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CHARACTERIZATION OF NANO K-CLOSED AND NANO K –OPEN MAPS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce the concept of a new class of closed and open maps namely Nano K – closed maps and Nano K –open maps in Nano topological spaces (briefly NK – Closed map & NK-open map). Also we analyze some of its properties and characterizations and the relations between the Nano K – closed maps and Nano K-open maps with the other existing nano generalized closed and open maps.

Keywords: NK-closed set, NK-open set, NK-Continuous, NK-closed map, NK-open map, NK – irresolute, Strongly NK – continuous.

1. INTRODUCTION

In [7],the specialists introduced a nano topological space with regard to a subset X of an universe which is characterized in terms of lower approximation, upper approximation and boundary region. He has also established nano closed sets and the weak forms of nano open sets namely, nano α open sets, nano semi open sets and nano pre open sets in a nano topological space. Since the advent of these notions several research papers with interesting results in different respects came to existence.

The purpose of the present paper is to introduce and investigate some of the fundamental properties of nano K – closed and Nano K - open maps.

DEFINITION 1.1[1]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as in discernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup\{R(X): R(X) \subseteq X, x \in U\}$ where R(X) denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X) = \cup\{R(X): R(X) \cap X \neq \Phi, x \in U\}$.
3. The boundary region of X with respect to R is the set of all objects which can be classified neither X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

PROPERTY 1.2[1]

If (U, R) is an approximation space and $X, Y \subseteq U$ then

1. $L_R(X) \subseteq X \subseteq U_R(X)$

2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(XUY) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
10. $U_R[U_R(X)] = L_R(U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

DEFINITION 1.3[1]

Let U be the universe, R be an equivalence relation on U and

$\tau_R(X) = \{\Phi, L_R(X), U_R(X), B_R(X), U\}$ where $X \subseteq U$. $U_R(X)$ satisfies the following axioms:

1. Φ and $U \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then, $\tau_R(X)$ is the topology on U called the nano topology on U with respect to X.

$(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. The complements of nano open sets are called nano closed sets.

Definition 1.4: Let $(U, \tau R(X))$ be a nano topological space and $\mathbf{H} \subseteq U$. Then **H** is said to be

(i) Nano semi-open [**9**] if $\mathbf{H} \subseteq Ncl(Nint(\mathbf{H}))$.

(ii) Nano pre-open [**4**] if $\mathbf{H} \subseteq Nint(Ncl(\mathbf{H}))$.

(iii) Nano semi pre-open [**4**] if $\mathbf{H} \subseteq Ncl(Nint(Ncl(\mathbf{H})))$.

(iv) Nano regular open [**10**] if $\mathbf{H} = Nint(Ncl(\mathbf{H}))$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition 1.5: Let $(U, \tau_R(X))$ be a nano topological space and $\mathbf{H} \subseteq U$. Then **H** is said to be

1. Ng – closed **[10]** if $NCl(\mathbf{H}) \subseteq \mathbf{V}$ whenever $\mathbf{H} \subseteq \mathbf{V}$ and **V** is nano open.

2. Ngsp- closed **[9]** if $Nspcl(\mathbf{H}) \subseteq \mathbf{V}$ whenever $\mathbf{H} \subseteq \mathbf{V}$ and **V** is nano open.

3. Nω- closed **[10]** if $Ncl(\mathbf{H}) \subseteq \mathbf{V}$ whenever $\mathbf{H} \subseteq \mathbf{V}$ and **V** is nano semi open.

4. N*g – closed **[10]** if $Ncl(\mathbf{H}) \subseteq \mathbf{V}$ whenever $\mathbf{H} \subseteq \mathbf{V}$ and **V** is nanoω open.

5. Ngs – closed **[10]** if $Nscl(\mathbf{H}) \subseteq \mathbf{V}$ whenever $\mathbf{H} \subseteq \mathbf{V}$ and **V** is nano open.

6. Nano regular closed **[9]** if **H** = Ncl(Nint(**H**)).

The complement of Ng –closed (resp. Ngsp-closed, N*g-closed, Ngs-closed, Nω- closed, Nr-closed) set is said to be Ng-open (resp.Ngsp-open, N*g-open, Ngs-open, Nω- open,Nr-open).

DEFINITION 1.6:

1. Let (U, τR(X)) and (**V**, τR′(Y)) be a nano topological spaces. Then the function f : (U,τR(X)) → (**V**, τR′(Y)) is said to be Nano continuous [**8**] on U if the inverse image of every nano open set in **V** is nano open in U.
2. Let (U, τR(X)) and (**V**, τR′(Y)) be a nano topological spaces. Then the function f : (U,τR(X)) → (**V**, τR′(Y)) is said to be Nano K- continuous on U if the inverse image of every nano open set in **V** is nano K-open in U.

