International Journal of Applied Engineering & Technology

DEGREE SPLITTING OF SQUARE HARMONIC MEAN GRAPHS

L. S. Bebisha Lenin[1] and M. Jaslin Melbha[2]
[1]Research Scholar and [2]Assistant Professor, Department of Mathematics, Women's Christian College, Nagercoil-629001, Tamil Nadu, India
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012.
[1]bebishalenin8497@gmail.com and [2]mjaslinmelbha@gmail.com

ABSTRACT

For a graph (V, E), the Degree Splitting Graph $DS(G)$ is obtained from G by adding a new vertex w_α for each partition of V_α that contains atleast two vertices and joining w_α to each vertex of V_α.

If there is an injective function $h : V(G) \to \{1,2, \dots, q+1\}$ such that an induced edge function $h^* : E(G) \to \{1,2, \dots, q\}$ defined by $h^*(e = uv) = \left\lceil \frac{2h(u)^2 h(v)^2}{h(u)^2 + h(v)^2} \right\rceil$ or $\left\lfloor \frac{2h(u)^2 h(v)^2}{h(u)^2 + h(v)^2} \right\rfloor$ is bijective, then a graph $G = (V, E)$ with p vertices and q edges is called a square harmonic labeling. A graph which admits a square harmonic mean labeling is called a square harmonic mean graph. In this paper we prove the Degree Splitting of Square Harmonic Mean graphs.

Keywords: Square harmonic mean graph, degree splitting (DS).

AMS Subject Classification: 05C09, 05C38

1. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected graph. A graph labeling is an assignment of integers to the vertices or edges or both based on certain conditions. The concept of square harmonic mean labeling was introduced by L. S. Bebisha Lenin, M. Jaslin Melbha [1]. We adhere to Harary's [3] conventions for all other terms and notations. Sandhya S.S, E. Ebin Raja Merly and Deepa [4] introduced Degree Splitting of Heronian Mean Graphs. The aforementioned studies served as our inspiration as we introduced a Square Harmonic Mean Labeling for Degree Splitting of special graphs.

Definition 1.1. The *Fork graph* is a tree with 5 vertices and 4 edges. It is also called *Chair graph*.

Definition 1.2. The *Bull graph* is a planar undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendant edges attached to disjoint vertices.

Definition 1.3. The *Fish graph* is a planar undirected graph with 6 vertices and 7 edges in the form of cycles C_3 and C_4 adjoined by a common vertex.

Definition 1.4. The *Butterfly graph* is a planar undirected graph with 5 vertices and 6 edges and can be constructed by joining 2 copies of the cycle graph C_3 with a common vertex.

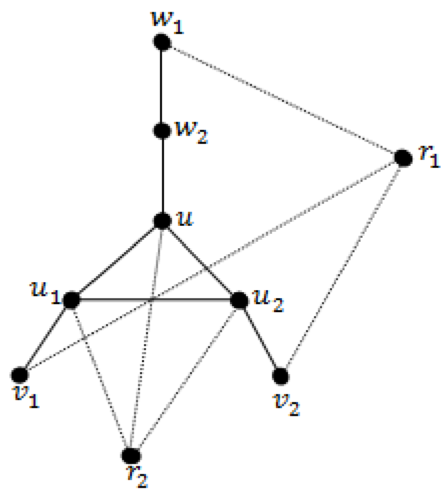
Definition 1.5. The *Cricket graph* is a planar undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendant vertices attached to same vertex.

Definition 1.6. The *Eiffel Tower graph* is a planar undirected tree with 7 vertices and 7 edges.

2. Main Results

Theorem 2.1. Degree splitting of Eiffel Tower graph admits a square harmonic mean graph.

Proof. The degree splitting of Eiffel tower graph is shown below.



