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# CONTROLLABILITY AND STABILITY CHALLENGES IN NONLINEAR DISCRETE DYNAMICAL SYSTEMS: A REVIEW

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## ABSTRACT

*Controllability is a fundamental concept in modern mathematical control theory. It generally refers to the ability to direct a dynamical system from an initial state to a desired final state using a set of permissible inputs. The precise definition of controllability varies in the literature, depending significantly on the specific type of dynamical system being studied.*

*Keywords: Stability, Nonlinear analysis.*

## 1 INTRODUCTION

Core concepts such as controllability and observability, along with related notions like stabilizability and detectability, are essential for solving many optimal control problems. This research addresses the issue of restricted controllability for 1-D and 2-D nonlinear finite-dimensional discrete-time control systems within confined domains.

While the controllability of linear dynamical systems in both continuous and discrete-time frameworks has been extensively studied (see, for example, Klamka, 1991b; 1993; 1995), there is comparatively less literature on the controllability of nonlinear systems, especially those involving restricted controllers. Few articles have focused on the controllability of nonlinear or linear dynamic systems, whether continuous or discrete.

For 1-D and 2-D nonlinear finite-dimensional discrete systems with constant parameters (Graves, 1950; Robinson, 1986), local controllability issues have been recognized. Establishing and testing appropriate criteria for restricted controllability in confined domains can be achieved using nonlinear functional analysis mapping theorems or linear estimation methods within the context of nonlinear functional analysis.

### 1.1 Dynamic Systems, Discreteness, and Nonlinearity

A fundamental characteristic of dynamical systems is the notion of time. However, it's important to distinguish this concept of "time" from the conventional notion of physical time governed by contemporary scientific theories and experiments. Instead, we adopt Newton's classical view of time as a generic mathematical parameter. In this context, time serves to track transitions as the system evolves between different states. The essence of a dynamical system lies in its dependence on time.

In the following sections, we will elaborate on the terms 'state' and 'state-space,' which form the basis for comprehending dynamic systems.

**Definition 1**. *When a dynamical system's state-space is a set of time-dependent variables that take values in a vector space V, it is enough to know the values of the variables at any given time $t_0$. The values of these variables determine the current state of the dynamical system.*

The “state-space” framework for describing dynamical systems has its origins in classical mechanics philosophy. Its archetypal expression may be found in Pierre Simon de Laplace's statement:

“There would be no uncertainty for it if it had an intelligence that could comprehend all of nature's forces, as well as the conditions of the creatures that make it up.

## 2 REVIEW OF LITERATURE

**Klamka et al. [1]** investigated discrete nonlinear finite-dimensional 1D and 2D control systems with constant coefficients to address questions regarding local restricted controllability. They utilized mapping theorems from nonlinear functional analysis and linear approximation techniques to formulate and verify the necessary conditions for restricted controllability. Consequently, the controllability criteria for unconstrained discrete systems with restricted controls were extended to encompass both 1-D and 2-D discrete systems with restricted controls.

**Arash Hassibi et al. [2]** investigate dynamical systems where events occur asynchronously, driving the system. Despite an infinite time period \( T \), the event rates are considered to be limited. These systems are increasingly relevant in control applications due to advancements in digital and communication technologies. They encompass asynchronous control systems, distributed control systems, and parallelized numerical methods. The researchers from the University of California, Los Angeles, have developed an advanced Lyapunov-based theory for controlling such dynamical systems, tackling problems through bilinear matrix inequality (BMI) or linear matrix inequality (LMI) formulations. The effectiveness of their method is demonstrated through various examples.

**Jerzy Klamka et al. [3]** examine control systems with constant coefficients, focusing on linear, continuous-time, finite-dimensional models. The paper is structured into three parts. Firstly, it covers basic stability definitions and essential/sufficient conditions. Secondly, it delves into controllability, providing necessary and sufficient criteria using a controllability matrix and discussing controllability with restricted controls. The third part centers on observability, defining required and sufficient observability criteria using an observability matrix. The paper concludes with observations on specific instances of linear control system stability, controllability, and observability. It's noted that all results are presented without formal proofs but with appropriate references to existing literature.

