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SPECIAL OPERATORS AND LEVEL SETS OF INTUITIONISTIC FUZZY D-IDEALS OF D-SUBALGEBRA

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ABSTRACT
In this paper, we discuss special operators and level sets of intuitionistic fuzzy d-ideals of d-subalgebra. Here we also investigate about some of its properties in detail by using the concepts of intuitionistic fuzzy d-ideals.

Keywords: d-subalgebra, intuitionistic fuzzy d-ideal, homomorphism, complement, necessity operator, possibility operator, level set.

AMS Subject Classification: 94D05,08A72

1. INTRODUCTION
Fuzzy set theory was discovered by Zadeh in 1965 [8]. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. Fuzzy mathematics is the branch of mathematics. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics.

Fuzzy set theory was guided by the assumption that the classical sets were not natural appropriate or useful notions in describing the real life problems because every object encountered in the real physical world carries some degree of fuzziness. Hence fuzzy set has become strong area of research in engineering, medical science, graph theory etc.

Fuzzy algebra is an important branch of fuzzy mathematics. J. Negger and H.S.Kim introduced the class d-algebra which is the generalization of BCK-algebras and investigated relation between d-algebras and BCK-algebras [5]. J. Negger, Y. B. Jun and H.S. Kim discussed ideal theory in d-algebra [4]. In 1986 Atanassov introduced the concept of intuitionistic fuzzy set as a generalization of fuzzy set [2]. Y.B. Jun , H.S.Kim and D.S. Yoo introduced the notion of intuitionistic fuzzy d-algebra [3]. M. Akram and K.H. Dar [1] introduced the concepts of fuzzy d-algebra, fuzzy subalgebra and fuzzy d-ideals of d-algebra.

2. PRELIMINARIES

Definition: 2.1
A d-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfies the following axioms:

i. $x * x = 0$

ii. $0 * x = 0$

iii. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, for all $x, y \in X$.

Definition: 2.2
A non-empty subset of a d-algebra X is called d-subalgebra of X if $x * y \in X$, for all $x, y \in X$.

Definition : 2.3
A mapping $f : X \to Y$ of d-subalgebra is called homomorphism if

$f(x * y) = f(x) * y(y), \quad for\ all\ x, y \in X.$

Definition: 2.4

For every intuitionistic fuzzy subset A of a set X, the following operators are defined

i) The necessity operator $\prod A = \{x, \mu_A(x), (\mu_A)^C(x)/x \in X\}$

ii) The possibility operator $\Omega A = \{x, \vartheta_A(x), (\vartheta_A)^C(x)/x \in X\}$.

Definition: 2.5

The complement of μ_A, denoted by $(\mu_A)^C$ is the fuzzy set in X given by $(\mu_A)^C(x) = 1 - \mu_A(x)$.

3. INTUITIONISTIC FUZZY D-IDEALS

Definition: 3.1

An intuitionistic fuzzy set $A = (\mu_A, \vartheta_A)$ in X is called intuitionistic fuzzy d-ideal of X if it satisfies the following axioms:

$\mu_A(0) \geq \mu_A(x)$

$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}\ for\ all\ x, y \in X.$

$\vartheta_A(0) \leq \vartheta_A(x)$

$\vartheta_A(x) \leq max\{\vartheta_A(x * y), \vartheta_A(y)\}$

$\vartheta_A(x * y) \leq max\{\vartheta_A(x), \vartheta_A(y)\}\ for\ all\ x, y \in X.$

Theorem: 3.2

Let $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X. Then so

$\prod A = \{x, \mu_A(x), (\mu_A)^C(x)/x \in X\}$.

Proof:

Since $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideals of X, then

$\mu_A(0) \geq \mu_A(x)$, $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}\ for\ all\ x, y \in X$

Now $\mu_A(0) \geq \mu_A(x)$

$1 - (\mu_A)^C(0) \geq 1 - (\mu_A)^C(x)$

$(\mu_A)^C(0) \leq (\mu_A)^C(x)$

Now for any $x, y \in X$

$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

$1 - (\mu_A)^C(x) \geq min\{1 - (\mu_A)^C(x * y), 1 - (\mu_A)^C(y)\}$

$(\mu_A)^C(x) \leq max\{(\mu_A)^C(x * y), (\mu_A)^C(y)\}$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

$1 - (\mu_A)^C(x * y) \geq min\{1 - (\mu_A)^C(x), 1 - (\mu_A)^C(y)\}$

$(\mu_A)^C(x * y) \leq max\{(\mu_A)^C(x), (\mu_A)^C(y)\}$

Hence $\prod A = \{x, \mu_A(x), (\mu_A)^C(x)/x \in X\}$ is an intuitionistic fuzzy d-ideal of X.