2, NANO K – CLOSED MAPS IN NANO TOPOLOGICAL SPACES

In this section we define and study the new class of maps, namely Nano K- Closed Maps (briefly, NK – Closed Map) in Nano topological spaces.

Definition 2.1: Let (U,τR(X)) and (**V**, τR'(Y)) be two nano topological spaces. A map

f: (U,τR(X)) → (**V**, τR'(Y)) is said to be Nano K-closed map (briefly NK – closed map) if the image of every nano closed set in (U, τR(X)) is NK – closed in (**V**, τR'(Y)).

Example: 2.2: Let U = $\{s_1, s_2, s_3, s_4\}$ with U/R = $\{\{s_1\}, \{s_4\}, \{s_2,s_3\}\}$ and X = $\{s_1,s_3\} \subseteq$ U. Then $\tau_R(X) = \{ \Phi, \{s_1\}, \{s_2,s_3\}, \{s_1,s_2,s_3\}, U\}$ which are nano open sets. Nano closed sets are

$\tau_R{}^c(X) = \{ \Phi, \{s_4\}, \{s_1, s_4\}, \{s_2, s_3, s_4\}, U\}$. Let **V** = $\{k_1, k_2, k_3, k_4\}$ with **V**/R' = $\{\{k_1\}, \{k_3\}, \{k_2,k_4\}\}$ and Y = $\{k_1,k_2\} \subseteq$ **V**. Then $\tau_{R'}(Y) = \{\Phi, \{k_1\}, \{k_2,k_4\}, \{k_1,k_2,k_4\},$**V**$\}$ which are nano open sets. Nano closed sets are $\tau_{R'}{}^c(Y) = \{ \Phi, \{k_3\}, \{k_1,k_3\}, \{k_2,k_3,k_4\},$ **V**$\}$. Nano K – closed sets are $\{ \Phi, \{k_1\}, \{k_3\}, \{k_1,k_3\}, \{k_2,k_4\}, \{k_2,k_3,k_4\},$ **V**$\}$. Nano K –open sets are $\{ \Phi, \{k_1\}, \{k_1,k_3\}, \{k_2,k_4\}, \{k_1k_2,k_4\}, \{k_2,k_3,k_4\},$ **V**$\}$.

Define the map f: $(U,\tau_R(X)) \rightarrow$ (**V**, $\tau_{R'}(Y)$) as $f(s_1) = k_1$, $f(s_2) = k_2$, $f(s_3) = k_4$, $f(s_4) = k_3$.

$f(\{s_4\}) = \{k_3\}$, $f(s_1,s_4)\} = \{k_1,k_3\}$, $f(\{s_2,s_3,s_4\}) = \{k_2,k_3,k_4\}$ $f\{\Phi\} = \Phi$, f(U) = **V** are the images of nano closed sets of U which are NK – closed in **V**.

Thus the map f: (U,τR(X)) → (**V**, τR'(Y)) is a NK – closed map.

Theorem 2.3: Every nano closed map is Nano K – closed map.

Proof: Let f: (U,τR(X)) → (**V**, τR'(Y)) be a nano closed map. Let **H** be a nano closed set in U. Since f is nano closed map then f(**H**) is nano closed set in **V**. Since every nano closed set is nano K- closed, then f(**H**) is nano K – closed in **V**. Therefore f is nano K – closed map.

Remark 2.4: The converse of the above theorem need not be true as seen from the following example.

Example 2.5: As in the previous example 2.2, Since **H** = $\{ k_3\}$ is nano K -closed in **V** but $f(\{k_3\}) = \{s_1\}$ is not nano closed in U.

Theorem 2.6: A map f: (U,τR(X)) → (**V**, τR'(Y)) is NK – clsed map if and only if NKCl(f(**H**)) ⊆

f(NCl(**H**)) for every subset **H** of (U, τR(X)).

Proof: Let f: (U,τR(X)) → (**V**, τR'(Y)) be a NK – closed map and **H** ⊆ U. Then NCl(**H**) is nano closed in U. Then f(NCl(**H**)) is NK – closed in **V**. Since **H**⊆ NCl(**H**) ⇒ f(**H**) ⊆ f(NCl(**H**)). Since NKCl(f(NCl(**H**)) is the NK – closed set containing f(**H**).

Therefore NKCl(f(**H**)) ⊆ NKCl(f(NCl(**H**)) ⊆ f(NCl(**H**)).