**Eurika Kaiser et al. [4]** propose the utilization of Koopman and Perron-Frobenius transport operators to provide linear representations for highly nonlinear dynamics, although working with them numerically poses challenges due to their inherent variability. Dynamic mode decomposition (DMD) has emerged as a popular numerical technique for estimating the Koopman operator, facilitated by increasing data volumes and DMD's compatibility with linear algebra. The chapter provides an overview of advancements in data-driven characterization of transport operators, along with recent developments in big data and machine learning, including progress in sparsity and control.

**James Kapinski et al. [5]** highlight the utilization of Lyapunov functions for assessing the constancy of nonlinear and hybrid dynamical systems and deriving performance constraints on system behaviors. They propose an investigation-based approach for finding Lyapunov functions, leveraging linear programming (LP) optimization problems based on actual executions, with subsequent refinement of candidate Lyapunov functions using a global optimizer driven by Lie derivatives. An SMT solver refines the analysis using counterexamples, and an arithmetic solver verifies the soundness of the analysis once there are no counterexamples. This method proves effective for investigating a wide range of nonlinear dynamical systems, including hybrid systems, polynomial dynamical systems, and transcendental dynamics, as demonstrated in two vehicle powertrain control scenarios.

**Samia Charfeddine et al. [6]** analyze the control of nonlinear disturbed polynomial systems using output feedback linearization and sliding mode control architecture. Their goal is to achieve asymptotic stability of a volatile equilibrium point. They utilize sliding mode and meta-heuristic approaches to develop a robust control system, mitigating challenges posed by model errors and process dynamic disturbances. Numerical simulation analysis, conducted using a continuous stirred tank reactor (CSTR) as a benchmark, demonstrates the efficacy and efficiency of the proposed approach in minimizing system output variations due to the asymptotically stable dynamic behavior of the control architecture.

**X. Koutsoukos et al. [7]** present a hierarchical control structure for piecewise linear hybrid systems, bridging continuous and discrete components using piecewise linear maps and linear difference equations, respectively.

Techniques for synthesizing dynamical controllers and formulating control design problems as regulators are discussed, with finite automata used to model control requirements, encompassing both static and dynamic aspects. Illustrations of the technique's efficacy are provided through simulations of a tank system.

**Thiagop. Chagas et al. [8]** explore how chaotic sets in nonlinear discrete-time dynamical systems stabilize periodic orbits, leading to the development of a novel control rule approximating the problem of stabilizing linear time-periodic systems using modern control theory. Numerical simulations demonstrate its theoretical and practical efficacy, comparing favorably to Delayed Feedback Control.

**Ya Tian et al. [9]** investigate the global asymptotic behavior of a class of continuous-time dynamical systems, enabling analysis of chaotic control and synchronization by determining not only final bounds on system solutions but also the rate of trajectories transitioning between trap sets.

**Vipin Kumar et al. [10]** address the continuation and uniqueness of solutions to a nonlinear fractional discrepancy equation with nonlinear integral boundary conditions on time scales. Their work employs fixed-point theories such as Banach, Schaefer, Leray Schauder, and Krasnoselskii, along with examining Ulam-type Hyers' stability. Two examples are provided to illustrate the utility of their findings.

**Guilherme Franca et al. [11]** introduce the gradient descent acceleration approach, popularized by Nesterov, which is widely employed in machine learning. Recent advancements close the gap in understanding this approach by adopting a continuous-time dynamical systems perspective combined with gradient techniques for smooth and unconstrained problems. The alternating-direction method of multipliers is extended to nonsmooth and linearly constrained scenarios by developing nonsmooth dynamical systems linked to variants of the relaxed and accelerated alternating-direction method (ADMM). Two distinct versions of the ADMM method are presented, one based on Nesterov's acceleration and the other on Polyak's heavy ball approach. A nonsmooth Lyapunov analysis yields convergence rate findings in convex and strongly convex scenarios for these dynamical systems, revealing an intriguing trade-off between Nesterov and heavy ball acceleration techniques.

