International Journal of Applied Engineering & Technology

COMPARITIVE STUDY OF BINOMIAL AND TRINOMIAL MODELS FOR AMERICAN OPTION PRICING

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ABSTRACT

The binomial model is a widely used method due to its simplicity and flexibility, while the trinomial model offers a refinement by introducing a third node for the underlying asset price movement. This paper presents theoretical working of binomial model for American Options. We present the empirical investigation of the efficacy of binomial and trinomial models for pricing American options. American options differ from European options in allowing early exercise, making their pricing more complex. Through empirical analysis using historical option pricing data, this study compares the convergence, accuracy, error and computational efficiency of these two models in estimating American option prices across various market conditions and option types. The findings provide insights into the strengths and limitations of each model, aiding practitioners and researchers in selecting the most suitable pricing approach for American options (AO) in different contexts.

Keyword: American Options, Call Options, Put Options, Binomial Model, Trinomial Model, Lattice Methods, Black Scholes Model, Black Scholes Merton Model.

1. INTRODUCTION

In the realm of financial derivatives, option pricing stands as a fundamental pillar, crucial for risk management and investment strategies. Black F. & Scholes M. introduced the PDE representing option pricing model in 1973[1]. They got a Noble prize for the same along with Merton C. [2]. After this discovery numerous methods were developed for pricing various options in stock market. Among various methodologies, binomial and trinomial models have emerged as prominent tools for pricing American options, offering insights into complex decision-making processes within dynamic markets [4]. Cox, Ross & Rubinstein in 1985 [3] gave a good portrayal of the Binomial Model where the prices jump at discrete intervals to one or two values as an alternative to changing continuously. Such models which are discrete, may not be precise in the real world but can give perception into financial problems. This empirical study delves into the efficacy of these models, aiming to provide clarity and guidance to practitioners and researchers navigating the intricacies of option valuation. The American option, distinguished by its flexibility to be exercised at any time before expiration, presents a unique challenge in pricing due to its early exercise feature. Traditional Black-Scholes model, albeit widely used for European options, falls short in capturing the nuances of American options [8], thereby necessitating alternative approaches such as binomial and trinomial models. The binomial model, rooted in the concept of risk-neutral valuation, simplifies the option pricing process by discretizing time and assuming a constant volatility. On the other hand, the trinomial model introduces an additional level of complexity by incorporating three possible price movements at each time step [5], offering a more nuanced representation of market dynamics. While, theoretical frameworks provide a solid foundation, empirical validation is indispensable for assessing the real-world applicability and performance of these models. This study undertakes a comprehensive analysis, utilizing historical market data and statistical techniques to evaluate the accuracy and reliability of both binomial and trinomial models in pricing American options across diverse market conditions [4]. By comparing the empirical results with theoretical expectations and benchmark models, this research aims to shed light on the strengths, limitations, and practical considerations associated with binomial and trinomial models for American option pricing [6,10,11]. Such insights are invaluable for market participants, risk managers, and academics seeking to make informed decisions and advancements in financial modeling and derivative pricing.

2. BLACK-SCHOLES MODEL FOR AMERICAN OPTIONS:

European call options and American call options are both types of financial derivatives that give the holder the right, but not the obligation, to buy an underlying asset (such as a stock) at a predetermined price (the strike price) on or before a specific expiration date. However, there is a fundamental difference between them [2].

2.1 American Options: With an American call option, the holder has the right to exercise the option at any time before or on the expiration date. This gives the holder more flexibility, as they can choose to exercise the option early if it is advantageous to do so. Despite the difference in exercise timing, under certain conditions, European call options and American call options can have the same value [5]. This occurs primarily when there are no dividends paid on the underlying asset during the option's life and there are no restrictions on short selling or borrowing against the asset. In such cases, the option holder would only exercise the American call option early if it were optimal to do so (e.g., if exercising early allows them to capture a dividend payment). Otherwise, they would wait until expiration, making the American call option equivalent to the European call option. This condition where the two options have the same value is known as "*no arbitrage*" However, when dividends are involved or other market constraints exist, the two options may have different values [10]

2.2 American Call option: The American call option's payoff is given by $C(S,t) = (S - K)_+$ if it is exercised when asset price is s and K is the *Strike price*. The payoff function is convex in the argument s for every K. The optimal exercise policy for American call Option with no dividend payment is at Expiration time T. $C(S,t)$ satisfises following Black-Scholes Model [9].