Theorem: 3.3

Let $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X. Then so

$\Omega A = \{x, \vartheta_A(x), (\vartheta_A)^C(x)/x \in X\}$.

Proof:

Since $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideals of X, then

$\vartheta_A(0) \geq \vartheta_A(x)$, $\vartheta_A(x) \geq min\{\vartheta_A(x * y), \vartheta_A(y)\}$

$\vartheta_A(x * y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$ $for\ all\ x, y \in X$

Now $\vartheta_A(0) \leq \vartheta_A(x)$

$1 - (\vartheta_A)^C(0) \leq 1 - (\vartheta_A)^C(x)$

$(\vartheta_A)^C(0) \geq (\vartheta_A)^C(x)$

Now for any $x, y \in X$

$\vartheta_A(x) \leq max\{\vartheta_A(x * y), \vartheta_A(y)\}$

$1 - (\vartheta_A)^C(x) \leq max\{1 - (\vartheta_A)^C(x * y), 1 - (\vartheta_A)^C(y)\}$

$(\vartheta_A)^C(x) \geq min\{(\vartheta_A)^C(x * y), (\vartheta_A)^C(y)\}$

$\vartheta_A(x * y) \leq max\{\vartheta_A(x), \vartheta_A(y)\}$

$1 - (\vartheta_A)^C(x * y) \leq max\{1 - (\vartheta_A)^C(x), 1 - (\vartheta_A)^C(y)\}$

$(\vartheta_A)^C(x * y) \geq min\{(\vartheta_A)^C(x), (\vartheta_A)^C(y)\}$

Hence $\Omega A = \{x, \vartheta_A(x), (\vartheta_A)^C(x)/x \in X\}$ is an intuitionistic fuzzy d-ideal of X.

4. LEVEL SETS OF INTUITIONISTIC FUZZY D-IDEALS:

Definition: 4.1

Let $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy set in X. For any $\alpha, \beta \in [0,1]$ then $\alpha -$ level cut of μ_A and $\beta -$ level cut of ϑ_A is as follows :

$\mu_{A,\alpha}^{\geq} = \{x \in X/\mu_A(x) \geq \alpha\}$

$\vartheta_{A,\beta}^{\geq} = \{x \in X/\vartheta_A(x) \leq \beta\}$

Theorem: 4.2

Let $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X, then $\mu_{A,\alpha}^{\geq}$ and $\vartheta_{A,\beta}^{\geq}$ are d-ideal of X, for any $\alpha, \beta \in [0,1]$.

Proof:

Let $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X and let $\alpha \in [0,1]$.

Then we have $\mu_A(0) \geq \mu_A(x) \quad for\ all\ x \in X$

But $\quad \mu_A(x) \geq \alpha \ \ for\ all\ x \in \mu_{A,\alpha}^{\geq}$

So $\quad \mu_A(0) \geq \alpha$

Therefore $\quad 0 \in \mu_{A,\alpha}^{\geq}$

Let $x, y \in X$ such that $(x * y), y \in \mu_{A,\alpha}^{\geq}$

Then $\mu_A(x * y) \geq \alpha$, $\mu_A(y) \geq \alpha$

Since μ_A is an intuitionistic fuzzy d-ideal of X, then

$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

$\geq \alpha$

$x \in \mu_{A,\alpha}^{\geq}$

Let $x, y \in X$ such that $x, y \in \mu_{A,\alpha}^{\geq}$

Then $\mu_A(x) \geq \alpha$, $\mu_A(y) \geq \alpha$

Since μ_A is an intuitionistic fuzzy d-ideal of X, then

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

$\geq \alpha$

$x * y \in \mu_{A,\alpha}^{\geq}$

Therefore, $\mu_{A,\alpha}^{\geq}$ is a d-ideal of X, for any $\alpha \in [0,1]$.

Clearly, this can be proved for maximal condition.

Therefore, $\mu_{A,\alpha}^{\geq}$, $\vartheta_{A,\beta}^{\leq}$ is a d-ideal of X, for any $\alpha, \beta \in [0,1]$.

Theorem: 4.3

An intuitionistic fuzzy set $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X, if and only if for all $\alpha, \beta \in [0,1]$, $\mu_{A,\alpha}^{\geq}$ and $\vartheta_{A,\beta}^{\geq}$ are either empty or d-ideal of X.

