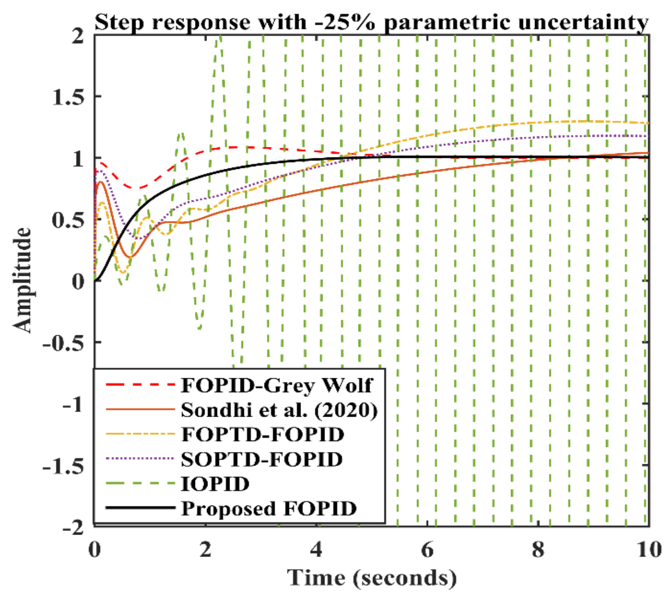

Figure 8: Step response under -25% uncertainty.

Table 2: Performance analysis based on various control strategies with disturbances

S.N	Controller	ISE	IAE	ITAE	t_r (sec)	t_s (sec)
1.	FOPID GWO	0.0668	0.5050	1.3118	0.2	2.3
2.	Sondhi et al. (2020)	1.5520	4.8580	48.2377	7.2	>10
3.	FOPDT	1.8551	6.3168	81.8687	3.8	>10
4.	SOPDT	1.0590	4.4164	50.8151	3.7	>10
5.	IOPID	1.28e+20	2.03e+10	9.78e+11	-	-
6.	Proposed FOPID	0.5243	1.0638	1.9081	2.1	2.9

Tabular comparison of numerous performance metrics and transient specifications are also undertaken, with the findings displayed in Tables 2 to 5. Tables 2 and 3 depicts the numerical values of various error indices for step response with and without disturbance, respectively. Tables 4 and 5 shows the error indices for +25% and -25% parametric uncertainty, respectively. It is evident from the simulation studies that the though proposed FOPID controller offers comparable performance indices and transient specifications, but the responses of proposed controllers are smooth which is demonstrating the efficacy of the chosen control method.

Table 3: Performance analysis based on various control strategies without disturbances

S.N	Controller	ISE	IAE	ITAE	t_r (sec)	t_s (sec)
1.	FOPID GWO	0.1183	0.6194	0.9504	0.2	2.3
2.	Sondhi et al. (2020)	1.5698	4.8687	48.4174	7.2	>10
3.	FOPDT	1.9460	6.5599	84.5854	3.8	>10
4.	SOPDT	0.1255	4.6349	52.6819	3.7	>10
5.	IOPID	1.34e+20	2.08e+10	9.9e+11	-	-
6.	Proposed FOPID	0.5310	1.1638	2.5519	2.1	2.9

Table 4: Performance analysis based on various control strategies with +25% uncertainty

S.N	Controller	ISE	IAE	ITAE	t_r (sec)	t_s (sec)
7.	FOPID GWO	0.0633	0.4616	0.9859	0.23	2.3
8.	Sondhi et al. (2020)	1.0781	3.2990	23.402	5.6	>10
9.	FOPDT	1.3920	4.7393	47.867	3.4	>10
10.	SOPDT	0.7326	3.0721	25.643	2.8	8.9
11.	IOPID	1.81e+38	1.78e+19	8.74e+20	-	-
12.	Proposed FOPID	0.5243	1.0638	1.9081	1.8	2.3

Table 5: Performance analysis based on various control strategies with -25% uncertainty

S.N	Controller	ISE	IAE	ITAE	t_r (sec)	t_s (sec)
13.	FOPID GWO	0.0625	0.4576	0.9740	0.22	4.3
14.	Sondhi et al. (2020)	1.0700	3.2916	23.353	5.9	10
15.	FOPDT	1.3807	4.7113	47.427	2.5	11
16.	SOPDT	0.7274	3.0622	25.564	2.4	8.8
17.	IOPID	1.61e+38	1.69e+19	8.26e+20	-	-
18.	Proposed FOPID	0.5243	1.0638	1.9081	1.9	1.9

5. CONCLUSIONS

The PHWR is a nonlinear, higher order system. The current study tries to control PHWR efficiently. First, using the GWO algorithm, the model of a higher order fractional system is reduced to a model of a lower order fractional system. Then, a FOPID controller has been developed to control the reduced order fractional model of PHWR. The parameter of FOPID has been tuned with the GWO algorithm. A performance analysis based on performance indices ISE, IAE, ITAE, rise time and setting time is performed. The step responses for set point changes and disturbance rejection have been simulated, there after step response under +25% uncertainty have also been recorded. It is concluded that the performance indices are much better than other methods except FOPID used with integer lower order system. The proposed controller shows the better transient responses.

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