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + (r - D_0) S \frac{\partial C}{\partial S} - rC = 0$$

Since the American option can be exercised at any time following is always true,

$$C(S,t) \geq (S - K)_+$$

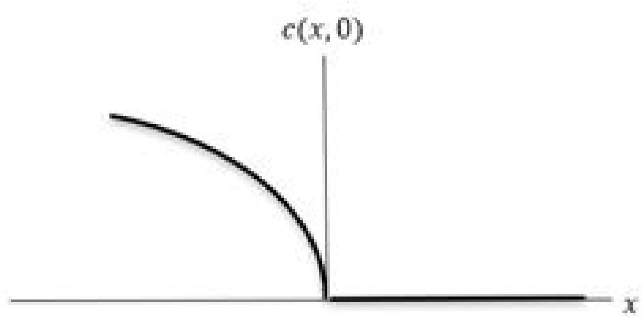

Figure 2.1: The American Call

2.3The Optimal Exercise Policy of American Call option: Let the modest non-trivial setting of American option be, such that the Black-Scholes Model, which is given by, [9]

$$S_t = S_0 e^{\left(rW_t + \left(r - \sigma^2/_2 t\right)\right)} \qquad (1)$$

with underlying asset without dividends and has a S_t as its asset price with price process that behaves like simple geometric motion under the risk neutral measure Q. W_t is the Brownian measure. Let us denote the value of American option as C_t.

$$C_t = \sup_{\tau: t \le \tau \le T} E\left(C(S_T)e^{-r(\tau-t)} \middle| \mathcal{F}_t\right) \quad (2)$$

$r \ge 0$	Risk free interest
W_t	Standard Wiener Process
C_t	Value of American Option at $t \le T$
$(\mathcal{F}_t)_{0 \le t \le T}$	Natural filtration of the Wiener Process W_t
σ	Volatility
$C(S_T)$	Payoff function at s and T

One of the lemma, in option pricing says that the optimal exercise policy of American option is to Exercise at T. If it's not, then $C(S_\tau) = (S_\tau - K)_+$ and is the payoff of American call option for $t \le \tau \le T$. Then value of the option at this pint for the holder is given by,

$$C(S_\tau - K)_+ e^{-r\tau} \le C(S_\tau e^{-r\tau} - Ke^{-rT})_+$$

Now the Martingale $\{S_t e^{-r\tau}\}_{t \ge 0}$ is the discounted price process since $\tau \le T$.

$$C(S_\tau - K)_+ e^{-r\tau} \le C(S_\tau e^{-r\tau} - Ke^{-rT})_+$$

So, for the option holder who is planning to exercise the option at τ, Value at time zero of American option is same as the European Call. Black-Scholes Model for American Call Option is given by,

$$C(S_T) = SN(d_1) - Ke^{-rt}N(d_2), \quad (3)$$

$$d_1 = \frac{\log(S/K) + \left(r + \sigma^2/2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

3. BINOMIAL MODEL FOR AMERICAN OPTIONS

In 1979, Cox, Ross and Rubinste introduced Binomial model in their research paper [3]. The discrete random walk models of the underlying asset gave rise to lattice methods option pricing and other derivative instruments. Therefore, the time duration up to T can be divided into M discrete and equal time steps, $\delta t = M/T$ it is assumed that the value of the asset can jump from its original value to up/down to finite time-step say N. Probability of the jump of up/down values is known [9]. For binomial model, let us assume that if $S_m$is the asset price time step $m\delta t$. Then there is a probability that it can either jump up to $u_m S_{m,}$ $u_m > 1$ or jump down to $d_m S_m$, $d_m < 1$ at time step $(m+1)\delta t$. Let the probability of up Jump be p_m and down jump be $1 - p_m$, therefore,

$$S_{m+1} = \begin{cases} u_m S_m, & with\ probability\ p_m \\ d_m S_m, & with\ probability\ 1-p_m \end{cases}$$

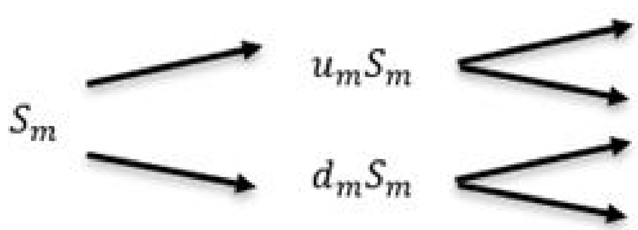

Figure 3.1 the Binomial Model

Let us assume that, the random walk S be lognormally distributed in risk neutral world. By approximating this continuous random walk with discrete random walk, where their mean and variance is same. Therefore,

$$S_m e^{r\delta t} = \varepsilon[S_{m+1}] = S_m(p_m u_m + (1-p_m)d_m)$$
$$var[S_{m+1}] = \varepsilon[(S_{m+1})^2] - (\varepsilon[S_{m+1}])^2$$