Let G be an Eiffel tower graph. Let u_1, u_2, u_3 be the vertices of cycle C_3 and v_1, v_2 be the pendant vertices. Let w_1, w_2, w_3 be the path P_n joined in cycle C_3. Let u be the common vertex of C_3 and P_n. Identify u_3 and w_3 as u. Let r_1, r_2 be the newly added vertices that contains atleast two vertices to form the same degree. Then $V(G) = \{u_\alpha, v_\alpha, w_\alpha, r_\alpha, u: \alpha = 1,2\}$ and $E(G) = \{u_1u_2, u_\alpha u, w_\alpha u, u_\alpha v_\alpha, w_1r_1, v_\alpha r_1, u_\alpha r_2, ur_2 : \alpha = 1,2\}$. A function $h : V(G) \to \{1,2, \dots, q+1\}$ is defined by $h(u_\alpha) = \alpha,\ \alpha = 1,2,\ h(u) = 5,\ h(v_\alpha) = \alpha + 2,\ \alpha = 1,2,$ $h(w_\alpha) = \alpha + 6, \alpha = 1,2,\ \ h(r_1) = 12,\ \ h(r_2) = 15$. The corresponding induced edge labels are $h^*(u_1u_2) = 1,\ \ h^*(u_\alpha u) = \alpha + 3,\ \alpha = 1,2,\ \ h^*(w_2u) = 7,\ \ h^*(u_\alpha v_\alpha) = \alpha + 1,\ \alpha = 1,2,$ $h^*(w_1w_2) = 6$, $h^*(w_1r_1) = 9$, $h^*(v_\alpha r_1) = \alpha + 9, \alpha = 1,2$, $h^*(u_\alpha r_2) = \alpha + 12,\ \alpha = 1,2$, $h^*(ur_2) = 12$. Thus h^* is bijective. Therefore, degree splitting of Eiffel tower graph admits a square harmonic mean graph.

Illustration 2.2. The image below displays a square harmonic mean labeling for degree splitting of Eiffel tower graph.

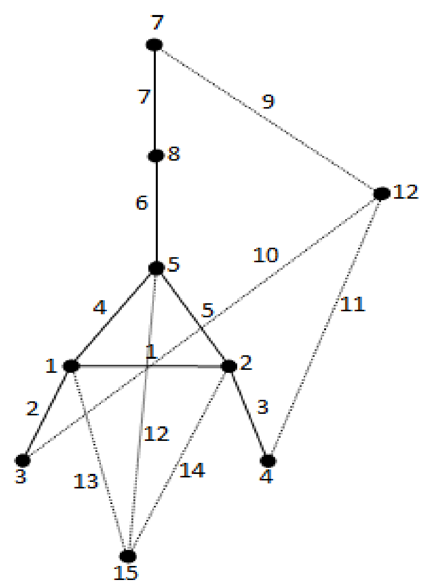
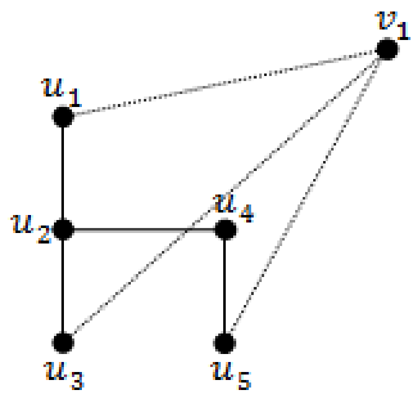


Figure I. Degree splitting of Eiffel Tower graph

Theorem 2.3. Degree splitting of fork graph admits a square harmonic mean graph.

Proof. The degree splitting of fork graph is shown below.