Conversely, Let **H** be a nano closed set in U. Then **H** = NCl(**H**). f(**H**) = f(NCl(**H**)) ⊆NKCl(f(**H**)). Therefore f(**H**) ⊆ NKCl(f(**H**)). Therefore f(**H**) = NKCl(f(**H**)). Therefore f(**H**) is NK – closed set in **V**. Therefore f: (U,τR(X)) → (**V**, τR'(Y)) is a NK – closed map.

Theorem 2.7: If f: (U,τR(X)) → (**V**, τR'(Y)) be a nano closed map and g: (**V**, τR'(Y)) →

(**W**, τR''(Z)) be NK closed mapping. Then their composition g o f: (U,τR(X)) → (**W**, τR''(Z)) is a NK – closed mapping.

Proof: Let **H** be a nano closed set in (U, τR(X)). Then f(**H**) is nano closed in (**V**, τR'(Y)). Then g o f (**H**) = g(f(**H**)) is NK – closed. Since g: (**V**,τR'(Y)) → (**W**, τR''(Z)) is NK – closed. Hence their composition is NK – closed mapping**.**

3. NANO K – OPEN MAPS IN NANO TOPOLOGICAL SPACES

Definition 3.1:A map f: (U, τR(X)) → (**V**, τR'(Y)) is said to be NK –open if the image of every nano open set in (U, τR(X)) is a nano K –open set in (**V**, τR'(Y)).

Example 3.2: Let U = {s_1, s_2, s_3, s_4} with U/R = {{s_2}, {s_3}, {s_1,s_4}} and X = {s_1,s_2} ⊆ U.

Then $\tau_R(X)$ = { Φ, {s_2}, {s_1,s_4}, {s_1,s_2,s_4}, U} which are nano open sets. Nano closed sets

Are $\tau_R{}^c(X)$ = { Φ, {s_3 }, {s_2 ,s_3 }, {s_1 ,s_2 ,s_4 }, U}. Let **V** = {k_1 , k_2 , k_3 , k_4 } with **V** /R' = {{k_1}, {k_3}, {k_2,k_4}} and Y = {k_1,k_2} ⊆ **V**. Then $\tau_{R'}(Y)$ = {Φ, {k_1}, {k_2,k_4}, {k_1,k_2,k_4},**V**} which are nano open sets. Nano closed sets are $\tau_{R'}{}^c(Y)$ = { Φ, {k_3}, {k_1,k_3}, {k_2,k_3,k_4}, **V**}. Nano K – closed sets are { Φ, {k_1}, {k_3}, {k_1,k_3}, {k_2,k_4}, {k_2,k_3,k_4}, **V**}.

Nano K –open sets are { Φ, {k_1}, {k_1,k_3}, {k_2,k_4}, {k_1k_2,k_4}, {k_2,k_3,k_4}, **V**}. Define the map f: (U,τ_R(X)) → (**V**, $\tau_{R'}$(Y)) as f(s_1) = k_2, f(s_2) = k_1, f(s_3) = k_3, f(s_4) = k_4. f({s_2}) = {k_1}, f(s_1,s_4)} = {k_2,k_4}, f({s_1,s_2,s_4}) = {k_1,k_2,k_4} f{Φ} = Φ, f(U) = **V** are the images of nano open sets of U which are NK – open in **V**. Thus the map f: (U,τR(X)) → (**V**, τR'(Y)) is a NK – open map.

Theorem 3.3: Every nano open map is NK – open map.

Proof: f: (U, τR(X)) → (**V**, τR'(Y)) be a nano open map. Let **H** be a nano open set in

(U, τR(X)).. Then the image of **H** is nano open in (**V**, τR'(Y)). Since every nano open set is Nk – open set. Therefore f(**H**) is NK –open set in (**V**, τR'(Y)). Hence f is a NK – open map.

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: As in the previous example 3.2, Since **H** = {k_1, k_3} is nano K -open in **V** but f({k_1, k_3}) = {s_2, s_3} is not nano open in U.

Definition 3.6: Let (U, τR(X)) and (**V**, τR (Y)) be two nano topological spaces. Then the functionf : (U,τR(X)) → (**V**, τR (Y)) is said to be Strongly Nano K⌜- continuous on U if the inverse image of every nano K - open set in **V** is nano open in U.

Definition 3.7: Let (U, τR(X)) and (**V**, τR′(Y)) be a nano topological spaces. Then the functionf : (U,τR(X)) → (**V**, τR′(Y)) is said to be Nano K – irresolute if the inverse image of every nano K - open set in **V** is nano K -open in U.