Therefore, for, $u_{m,} d_m$ & p_m we have,

$$(p_m u_m + (1-p_m)d_m) = e^{r\delta t}$$
$$(p_m(u_m)^2 + (1-p_m)(d_m)^2) = \sigma^2\delta t + e^{2r\delta t}, \sigma = volatility$$

We assume that u_m, d_m & $p_m$be constant for the entire random walk, even though there is a possibility that they can very vary with m

$$\begin{aligned} pu + (1-p)d &= e^{r\delta t} \\ pu^2 + (1-p)d^2 &= \sigma^2\delta t + e^{2r\delta t} \end{aligned} \quad (4)$$
$$p_m = p, u_m = u \,\&\, d_m = d, for\ all\ m$$
$$\text{And } u = {}^1/_d \quad (5)$$

Therefore, to calculate u, d & p we can use equation no. 4 & 5. Let δt be small enough so that we can neglect terms of $O\left((\delta t)^{3/2}\right)$ and other higher terms too.

$$\begin{aligned} p &= \frac{1}{2}\left(1 + \left(\left(r - \frac{1}{2}\sigma^2\right)/\sigma\sqrt{\delta t}\right) + \frac{\sigma^2}{6}\delta t + O\left((\delta t)^{3/2}\right)\right), \\ u &= 1 + \sigma\sqrt{\delta t} + \frac{1}{2}\sigma^2\delta t + O\left((\delta t)^{3/2}\right), \\ d &= 1 - \sigma\sqrt{\delta t} + \frac{1}{2}\sigma^2\delta t + O\left((\delta t)^{3/2}\right) \end{aligned} \quad (6)$$

Since we can neglect the terms of $O\left((\delta t)^{3/2}\right)$,we have following,

$$p = \frac{r\delta t - e^{-\sigma\sqrt{\delta t}}}{e^{\sigma\sqrt{\delta t}} - e^{-\sigma\sqrt{\delta t}}}$$
$$u = e^{\sigma\sqrt{\delta t}}, d = e^{-\sigma\sqrt{\delta t}}$$

Now, for American options, the possibility of early exercise can be easily incorporated into a Binomial Model [9]. Let T be divided into Mequal time steps $t = M/T$. From Eq. 4,5& 6 we get

$$S_{m,n} = u^{2n-m}S_{0,0}, n = 0,1,2 \dots \dots, m$$

Here, $S_{0,0}$is the current value and $S_{m,n}$ is the value at time m. The possible values of the option at time $m\delta t$ can be calculated for Put and Call as,

$$P_{M,n} = (K - S_{m,n})_+, C_{M,n} = (S_{m,n} - K)_+$$

$$V_{M,n} = \begin{cases} 0 & S_{m,n} < K \\ B & S_{m,n} \geq K \end{cases}$$

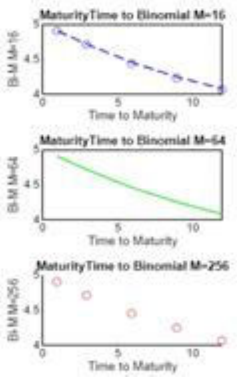

Figure 4.1: Comparison American Put Option by Binomial Model for different steps

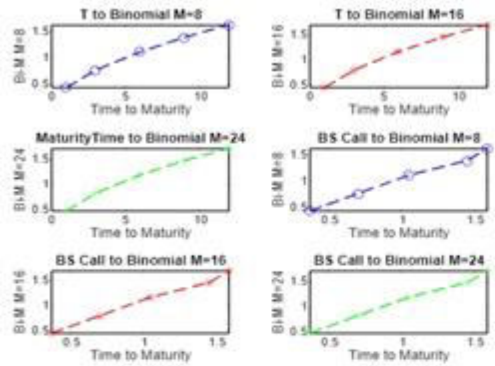

Figure 4.2: Comparison American Call Option by Binomial Model for different steps

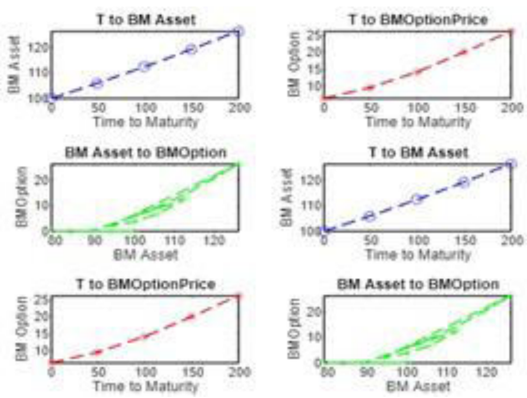

Figure 4.3: Comparison American Call Asset Price to Option Price by Binomial Model