Let $u_1, u_2, \dots, u_5$ be the vertices and v_1 be the newly added vertex that contains atleast two vertices to form the same degree. Let $V(G) = \{u_\alpha : \alpha = 1, 2, \dots, 5\} \cup \{v_1\}$ and $E(G) = \{u_\alpha u_{\alpha+1} : \alpha = 1, 2\} \cup \{u_2u_4, u_4u_5\} \cup \{u_\alpha v_1 : \alpha = 1, 3, 5\}$. A function $h : V(G) \to \{1,2, \dots, q+1\}$ is defined by $h(u_\alpha) = \alpha, 1 \le \alpha \le 5$, $h(v_1) = 8$. The corresponding induced edge labels are $h^*(u_\alpha u_{\alpha+1}) = \alpha, \alpha = 1,2$, $h^*(u_2u_4) = 3$, $h^*(u_4u_5) = 4$, $h^*(u_\alpha v_1) = \begin{cases} 5 & if \alpha = 1 \\ 6 & if\ \alpha = 3 \\ 7 & if\ \alpha = 5 \end{cases}$.

Thus h^* is bijective. Therefore, degree splitting of fork graph admits a square harmonic mean graph.

Illustration 2.4. The image below displays a square harmonic mean labeling for degree splitting of fork graph.

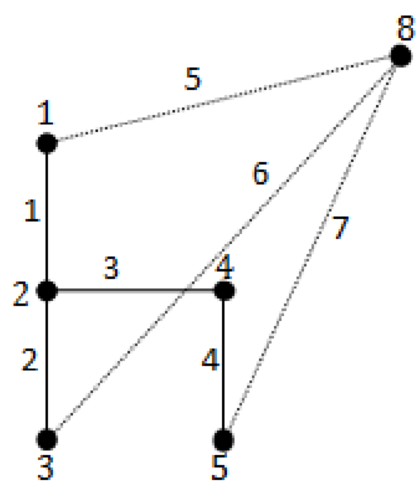
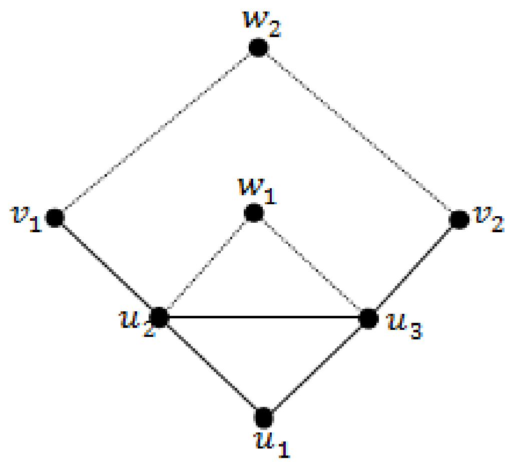


Figure II. Degree splitting of Fork graph

Theorem 2.5. Degree splitting of Bull graph admits a square harmonic mean graph.

Proof. The degree splitting of bull graph is shown below.



Let u_1, u_2, u_3 be the vertices of C_3. Let $v_\alpha: \alpha = 1,2$ be the pendant vertices attaching to any two vertices of C_3 and w_1, w_2 be the newly added vertices that contains atleast two vertices to form the same degree. Let $V(G) = \{u_\alpha: \alpha = 1, 2, 3\} \cup \{v_\alpha, w_\alpha: \alpha = 1,2\}$ and $E(G) = \{u_\alpha u_{\alpha+1}, u_3 u_1, v_\alpha u_\alpha: \alpha = 1,2\} \cup \{u_\alpha w_1, v_\alpha w_2: \alpha = 1,2\}$. A function $h : V(G) \to \{1,2, \dots, q + 1\}$ is defined by $h(u_\alpha) = \alpha, \alpha = 1,2,3$, $h(v_\alpha) = 2\alpha + 2, \alpha = 1,2$, $h(w_\alpha) = 2\alpha + 6, \alpha = 1,2$. The corresponding induced edge labels are $h^*(u_\alpha u_{\alpha+1}) = \alpha,$ $\alpha = 1,2$, $h^*(u_3 u_1) = 3$, $h^*(v_\alpha u_{\alpha+1}) = \alpha + 3, \alpha = 1,2$, $h^*(u_{\alpha+1} w_1) = \alpha + 5, \alpha = 1,2$, $h^*(v_\alpha w_2) = \begin{cases} 8 & if\ \alpha = 1 \\ 9 & if\ \alpha = 2 \end{cases}$. Thus h^* is bijective. Therefore, degree splitting of bull graph admits a square harmonic mean graph.