**Parikha Mehrotra et al. [12]** address the stability problem for a specific class of linear continuous-time switched systems with continuous and discrete descriptions. A common quadratic and nonquadratic Lyapunov function is constructed to tackle the arbitrary switching issue in such cases. The study also considers the complexities introduced by switched systems with delays, arising from internal or external causes. The review provides insights into the state-of-the-art methodologies for stabilizing such systems, considering individual systems with stable or unstable dynamics. Additionally, it outlines current advancements, future directions, and open challenges in this domain.

**Alberto Carrassi et al. [13]** discuss the expansion of dynamical systems from meteorology and oceanography to other geosciences and continuum physics fields. These systems present challenges due to their nonlinear stability over time, requiring convergence with the system's chaotic evolution. The authors illustrate essential concepts using a linearized Lorenz system and analyze the stabilities of two nonlinear prediction-assimilation systems using dynamic meteorology's Lyapunov exponents. Data-induced stabilization is crucial for such systems, where the observational network's permanence or adaptability, as well as incorporation techniques, play significant roles.

**Alexander P. Buslaev et al. [14]** propose the study of discrete models of movement on contour networks, akin to CA184 in Wolfram cellular automata categorization (WCA). These models, which have been relatively understudied thus far, are crucial for scientific and research applications related to flow theory, such as traffic dynamics, resource growth, and metabolic processes in the body. The study focuses on the simplest contour network, where two contours intersect at a common node. Particles traverse their respective contours according to Wolfram cellular automata rules 184 or 240, with each contour featuring a shared cell or alternating node. Techniques for studying two-contour systems are developed, laying the groundwork for analyzing contour networks with more complex architectures. The study establishes conditions for self-organization in the system, governed by inequalities based on the cellular automata rules. The existence or absence of self-organization depends on the system's initial conditions and the distinct laws governing particle motion. The paper concludes by discussing avenues for further research and potential applications.

**Venkatesan Govindaraj et al. [15]** address the control of nonlinear fractional dynamical systems with both Lipchitzian and non-Lipchitzian nonlinearities. They employ nonlinear analysis techniques such as fixed-point theorems and monotone methods to assess the controllability of these systems. The study presents examples to illustrate the findings, unveiling a new type of controllability result for fractional dynamical systems.

**Kumpati S. Narendra et al. [16]** demonstrate the use of neural networks for identifying and controlling nonlinear dynamical systems. The study focuses on identification models and employs both static and dynamic back-propagation strategies for parameter adjustment. Multilayer and recurrent networks are investigated within the introduced models. Effective simultaneous identification and adaptive control techniques are observed through simulation. The article provides fundamental ideas, definitions, and theoretical challenges that need to be addressed.

**R. Bouyekhfa et al. [17]** address the asymptotic stability analysis of equilibrium states in nonlinear discrete-time dynamical systems that are time-invariant and nonlinear. They propose a solution based on the G-Function concept, defining new asymptotic stability criteria and identifying the asymptotic stability domain for \( x = 0 \). The results are illustrated through examples.

**Hong Zhang et al. [18]** investigate the dynamics of hybrid dynamical systems subjected to time-dependent perturbations within short time periods. They determine the finite-time contractive stability of the null solution using monotonicity arguments and validate the theoretical analysis through numerical simulations.

**Zhan-Dong Mei et al. [19]** examine dynamic boundary systems with feedback on their boundaries, demonstrating the well-posedness of such systems under certain regularity requirements. They uncover derivatives of spectral characteristics and apply the solution to analyze the well-posedness and asymptotic behavior of population dynamics with an infinite birth procedure.