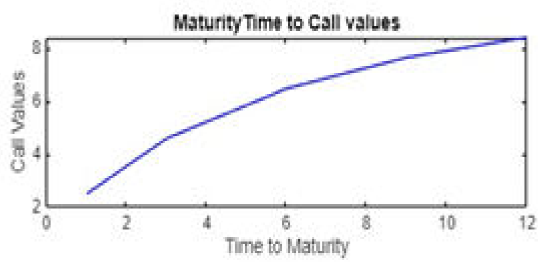

Figure 4.4: Comparison of Maturity Time to American call by Binomial Model

4. TRINOMIAL MODEL FOR AMERICAN OPTION:

Trinomial Models are also one of the Lattice Methods, like in Binomial Model trinomial models have 3 steps at each node Up, Middle and Down. Although Binomial Model method is very famous and useful for option pricing. Trinomial models also work well for the same [9]. Let the asset value take S_u, S_q and S_d after a time step δt. p_u is the probability that the asset is S_u

At δt. Similarly, p_q is the probability for S_q and p_d is the probability for S_d. $0 < d < q < u$. Also, there are only three possibilities, we have,

$$p_u + p_q + p_d = 1, 0 \leq p_d \leq 1, 0 \leq p_q \leq 1, 0 \leq p_d \leq 1$$

Let Asset take lognormally distributed values in risk neutral world then,

$$p_u u + p_q q + p_d d = e^{r\delta t}$$
$$p_u u^2 + p_q q^2 + p_d d^2 = \sigma^2 \delta t - e^{2r\delta t}$$

After sufficient calculations and assumptions of the above 3 equations in 6 unknown and replacing $\delta t = \delta t/2$ we get,

$$u = e^{2\sigma\sqrt{\delta t/2}}, \qquad q = 1, \qquad d = e^{-2\sigma\sqrt{\delta t/2}}$$

With,

$$p = \frac{e^{r\delta t/2} - e^{-\sigma\sqrt{\delta t/2}}}{e^{\sigma\sqrt{\delta t/2}} - e^{-\sigma\sqrt{\delta t/2}}}, \qquad p_u = p^2, \qquad p_q = 2p(1-p), \qquad p_d = (1-p)^2$$

Then the expected value of portfolio at time step m form time step $m+1$ is,

$$\varepsilon\,[V_{m+1}] = p_u V_{m+1,n+1} + p_q V_{m+1,n} + p_d V_{m+1,n-1}$$

5. Comparison between Binomial Model & Trinomial model for pricing American options

5.1 Comparison of Binomial and Trinomial methods based on the accuracy:

The main difference between the Binomial and Trinomial models lies in the number of possible price movements. The Binomial model assumes two possible outcomes (up or down), while the Trinomial model considers three outcomes (up, down, or stay the same). The Trinomial model can offer more accuracy by allowing for an additional movement, providing a closer approximation to actual market conditions. However, this added complexity may come with increased computational demands. The choice between them depends on the specific requirements of the option pricing scenario and the trade-off between accuracy and computational efficiency.

Let us consider values of American Put Option by Binomial and Trinomial Models for, $k = 10, S = 9, r = 0.12, \sigma = 0.5, T$ = measured in months

Table 5.1.1: Comparison of American Put Values of Binomial and Trinomial Models

T	Black Scholes Model	Binomial Model				Trinomial Model			
		$M = 16$	32	64	128	$M = 16$	32	64	128
1	1.1311	1.1376	1.1308	1.1311	1.1317	1.1299	1.1307	1.1314	1.1315
3	1.3805	1.3815	1.3833	1.3822	1.3814	1.3807	1.3810	1.3809	1.3803
6	1.6166	1.6342	1.6191	1.6196	1.6185	1.6151	1.6176	1.6175	1.6173
9	1.7835	1.8078	1.7906	1.7814	1.7847	1.7870	1.7795	1.7831	1.7810
12	1.9086	1.9399	1.9216	1.9112	1.9106	1.9177	1.9085	1.9088	1.9087

From the above table it is very clear to see that the convergence of trinomial model is faster than Binomial model. But as the number of steps increase Binomial model values go very near to Black-Scholes Model [10,11].

5.2 Comparison of Binomial and Trinomial methods:

The convergence of the Binomial and Trinomial models for pricing American options depends on the number of steps (n) used in the tree. Generally, increasing the number of steps improves convergence by providing a more accurate approximation to the continuous-time process. However, this comes at the cost of increased computational complexity.