Illustration 2.6. The image below displays a square harmonic mean labeling for degree splitting of bull graph.

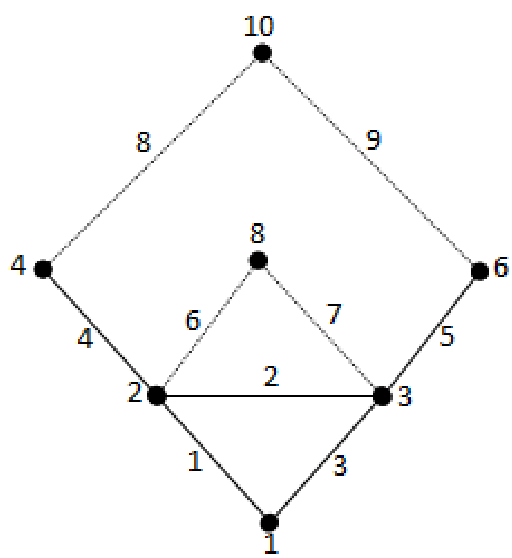
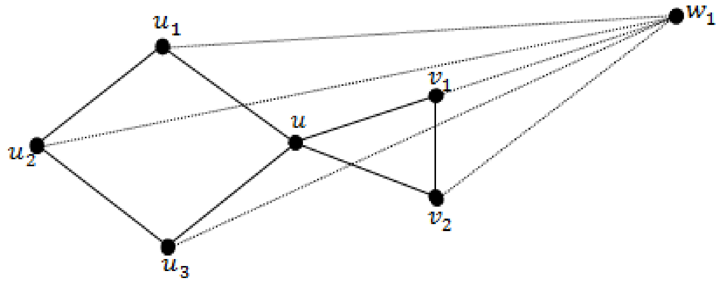


Figure III. Degree splitting of Bull graph

Theorem 2.7. Degree splitting of fish graph admits a square harmonic mean graph.

Proof. The degree splitting of fish graph is shown below.



Let G be a fish graph. Let u_1, u_2, u_3, u_4 be the vertices of cycle C_4 and v_1, v_2, v_3 be the vertices of cycle C_3. Let u be the common vertex of C_4 and C_3. Identify u_4 and v_3 as u. Let w_1 be the newly added vertex that contains atleast two vertices to form the same degree. Then $V(G) = \{u_\alpha : \alpha = 1, 2, 3\} \cup \{v_\alpha, w_1 : \alpha = 1,2\}$ and $E(G) = \{u_\alpha w_1 : \alpha = 1, 2, 3\} \cup \{u_\alpha u_{\alpha+1}, u_3 u, u_1 u, u v_\alpha, v_\alpha w_1, v_1 v_2 : \alpha = 1,2\}$. A function $h : V(G) \to \{1,2, \dots, q+1\}$ is defined by $h(u_\alpha) = \alpha, \alpha = 1, 2, 3$, $h(u) = 5$, $h(v_\alpha) = 2\alpha + 4, \alpha = 1,2$, $h(w_1) = 13$. The corresponding induced edge labels are $h^*(u_\alpha u_{\alpha+1}) = \alpha, \alpha = 1,2$, $h^*(u_3 u) = 3$, $h^*(u_1 u) = 4$, $h^*(u v_\alpha) = \alpha + 5, \alpha = 1, 2$, $h^*(v_1 v_2) = 5$, $h^*(u_\alpha w_1) = \alpha + 7, \alpha = 1, 2, 3$, $h^*(v_\alpha w_1) = \alpha + 8, \alpha = 1, 2$. Thus h^* is bijective. Therefore, degree splitting of fish graph admits a square harmonic mean graph.