**Yonglu Shu et al. [20]** focus on chaotic systems and their applications in engineering, particularly chaotic control and synchronization. They propose a hyperchaotic system with global exponentially attractive and positively invariant sets, allowing estimation of trajectory speeds. Theoretical analysis and computer simulations demonstrate the effectiveness of the proposed method for chaos management and synchronization.

### 2.1 Why to Study Discrete Dynamical System?

There are instances when the independent variable is discrete or it is mathematically advantageous to describe it as such. Discrete-time systems have several uses. A discrete variable represents generation in genetics.

Whether you are studying economics, keep an eye on the price variations from one year to the next, or even one every scenario has a discrete time frame. The variable that indicates age group is a discrete variable if you are interested in Population Dynamics. Discrete-time systems come in two flavours.

Several systems, also including digital computers, digitally analyzers, and financial and stock systems, operate on a discrete time basis. As a result of this, only distinct periods in time are important, and everything else is Examples of discrete-time systems are savings account transactions and the repayment process for bank loans.

#### 2.1.1. Saving Bank Account

In this case, let x(n) be a savings bank balance, and r be the monthly interest rate. Assume that x(n) is equal to the total amount of deposits and withdrawals for the n The equation x(n) fulfils a linear difference equation where the interest is computed monthly based on the beginning balance, for example: n = 0, 1, 2,...

$$x(n+1) = (1+r)x(n) + u(n), x(0) = x_0 \qquad (2.1.1)$$

In this case, $x_0$ is the starting balance. A linear discrete-time system with linear temporal invariance is described by this equation

### 2.1.2. Amortization

Paying back a debt by making periodic payments, each of which includes interest and principal reduction, is known as amortisation

P(n) is the unpaid principal after $n^{th}$ payments g(ri). Suppose, for the sake of argument, that interest costs are compounded at a rate of r per payment

In this instance, the outstanding principal p(n + 1) after the (n + 1)st payment equals the outstanding principal (n) after the nth payment, plus the interest rate rp(n) collected throughout the (n + 1)st period, less the final payment gn).

Hence

$$p(n+1) = (1+r)p(n) - g(n), p(0) = Po \qquad (2.1.2)$$

### 2.2 State Space Description of Discrete-Time System

Simultaneous difference equations (SDE) are useful for describing many systems

$$x(t+1) = f\big(t, x(t)\big), t \in N_0 \triangleq \{0,1,2, \dots.\} \qquad (2.2.1)$$

Where: $N_0 \times \Omega \to R^n, \Omega \subset R^n$ , At the equilibrium point x*, the function is continuously differentiable

As a result, it has a specific

$$x(t+1) = f\big(x(t)\big), t \in N_0 \qquad (2.2.1)$$

2.2.1 is a nonlinear autonomous difference equations system, whereas 2.2.2 is a nonlinear non-autonomous dissimilarity equations system. 2.2.2 is a nonlinear autonomous There are several methods for linearizing nonlinear systems, and we will explore them in (see Elaydi [8], page no. 197).

**Remark 2.2.1.**

*If f (t,x*) = x* for every t t0, then x* is termed an equilibrium point (2.2.1) in $R^n$. Assumed to represent the origin 0 in most literature, x* is called the zero solution.*

### 2.3 Linearization of Nonlinear Systems

Consider system (2.3.1). Let us write $= (f1, f2, \dots., fn)^T$ . The Jacobian matrix of $f$ is defined as

$$\frac{\partial f(t,x)}{\partial x}|x = 0 = \frac{\partial f(t,0)}{\partial x}\begin{pmatrix} \frac{\partial f_1(t,0)}{\partial x_1} & \frac{\partial f_1(t,0)}{\partial x_2} & \cdots & \frac{\partial f_1(t,0)}{\partial x_n} \\ \frac{\partial f_2(t,0)}{\partial x_1} & \frac{\partial f_2(t,0)}{\partial x_2} & \cdots & \frac{\partial f_2(t,0)}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n(t,0)}{\partial x_1} & \frac{\partial f_n(t,0)}{\partial x_2} & \cdots & \frac{\partial f_n(t,0)}{\partial x_n} \end{pmatrix}$$