5.2.1 Nature of Convergence of Binomial and Trinomial Model for American call option.

The Binomial model is a discrete-time model used for pricing options, including American call options. In the Binomial model, the price of the underlying asset can move either up or down in each time step, and the option's value is calculated recursively at each step until expiration. The nature of convergence of the Binomial model for American call options depends on the number of steps in the binomial tree [11]. As the number of steps increases, the binomial model approaches the continuous-time Black-Scholes model, which is widely used for option pricing. In other words, the option price calculated using the However, it's important to note that the convergence may not be uniform across all situations. In some cases, the convergence might be rapid, while in others, it might be slower, especially if there are significant changes in volatility or interest rates. Additionally, the convergence might be affected by factors such as dividend yield and the proximity of the option's strike price to the current stock price. Despite these considerations, the Binomial model is still a powerful tool for pricing American call options, especially when it's not feasible to use continuous-time models like Black-Scholes. The trinomial model is an extension of the binomial model, where instead of two possible price movements (up and down), there are three possible movements (up, down, and stay the same) in each time step [10]. This additional level of complexity allows for a more accurate representation of price movements, particularly when dealing with assets that exhibit characteristics such as mean reversion or volatility skew. Similar to the binomial model, the nature of convergence of the trinomial model for American call options depends on the number of steps in the trinomial tree. As the number of steps increases, the trinomial model approaches the continuous-time models such as the Black-Scholes model. However, the convergence of the trinomial model to continuous-time models like Black-Scholes may generally be slower compared to the binomial model due to the additional complexity introduced by the third possible movement [8]. Nevertheless, as the number of steps increases, the option price calculated using the trinomial model tends to converge to the option price calculated using continuous-time models [6,7]. It's important to note that while the trinomial model offers more accuracy in capturing complex price movements, it also comes with increased computational complexity compared to the binomial model. Therefore, the choice between binomial and trinomial models often depends on the specific characteristics of the underlying asset and the level of precision required in the pricing model. The Trinomial model may offer better convergence due to the additional state but could be computationally more demanding than the Binomial model. The optimal choice depends on the specific requirements of the option pricing scenario and the available computational resources. It's advisable to experiment with different step sizes and models to find a balance that suits your accuracy and efficiency needs. In most of the study, to diminish the approximation errors more steps are used. Bur doing so nodes in Lattice models increase close to exponential rate [9]. Therefore, the computational time and number of calculations plays a vital role in efficiency of these models.

Table No. 5.2.2:

Binomial		Trinomial	
Nodes at Step n	Total Nodes	Nodes at Step n	Total Nodes
$n+1$	$(N^2+3N+2)/2$	$2n+1$	$(N+1)^2$

Therefore, if we want more accuracy then computational time will increase greatly.

Table No. 5.2.3: Comparing no. of nodes for various no. of steps

Number of steps	Binomial		Trinomial	
	Nodes at each node	Total No. of nodes	Nodes at each node	Total No. of nodes
2	3	6	5	21
4	5	15	9	55
8	9	45	17	171
16	17	153	33	595
32	33	561	65	2211
64	65	2145	129	8515
128	129	8385	257	33411
256	257	33153	513	132355
512	513	131841	1025	526851
1024	1025	525825	5121	13120003
5120	5121	13114881	10241	52454403

10240	10241	52444161	20481	209766403

It can be seen easily from table 4.2.3 that the Binomial Model (BM) has half the nodes than Trinomial Model (TM). Also, the total no of nodes increase must faster in Trinomial Model as compared to Binomial Model from step 128 onwards. But this comparison is not enough to compare accuracy and efficiency of these models. Black-Scholes (BS) Model and Binomial Models give similar values when the number of nodes become significantly large. Trinomial model on the other hand has much better performance than Binomial and BS Model [11]. Following are the graphs of Comparison of Binomial and Trinomial Method w.r.t. no of steps and no of Nodes

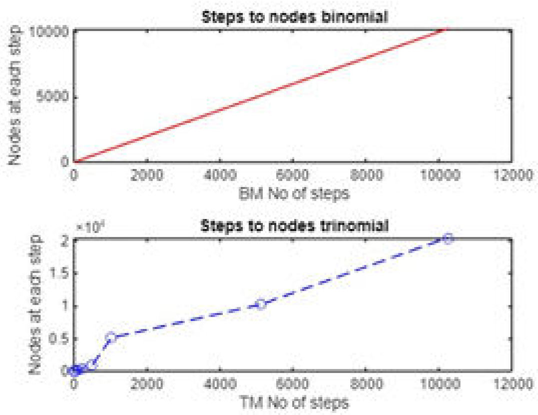

Figure 5.2.1: Comparison of Binomial and Trinomial for No. of Steps to Each Nodes