Illustration 2.8. The image below displays a square harmonic mean labeling for degree splitting of fish graph.

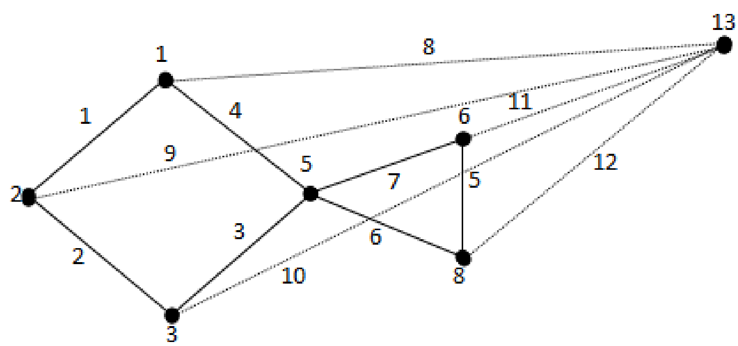
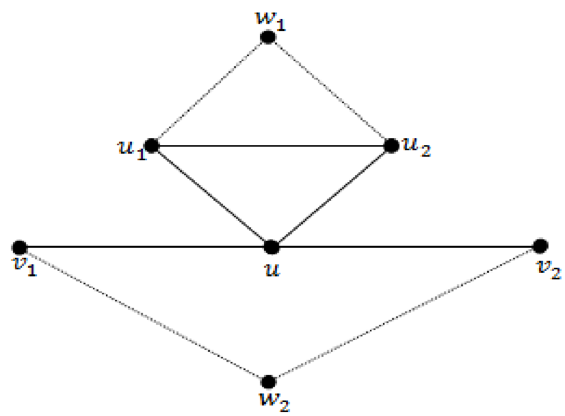


Figure IV. Degree splitting of Fish graph

Theorem 2.9. Degree splitting of Cricket graph admits a square harmonic mean graph.

Proof. The degree splitting of cricket graph is shown below.



Let G be a cricket graph. Let u_1, u_2, u_3 be the vertices of triangle and v_1, v_2 be the two disjoint pendant vertices attached to the same vertex u_3. Identify u_3 as u. Let w_1, w_2 be the newly added vertices that contains atleast two vertices to form the same degree. Then $V(G) = \{u_\alpha, v_\alpha, w_\alpha, u: \alpha = 1,2\}$ and $E(G) = \{u_1u_2, u_\alpha u, v_\alpha u, v_\alpha w_2, u_\alpha w_1 : \alpha = 1,2\}$. A function $h : V(G) \rightarrow \{1,2, \ldots, q+1\}$ is defined by $h(u_\alpha) = \alpha, \alpha = 1,2,$ $h(u) = 3,$ $h(v_\alpha) = \alpha + 4, \alpha = 1,2,$ $h(w_\alpha) = 2\alpha + 6, \alpha = 1,2$. The corresponding induced edge labels are $h^*(u_1u_2) = 1,$ $h^*(u_\alpha u) = \alpha + 1,$ $\alpha = 1,2,$ $h^*(v_\alpha u) = \alpha + 3, \alpha = 1, 2,$ $h^*(u_\alpha w_1) = \alpha + 5, \alpha = 1,2,$ $h^*(v_\alpha w_2) = \alpha + 7, \alpha = 1,2$. Thus h^* is bijective. Therefore, degree splitting of cricket graph admits a square harmonic mean graph.

Illustration 2.10. The image below displays a square harmonic mean labeling for degree splitting of cricket graph.