Proof:

Let $\mu_{A,\alpha}^{\geq}$ and $\vartheta_{A,\beta}^{\geq}$ be either or d-ideal of X, for $\alpha, \beta \in [0,1]$

For any $x \in X$,

Let $\mu_A(x) = \alpha$

$\vartheta_A(x) = \beta$ then $x \in \mu_{A,\alpha}^{\geq}\ \ and\ \vartheta_{A,\beta}^{\geq}$

So, $\mu_{A,\alpha}^{\geq} \ \neq \emptyset \neq \ \vartheta_{A,\beta}^{\geq}$

Since $\mu_{A,\alpha}^{\geq}\ \ and\ \vartheta_{A,\beta}^{\geq}$ are d-ideals of X, therefore $0 \in \mu_{A,\alpha}^{\geq}\ \ and\ \vartheta_{A,\beta}^{\geq}$

Hence $\mu_A(0) \geq \alpha = \mu_A(x)$ and

$\vartheta_A(0) \leq \beta = \vartheta_A(x)$ where $x \in X$

If there exist $x', y' \in X$ such that

$\mu_A(x') < min\{\mu_A(x' * y'), \mu_A(y')\}$

Then by taking

$$\alpha_0 = \frac{1}{2}[\mu_A(x') + min\{\mu_A(x' * y'), \mu_A(y')\}]$$

We have,

$min\{\mu_A(x' * y'), \mu_A(y')\} > \alpha_0 > \mu_A(x')$

Hence $x' \notin \mu^{\geq}_{A,\alpha_0}$

$(x' * y') \in \mu^{\geq}_{A,\alpha_0}$, $y' \in \mu^{\geq}_{A,\alpha_0}$

That is $\mu^{\geq}_{A,\alpha_0}$ is not d-ideals of X

Which is a contradiction.

Therefore $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

If there exist $x'', y'' \in X$ such that

$\mu_A(x'' * y'') < min\{\mu_A(x''), \mu_A(y'')\}$

Then by taking

$$\alpha_1 = \frac{1}{2}[\mu_A(x'' * y'') + min\{\mu_A(x''), \mu_A(y'')\}]$$

We have,

$min\{\mu_A(x''), \mu_A(y'')\} > \alpha_1 > \mu_A(x'' * y'')$

Hence $x'' * y'' \notin \mu^{\geq}_{A,\alpha_1}$

$x'' \in \mu^{\geq}_{A,\alpha_1}$, $y'' \in \mu^{\geq}_{A,\alpha_1}$

That is $\mu^{\geq}_{A,\alpha_1}$ is not d-ideal of X

Which is a contradiction.

Therefore $\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

Clearly, this can be proved for maximal condition.

Therefore $\vartheta_A(x * y) \leq max\{\vartheta_A(x), \vartheta_A(y)\}$.

Conversely,

Assume that $A = (\mu_A, \vartheta_A)$ be an intuitionistic fuzzy d-ideal of X

To Prove: $\mu^{\geq}_{A,\alpha}$ and $\vartheta^{\geq}_{A,\beta}$ are either empty or d-ideal of X.

Suppose that $\mu^{\geq}_{A,\alpha}$ and $\vartheta^{\geq}_{A,\beta} \neq \varphi$ for any $\alpha, \beta \in [0,1]$

It is clear that $0 \in \mu_{A,\alpha}^{\geq}$ and $\vartheta_{A,\beta}^{\geq}$

Since $\mu_A(0) \geq \mu_A(x) = \alpha$

$\vartheta_A(0) \leq \vartheta_A(x) = \beta$

Let $x, y \in X$ such that

$x * y \in \mu_{A,\alpha}^{\geq}$, $y \in \mu_{A,\alpha}^{\geq}$

Then

$\mu_A(x * y) \geq \alpha$ and $\mu_A(y) \geq \alpha$

$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\} \geq \alpha$

and hence $x \in \mu_{A,\alpha}^{\geq}$ $for\ all\ x, y \in X$

Therefore, $\mu_{A,\alpha}^{\geq}$ is a d-ideal of X for $\alpha \in [0,1]$

Let $x, y \in X$ such that

$x \in \mu_{A,\alpha}^{\geq}$, $y \in \mu_{A,\alpha}^{\geq}$

Then

$\mu_A(x) \geq \alpha$ and $\mu_A(y) \geq \alpha$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\} \geq \alpha$

and hence $x * y \in \mu_{A,\alpha}^{\geq}$ $for\ all\ x, y \in X$

Therefore, $\mu_{A,\alpha}^{\geq}$ is a d-ideal of X for $\alpha \in [0,1]$

Clearly, this can be proved for maximal condition.

Therefore, $\mu_{A,\alpha}^{\geq}\vartheta_{A,\beta}^{\leq}$is a d-ideal of X for $\alpha, \beta \in [0,1]$

4. CONCLUSION

In this paper, we have given some ideas on special operators and level set of intuitionistic fuzzy d-ideals of d-subalgebra. Here we also investigate about some of its properties in detail by using the concepts of intuitionistic fuzzy d-ideals.

5. REFERENCES

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