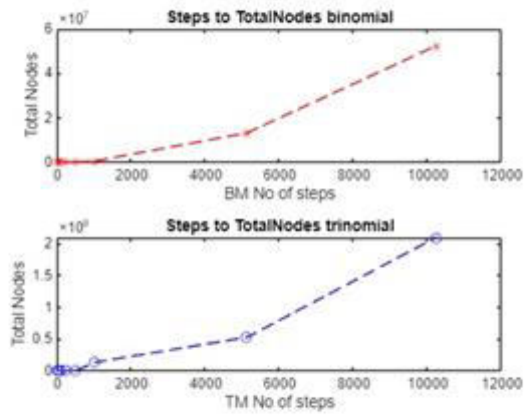

Figure 5.2.2: Comparison of Binomial and Trinomial for No. of Steps to Total Nodes

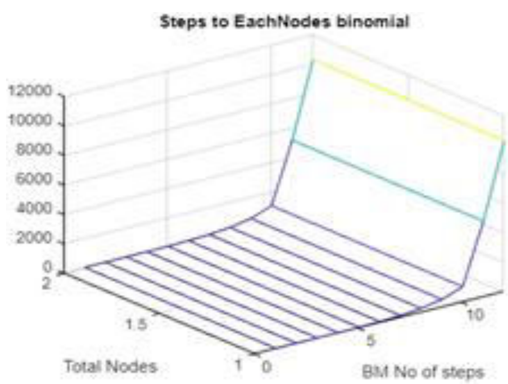

Figure 5.2.3: Comparison of Total Nodes to No. of Steps Binomial

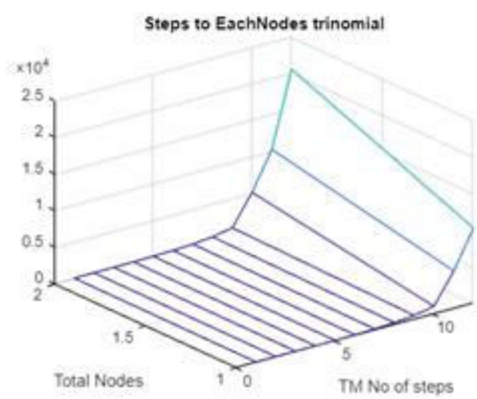

Figure 5.2.4: Comparison of Total Nodes to No. of Steps Trinomial

In figure 5.2.1 we can see that Binomial graph is linear for No. of Steps to Nodes at each step, but Trinomial graph looks exponential. In figure 5.2.2 we can see that in total nodes in binomial methods increase slowly but for trinomial methods the graph moves very fast. It means that trinomial methods converge faster. In graph 5.2.3 & 5.2.4 it is quite obvious that trinomial converges faster than Binomial method.

Table 5.2.4: Comparison of Binomial and Trinomial with respect to different Asset Price.

Values of American call at

$K = 100, S = 110,115,120,125,130, r = 0.1\ \sigma = 0.25, T = 0.5$

Asset Price	Method	Number of Steps								Actual Price Using Black-Scholes Method
		8	16	32	128	512	1024	5120	10240	

110	Binomial	17.1114	17.1107	16.7845	16.9754	16.9613	16.9623	16.9630	16.9629	16.9629
	Trinomial	16.8314	16.8399	16.8096	16.9567	16.9625	16.9627	16.9629	16.9629	16.9629
115	Binomial	21.2580	21.2068	21.2109	21.2139	21.2114	21.2110	21.2108	21.2104	21.2104
	Trinomial	21.2424	21.2348	21.2208	21.2109	21.2107	21.2107	21.2104	21.2104	21.2104
120	Binomial	25.8341	25.8164	25.8103	25.7092	25.7157	25.7102	25.7090	25.7090	25.7090
	Trinomial	25.6574	25.6777	25.6934	25.7028	25.7098	25.7094	25.7091	25.7090	25.7090
125	Binomial	31.3390	31.3375	31.3490	31.3678	31.3699	31.3767	31.3850	31.3849	30.3849
	Trinomial	31.3376	31.3361	31.3674	31.3697	31.3758	31.3750	31.3849	31.3849	30.3849
130	Binomial	34.7865	34.8223	34.9807	35.0007	35.1656	35.1798	35.1809	35.1810	35.1810
	Trinomial	34.6544	34.8873	34.9007	35.0012	35.1787	35.1806	35.1810	35.1810	35.1810

In table 5.2.4 we can see that the Trinomial Method values converge to Actual Price of Black-Scholes model for every Asset price level as compared to Binomial method values. Both models produce best values from step 512 onwards. In most of the cases at step 5120 & 10240 we obtain the actual value of option for both models.