Theorem 3.8: For any bijection f: (U, τR(X)) → (V, τR'(Y)), the following statements are equivalent.

(i) f^{-1}: (**V** , τR'(Y)) → (U, τR(X)) is NK – continuous.

(ii) f is NK –open

(iii) f is NK – closed

PROOF: (I) ⇒ (II)

Let **H** be a nano open set in (U, τR(X)). Since f^{-1}: (**V** , τR'(Y)) → (U, τR(X)) is NK – continuous, $(f^{-1})^{-1}$(**H**) = f(**H**) is NK –open in (**V**, τR'(Y)). That is f(**H**) is NK- open in **V** for every nano open set in **H** in U. Therefore f is NK – open.

(II) ⇒ (III)

Let **F** be a nano closed set in (U, τR(X)). Then $\mathbf{F}^c$ is nano open set in (U, τR(X)). Therefore f($\mathbf{F}^c$) is NK –open in (**V**, τR'(Y)). Therefore f($\mathbf{F}^c$) = $[f(\mathbf{F})]^c$ is NK –open in (**V**, τR'(Y)). That is f(**F**) is NK – closed in (**V**, τR'(Y)). Therefore f is NK – closed.

(iii) ⇒ (i)

Let **F** be a nano closed set in (U, τR(X)). Therefore f(**F**) is NK – closed in (**V**, τR'(Y)).

Now f(**F**) = $(f^{-1})^{-1}$(**F**) is NK – closed in (**V**, τR'(Y)). Therefore f^{-1}: (**V**, τR'(Y)) → (U, τR(X)) is NK – continuous.

Theorem 3.9: Let f: (U, τR(X)) → (**V**, τR'(Y)) and g: (**V**, τR'(Y)) → (**W**, τR''(Z)) be two mappings such that their composition g o f : (U, τR(X)) → (**W**, τR''(Z)) is a NK – closed mapping. Then the following statements are true.

(i) If f is nano continuous and surjective, then g is NK – closed.

(ii) If g is NK – irresolute and injective then f is NK – closed.

(iii) If g is strongly NK – continuous and injective then f is nano closed.

Proof: (i) Let **H** be a nano closed set in (**V**, τR'(Y)). Since f: (U, τR(X)) → (**V**, τR'(Y)) is nano continuous, f^{-1}(**H**) is nano closed in (U, τR(X)). Since g o f : (U, τR(X)) → (**W**, τR''(Z)) is NK –

closed in (**W**, τR''(Z)). (g o f)[f^{-1}(**H**)] is NK – closed in (**W**, τR''(Z)). g(**H**) is NK – closed in (**W**,

τR''(Z)). Since f: (U, τR(X)) → (**V**, τR'(Y)) is surjective then g: (**V**, τR'(Y)) → (**W**, τR''(Z)) is NK – closed.

(ii) Let **H** be a nano closed set in (U, τR(X)). Since the function g o f: (U, τR(X)) → (**W**, τR''(Z)) is NK – closed, then (g o f) (**H**) is NK – closed in (**W**, τR''(Z)). Since g is NK – irresolute, g^{-1}[(g o f)(**H**)] is NK – closed in (**V**, τR'(Y)). That is the set f(**H**) is NK – closed in (**V**, τR'(Y)). Since the function g: (**V**, τR'(Y)) → (**W**, τR''(Z)) is injective, then the image of a nano closed set in (U, τR(X)) is NK – closed in (**V**, τR''(Y)). Therefore f: (U, τR(X)) → (**V**, τR'(Y)) is NK – closed.

(iii) Let **H** be a nano closed set in (U, τR((X)). Since the function g o f:(U, τR(X)) → (**W**, τR''(Z)) is NK – closed in (**W**, τR''(Z)). g o f (**H**) is NK – closed in (**W**, τR''(Z)). Since the function

g:(**V**, τR'(Y)) → (**W**, τR''(Z)) is strongly NK – continuous, g^{-1}[g o f(**H**)] is nano closed in (V, τR'(Y)). Since the function g: (**V**, τR'(Y)) → (**W**, τR''(Z)) is injective, f(**H**) is nano closed in (**V**, τR'(Y)) for every nano closed set **H** in (U, τR(X)). Therefore f: (U, τR(X)) → (V, τR'(Y)) is nano closed.

CONCLUSION

In this paper we defined and studied Nano K – closed and Nano K –open maps in Nano topological spaces. Also we studied the basic properties and the relations between the Nano K- closed and Nano K – open maps with the other existing nano closed maps. There may be many interesting results can be analyze in future.

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