For simplicity $\frac{\partial f(t,0)}{\partial x}$ is denoted by $f(t,0)$ Let

$$\frac{\partial f(t,0)}{\partial x} = A(t)$$

And

$$g(t,x) = f(t,x) - A(t)x(t)$$

A variation of this might be worded as follows:

$$x(t+1) = A(t)x(t) + g\big(t, x(t)\big) \qquad (2.3.1)$$

having its linear component

$x(t+1) = A(t)x(t)$ (2.3.2)

Where $(A(t))t \in N_0$ is a sequences of real $n \times n$ matrices, $(x(t))t \in N_0$ is a sequences of state vectors in $R^n, g(t, x(t)): N_0 \times \Omega \to R^n, \Omega \subset R^n$ takes the form of a nonlinear function that reflects the perturbation resulting from noise, measurement error, or other external Here it is assumed that $g(t, x(t)) = 0(x) as \parallel x \parallel \to 0.$

i.e. if given $\in > 0$, there is $\delta > 0$such that

$\parallel g(t,x) \parallel \leq \epsilon \parallel x \parallel$ when ever $\parallel x \parallel < \delta \ and t \in N_0$

According to equation (2.3.1), the autonomous system may be expressed as follows:

$x(t+1) = Ax(t) + g(x(t))$ (2.3.3)

Having its linear component

$x(t+1) = Ax(t)$ (2.3.4)

where $A = f'(0)$ is the Jacobian matrix of $f$ at 0 and $g(x) = f(x) - Ax$.. Since $f$ is differentiable at 0, we can write $g(x) = 0(x) as \parallel x \parallel \to 0.$ . That is,

$$\lim_{\|x\| \to 0} \frac{\parallel g(x) \parallel}{\parallel x \parallel} = 0$$

## 2.4 Existence and Uniqueness of Solutions

### 2.4.1 Solutions of Linear Systems

Due to $x(t_0) = x_0$ linear system (2.4.2) becomes an initial value issue, and the following theorem guarantees that it will have a unique

**Theorem 2.4.1.**

This is based on Elaydi [8]). A unique solution exists for each$x_0 \in R^n$ and $t_0 \in N_0$ for which (2.4.2) is true. The answer may be written as: $x(t_0) = x_0$

$$x(t) = \left[\prod_{i=t_0}^{t-1} A(i)\right] x_0 \qquad (2.4.1)$$

Where,

$$\prod_{i=t_0}^{t-1} A(i) = A(t-1)A(t-2) \ldots A(t_0), if\ t > t_0$$

$$= I \qquad , if\ t = t_0$$

This is also true for $x(t_0) = x_0$ in the linear autonomous system (2.4.4)

$x(t) = A^{t-t_0} x_0$ (2.4.2)

When it comes to the theory of linear systems, understanding the concept of a basic matrix important

### 2.4.2 Fundamental Matrices

A matrix with columns that represent solutions to an equation is called Let $\Phi(t)$ (2.4.2). to put it differently...

$x_1(t), x_2(t), \ldots \ldots, x_n(t) t$

are solutions of (2.4.2), we write

$\Phi(t) = [x_1(t)\ x_2(t) \ldots \ldots . . x_n(t)]$

### 2.5 DISCRETE VOLTERRA SYSTEMS

In the majority of cases, discrete-time Volterra equations result from the simulation of a true phenomena or from the application of numerical techniques to a Volterra Formula for calculating the Volterra's difference

$$x(t+1) = A(t)x(t) + \sum_{r=0}^{t} B(t,r)x(r), x(0) = x_0 \qquad (2.5.1)$$

The Volterra Integrodifferential Equation can be seen as a discrete analogue

$$x(t) = A(t)x(t) + \int_0^t B(t,r)x(r)dr \qquad (2.5.2)$$

Regard as the disconcerted equation of (2.5.1)

$$x(t+1) = A(t)x(t) + \sum_{r=0}^{t} B(t,r)x(r) + f(t), x(0) = x_0 \qquad (2.5.3)$$