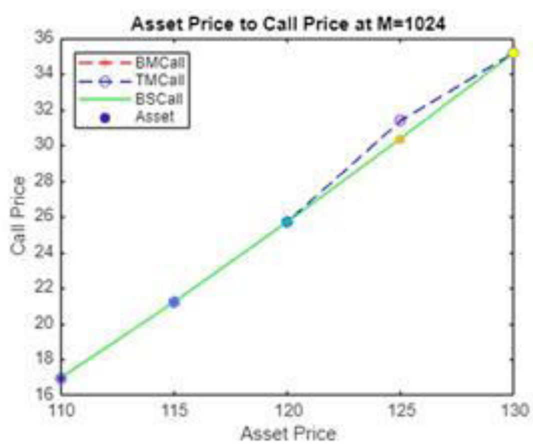

Figure 5.2.5: Asset Price to Call Prices at Step 1024

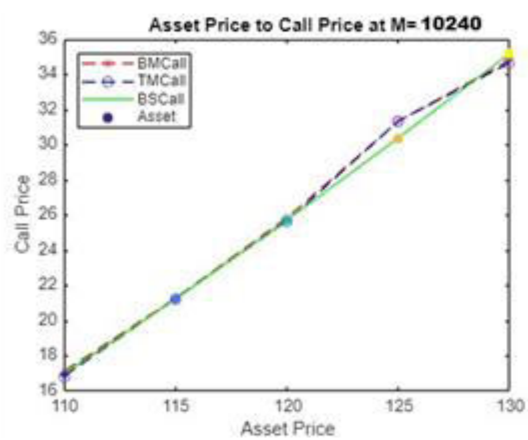

Figure 5.2.6: Asset Price to Call Prices at Step 10240

5.3 Error Calculation between American call option using BS Aodel and Binomial, Trinomial Models.

To measure the error between American call option prices obtained from the Black-Scholes method and the binomial model, you can use various statistical metrics such as, Mean Absolute Error (MAE): Calculating the absolute difference between the option prices obtained from the two models for each observation, then taking the average of these differences. By employing these metrics, you can assess the disparity between the option prices generated by the Black-Scholes method and the trinomial model, enabling a comparison of their performance and accuracy in pricing American call options.

Table 5.3.1: Error Comparison of Binomial and Trinomial Models

No. of Steps	Model	Option Price	Error
8	Binomial	25.8341	0.1251
	Trinomial	25.6574	0.0516
32	Binomial	25.8103	0.1013
	Trinomial	25.6934	0.0156
128	Binomial	25.7092	0.0002
	Trinomial	25.7028	0.0062
512	Binomial	25.7157	0.0067
	Trinomial	25.7098	0.0008
1024	Binomial	25.7102	0.0012
	Trinomial	25.7094	0.0004
5120	Binomial	25.7090	0
	Trinomial	25.7091	1E-04
10240	Binomial	25.7090	0
	Trinomial	25.7090	0

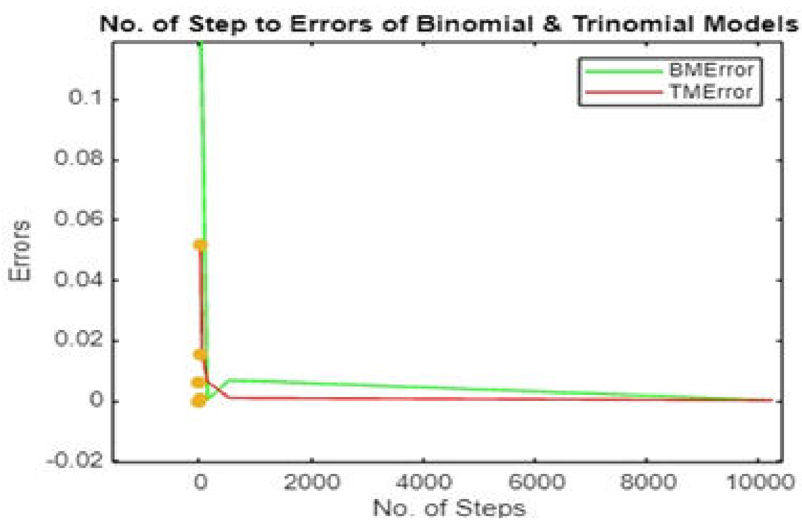

Figure 5.3.1: Error Analysis of Binomial & Trinomial Models

Table 5.3.2: Comparison of Computational time for Binomial & Trinomial Models

No. of steps	Computer Time in seconds	
	Binomial	Trinomial
512	1	1
1024	1	3
5120	40	72
10240	178	230