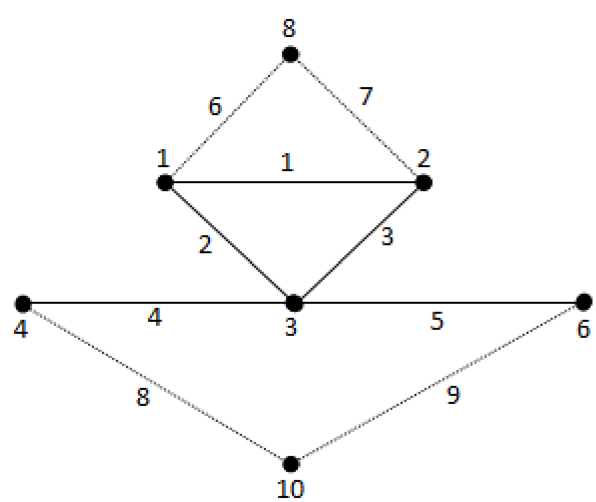
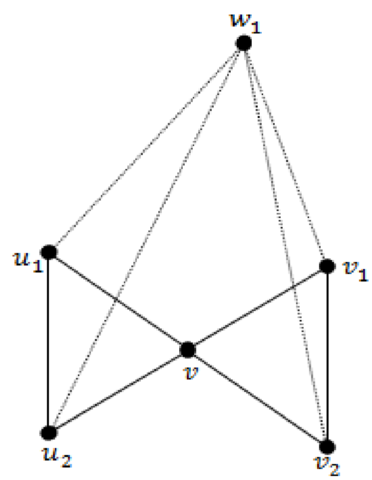


Figure V. Degree splitting of Cricket graph

Theorem 2.11. Degree splitting of Butterfly graph admits a square harmonic mean graph.

Proof. The degree splitting of butterfly graph is shown below.



Let G be a butterfly graph. Let u_1, u_2 and v_1, v_2 be the vertices of 2 copies of cycle C_3 and v be the common vertex of G. Let w_1 be the newly added vertex that contains atleast two vertices to form the same degree. Then $V(G) = \{u_\alpha, v_\alpha, w_1, v: \alpha = 1,2\}$ and $E(G) = \{u_1u_2, u_\alpha v, v_\alpha v, u_\alpha w_1, v_\alpha w_1 : \alpha = 1,2\}$. A function $h : V(G) \to \{1,2, \dots, q+1\}$ is defined by $h(u_\alpha) = \alpha,\ \alpha = 1,2,$ $h(v) = 4$, $h(v_\alpha) = \alpha + 5, \alpha = 1,2,$ $h(w_1) = 11$. The corresponding induced edge labels are $h^*(u_1u_2) = 1$, $h^*(u_\alpha v) = \alpha + 1,\ \alpha = 1,2,$ $h^*(v_\alpha v) = \alpha + 3,\ \alpha = 1,2,$ $h^*(v_1v_2) = 6$, $h^*(u_\alpha w_1) = 3\alpha + 4, \alpha = 1, 2,$ $h^*(v_\alpha w_1) = \alpha + 7, \alpha = 1,2$. Thus h^* is bijective. Therefore, degree splitting of butterfly graph admits a square harmonic mean graph.

Illustration 2.12. The image below displays a square harmonic mean labeling for degree splitting of butterfly graph.

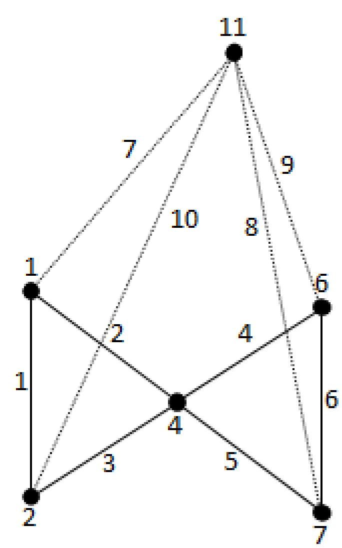


Figure VI. Degree splitting of Butterfly graph

CONCLUSION

Here we have investigated the behaviour of degree splitting of some graphs like Fork graph, Bull graph, Fish graph, Butterfly graph, Cricket graph, Eiffel Tower graph are square harmonic mean labeling.

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