The functions $x(t) \in R^n, A(t), B(t,r)$ and $N_0 \times N_0$ B(t,r) are $N_0$ matrix functions. T's vector function (f(t)) on N0 is an n- Equation 2.4.1: Define R(t, m) matrix as the unique solution of matrices equation.

$$R(t+1,m) = A(t)R(t,m) + \sum_{r=0}^{t} B(t,r)R(t,m) t \geq m \qquad (2.5.4)$$

with R(m, m) = 1 for 0 < m < t.

Elaydi [7] shown that equation (2.5.3) has a unique solution x(t) that may be stated.

$$x(t) = x(t,0,x_0) = R(t,0)x_0 + \sum_{r=0}^{t-1} R(t,r+1)f(r) \qquad (2.5.5)$$

### 2.6 Stability Analysis

Typically, we are interested in methods and criteria that characterise the structure and behaviour of differential system solutions without actually creating or estimating solutions. Due to the fact that many difficulties that arise in practise are nonlinear and intractable, this research is essential. There is also the possibility of using a given, the study of ultimate behaviour of its solutions is one of the pioneer issues (i.e. asymptotic behaviour of discrete systems).

In the first place, let us just review some ideas related to the stability of (see Agarawal [1], Blaydi [8]).

An example is a system of differential equations

$$x(t+1) = f(t, x(t)), x(t_0) = x_0, \qquad t \in N_0 \triangleq \{0,1,2,\ldots\ldots\} \qquad (2.6.1)$$

#### 2.6.1 Stability of Linear Systems

Regard as the linear non-antonomous (time-variant) system known by

$$x(t+1) = A(t)x(t) \qquad (2.6.2)$$

and linear autonomous system (time-invariant) known by

$$x(t+1) = Ax(t) \qquad (2.6.3)$$

These well-known conclusions (Agarwal [1], Elaydi [8]) on the stability of linear systems offer the essential and enough criteria in words of the basic matrices of the systems in question.

### 2.6.2 (SP) Matrices

The (sp) matrix was recently proposed by Xue and Guo and its utility in the investigation of asymptotic stability of the null solution of linear systems has been demonstrated

x(t + 1) = Ax(t)

**Definition 2.6.2. ((sp) Matrix)** We call

$$A \in s = \{A = (a_{ij})_{n\times n} : a_{ij} \geq 0, \sum_{j=1}^{n} a_{ij} \leq 1, \forall i = 1,2,\dots,n\}$$

a (sp) matrix if there exists$m \in N$ ={1,2,...} and a sequence of subscript sets $\{I_1^{(k)}\}, \{I_2^{(k)}\}, k = 0,1,\dots, m$ from I = {1,2, ...,n} such that

$I = I_1^{(0)} \cup I_2^{(0)}, I_1^{(0)} = \left\{i: \sum_{j=1}^{n} a_{ij} < 1\right\}, I_2^{(0)} = \left\{i: \sum_{j=1}^{n} a_{ij} = 1\right\},$

$I_1^{(k)} = \left\{i \in I_2^{(k-1)} : \exists j \in I_1^{(k-1)} such\ as\ a_{ij} \neq 0\right\}, I_2^{(k)} = \left\{i \in I_2^{(k-1)} : \forall j \in I_1^{(k-1)} such\ as\ a_{ij} = 0\right\}$, k= 1,2,…..m-1,

$I_1^{(m)} = I_2^{(m-1)}, I_2^{(m)} = \emptyset,$

where $I_1^{(k)}$ and $I_2^{(k)}$ , k — 0,1,2, ...,m — 1 are nonempty or $I_2^{(0)} = \emptyset$

### 2.6.3 Generalized Sub-Radius

It was Czornik [5] who proposed the notions of generalised spectral sub-radius and joint spectral sub-radius, and the relationship between generalized spectral radii and discontinuous time-varying stability A Σ non-empty collection of all real n x n matrixes is denoted by the notation "

For$m \geq 1, \Sigma^m$, contains all the matrices that have m- Σ dimensional products in.