In table 5.3 we compare the performance of the two models based on the error with respect to Actual option prices obtained from Black-Scholes model using MATLAB. The price of option for two lattice models with respect to the cumulative number of steps. For each step the error is calculated as the absolute difference between actual price and price of two models. From the table above we can check that the error from step 1024 onwards become very less as the model prices are converging to the actual price. Therefore 10000 steps are enough to get the best answer for option price. We also observe that Binomial model have greater errors than Trinomial models. the size of the errors formed by the two different models is compared in order to rank the efficiency of the lattice models.

We can do the equivalent investigation by calculating the option values obtained roughly at each number of steps and illustrating in a figure. Reordering the two lines together it is clear that the trinomial model converges much more rapidly than the binomial model.

6. CONCLUSION

In this study, we conducted a comparative analysis of binomial and trinomial models for pricing American options. The aim was to assess the effectiveness of each model in capturing the complex dynamics of American options and to determine which model provides more accurate pricing and derivative estimates. First, we reviewed the theoretical foundations of both the binomial and trinomial models, highlighting their assumptions and key features. The binomial model assumes only two possible price movements at each time step, while the trinomial model allows for three possible movements, providing a more flexible framework to capture the underlying asset's price dynamics. Next, we implemented both models in MATLAB and conducted numerical experiments to price American options under various scenarios. We considered factors such as the number of steps in the lattice structure, number of nodes, volatility, interest rates, and time to expiration. Through these experiments, we observed how the option prices varied between the two models. Our findings suggest that while both models can be effective in pricing American options, the trinomial model generally provides more accurate results, especially for assets with complex price dynamics. The trinomial model's ability to accommodate an additional price movement at each time step allows for a more refined approximation of the underlying asset's behavior compared to the binomial model. However, it is essential to note that the trinomial model comes with increased computational complexity compared to the binomial model. As such, the choice between the two models depends on various factors, including the desired level of accuracy, computational resources, and the complexity of the underlying asset's price dynamics.

REFERENCES

1. Black, F. and Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), pp.637-654.
2. Merton, R.C., 1973. Theory of rational option pricing. *The Bell Journal of economics and management science*, pp.141-183.
3. Cox John, C., Ross Stephen, A. and Ross Rubinstein, M., 1979. Option pricing: A simplified approach. *Journal of financial Economics*, *7*(3), pp.229-263.
4. Xiaoping, H. and Jie, C., 2014. Randomized binomial tree and pricing of American-style options. *Mathematical Problems in Engineering*, *2014*.
5. Langat, K.K., Mwaniki, J.I. and Kiprop, G.K., 2019. Pricing options using trinomial lattice method. *Journal of Finance and Economics*, *7*(3), pp.81-87.
6. Horasanlı, M., 2007. A comparison of lattice based option pricing models on the rate of convergence. *Applied mathematics and computation*, *184*(2), pp.649-658.
7. Dai, T.S. and Lyuu, Y.D., 2010. The bino-trinomial tree: A simple model for efficient and accurate option pricing. *The Journal of Derivatives*, *17*(4), pp.7-24.
8. Ratibenyakool, Y. and Neammanee, K., 2020. Rate of convergence of binomial formula for option pricing. *Communications in Statistics-Theory and Methods*, *49*(14), pp.3537-3556.
9. Wilmott, P., 2013. *Paul Wilmott on quantitative finance*. John Wiley & Sons.
10. Josheski, D. and Apostolov, M., 2020. A review of the binomial and trinomial models for option pricing and their convergence to the Black-Scholes model determined option prices. *Econometrics Ekonometria Advances in Applied Data Analysis*, *24*(2), pp.53-85.

11. Boyarchenko, S. and Levendorskii, S., 2007. Options in the binomial and trinomial models. *Irreversible Decisions under Uncertainty: Optimal Stopping Made Easy*, pp.59-78.

12. Tian, Y.S., 1999. A flexible binomial option pricing model. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *19*(7), pp.817-843.

13. Shreve, S.E., 2004. *Stochastic calculus for finance II: Continuous-time models* (Vol. 11). New York: springer.

14. Wilmott, P., Howison, S. and Dewynne, J., 1995. *The mathematics of financial derivatives: a student introduction*. Cambridge university press.

15. Tretyakov, M.,2009, *Lecture notes on Financial Mathematics*, University of Leicester, UK