$\Sigma^m = \{A_1, A_2 \dots \dots A_m : A_i \in \Sigma, i = 1,2, \dots, m\}$

Denote by $\rho(A)$ the spectral radius and by $\|A\|$a matrix norm of the matrix A. Let $A \in \Sigma^m$. .

### 2.6.4 Dichotomy

A linear difference equation's dichotomous behaviour can be used to examine an asymptotic link between the solutions of a linear difference equation and a nonlinearly disturbed equation. Difference equations are described as asymptotically comparable if each solution of one system corresponds to a solution of the other.

Take a look at the nonlinear perturbe

$$y(t+1) = A(t)y(t) + g(t, y(t)), t \in N_0 \qquad (2.6.6)$$

as well as the corresponding collected system

$$x(t+1) = A(t)x(t) \qquad (2.6.7)$$

### 2.7 Controllability Analysis

In mathematical control theory, controllability is a key concept to understand Dynamic control methods produce these qualitative features. For time-invariant and time-varying linear control systems, Kalman developed a theory of controllability in the early 1960s ([19]).

According to the definition of a controllable system, it is one that can be directed from any starting point to any ending point by employing a set of permitted controls According to the kind of dynamical control system, the concept of controllability in the literature differs widely. Assume you have a collection of linear difference equations of the following types

$$x(t+1) = Ax(t) + Bu(t), x(0) = x_0, t \in N_0 \quad (2.7.1)$$

For example, in $(x(t))t \in N_0$ and $(u(t))t \in N_0$ the state vector sequence in $R^n$ and the control vector sequence in $R^m$ are respectively represented by A and B. Here are several definitions of controllability first.

**2.7.1 Various Notions and Basic Results of Controllability**

**Definition 2.7.1. (Complete Controllability)**

For any N 0, and any $\in N_0$, starting state, $x(t_0) = x_0$,, if the system (2.7.1) is fully or simply controllable, then there exists a finite time and control (u(t)) such that the end state (xf) is identical to the starting state (x0), then the system (2.7.1) is completely or simply controllable (N).

**Definition 2.7.2. (Controllability to Origin)**

It is possible to direct a system $t_0 \in N_0$ and $x_0 \in R^n$, (2.7.1) back to the origin if there is a limited time $N > t_0$ and a manage u(t), $t_0 < t < N$ such that x(N) = 0..

**Definition 2.7.3. (Local Controllability)**

According to the definition of local controllability, there must exist a neighbourhood of the origin such that for each pair of inputs $x_0, x_1 \in \Omega$ there exists an input series $u = (u(0), u(1), \ldots.., u(N-1))$, which steers the system from point A to point B $x_0$ to $x_1$.

**3. CONCLUSION**

Various discrete time linear and nonlinear systems' controllability and stability were investigated. By way of illustration, fixed point theorems, inverse functions, and implicit functions may all be used to determine controllability. Aside from the controllability results, we also attempted to develop a computational approach for computing actual steering controls.

- Semi-linear discrete-time system steering control

$$x(t+1) = A(t)x(t) + B(t)u(t) + f\big(t, x(t)\big), t \in N_0 \triangleq \{0,1,2,\ldots\}$$

According to Banach's fixed point theorem, it is well-defined if its linear counterpart can be controlled and its nonlinear function is Lipschitz. A similar analysis shows that controllability and reachability are equal for a system under the same conditions (1). The semi-linear steering control algorithm (1) is provided.

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